INFLATION-OUTPUT TRADEOFF AS EQUILIBRIUM OUTCOME OF GLOBALIZATION

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The paper provides an integrated analysis of globalization effects on the inflation-output tradeoff and monetary policy in the New-Keynesian framework. The prediction of the analysis is threefold. First, labor, goods, and capital mobility flatten the Phillips curve, the tradeoff between inflation and activity. Second, the same globalization forces lead the welfare-based monetary policy to be more aggressive with regard to inflation fluctuations, and at the same time, more benign with respect to the output-gap fluctuations. Third, the equilibrium response of inflation to supply and demand shocks is more moderate, and the response of the output gap to these shocks is more pronounced – when the economy opens up – under such welfare-based monetary policy.

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1. INTRODUCTION

It has been observed, by economists and policy makers, that the short-run tradeoff between inflation and activity has recently become flatten.

Bean (2006) writes succinctly about this trend, as follows.

"One of the most notable developments of the past decade or so has been the apparent flattening of the short-run trade-off between inflation and activity. The seventies were characterized by an almost vertical relationship in the United Kingdom, in which attempt to hold unemployment below its natural rate resulted in rising inflation. In the eighties, the downward sloping relationship reappears, as inflation was squeezed out of the system by the slack of the economy. However, since the early Nineties, the relationship looks to have been rather flat. Three factors - increased specialization; the intensification of product market competition; and the impact of that intensified competition and migration on the

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behavior of wages—should all work to flatten the short-run tradeoff between inflation and domestic activity.”  

Recent evidence on the decline in the sensitivity of U.S. inflation to unemployment, and other measures of resource utilization, includes also Roberts (2006) and Williams (2006). Work by staff at the Federal Reserve Board indicates that this result generally holds across a variety of regression specifications, estimation methods, and data definitions. (See Ihrig et al (forthcoming)).

A massive globalization process has swept emerging markets in Latin America, the European transition economies, and the East Asia emerging economies in the past two decades. The 1992 single-market reform in Europe, and the formation of the Euro zone, is remarkable episodes of globalization. Similarly, emerging markets, including China and India, became significantly more open.

Wynne and Kersting (2007) note that in the 1970s, more than three quarters of industrial countries had restrictions of some sort on international financial transactions. By the 2000s, none did. Likewise, restrictions on these transactions among emerging markets fell from 78 percent in the 1970s to 58 percent in the 2000s.

An important aspect of openness relates to labor flows. International migrants constitute 2.9 percent of the world population in the 2000s, up from 2.1 percent in the 1975. In some countries changes have been more dramatic. In Israel in the 1990s there was a surge of immigrants of up to 17 percent of the population, and the central Bank accomplished also a sizable decline of inflation. It is possible that the two episodes are related.² In Spain in 1995, the percentages of foreigners in the population and in the labor force were, respectively, below 1% and below 0.5%. At the end of 2006, these rates were around 9% and 14%, respectively. The impact of the Spanish immigration boom on the Phillips curve has been recently addressed by Bentolila, Dolado and Jimeno (2007).

Recently, inflation around the world reduced substantially. The average annual inflation rate, among developing countries, was 41 percent in the early 1980s, and came down to 13 percent towards the end of the 1990s. Global inflation in the 1990s has dropped from 30 percent a year to about 4 percent a year.

Indeed, Rogoff (2003, 2004) is one of the first to observe that favorable factors have been helping to drive down global inflation in the last two decades. A hypothesis, which he put forth, is that "globalization — interacting with deregulation and privatization — has played a strong supporting role in the past decade’s disinflation."³

³ Similarly, Mishkin (2007) writes about the US inflation-output tradeoff: "The finding that inflation is less responsive to the unemployment gap, suggests that fluctuations in resource utilization will have smaller implications for inflation than used to be the case. From the point of view of policymakers, this development is a two-edged sword: On the plus side, it implies that an overheating economy will tend to generate a smaller increase in inflation. On the negative side, however, a flatter Phillips curve also implies that a given increase in inflation will be more costly to wring out of the system".

² For some related literature see Arststein (2002) and Friedman and Suchoy (2004).

³ Borio and Filardo (2007) present cross country evidence, in support of their contention, that global factors have recently become empirically more relevant, for domestic inflation determination. But Ihrig et al (forthcoming) have shown that their result is very specific to the econometric method used. Based on cross country analysis, Badinger (2007) find that globalization is also correlated with more aggressive policy

The purpose of this paper is to provide a unified analysis, of the effects of various features of globalization on the inflation-output tradeoff, in a New-Keynesian framework. Globalization features are international capital mobility, international trade in goods, and international migration. We demonstrate a common effect of these different channels of openness on the trade-off. That is, each one of these channels helps to flatten the Phillips curve.

The reason why New-Keynesian framework is capable of generating a trade off, between inflation and economic activity, is that producer desired prices rise with the economy's output, when marginal costs slope upward due to diminishing returns to scale. Furthermore, because labor supply increases, workers experience increasing marginal disutility of labor. As a result, real-wage demands could rise. Increased wage demands put an upward pressure on the marginal cost, and consequently on the producer desired prices. Thus, our analytical challenge is to find how trade in goods, financial openness, and migration affect economic output utilization and wage demands.

To accomplish our task we extend the New-Keynesian model in the following directions: (1) International labor mobility; both inward- and outward-migration. The presumption is that labor flows tend to mitigate wage demands, because they introduce a substitution between domestic and foreign labor; (2) International trade in goods. The presumption is that trade leads to specialization in domestic production and diversification in domestic consumption. Therefore, trade tends to weaken the link between domestic production and domestic consumption. As a result, the effect of the fluctuations of domestic production on inflation is also weakened by the presence of international trade in goods; (3) financial integration with the rest of the world. International trade in financial assets allows households to smooth their consumption over time and over states of nature. Such consumption smoothing also mitigates the fluctuations in the representative household labor supply. Smoothed fluctuations weaken the link between domestic output fluctuations and those associated with inflation.

The organization of the paper is as follows. Section 2 describes the analytical framework and analyzes the effect of migration on the marginal cost. Section 3 derives the aggregate supply relationship for various aspect of openness. Section 4 derives a utility-based quadratic loss function, from the representative household utility function, for different regimes of openness. Section 5 derives the optimal monetary policy rule under discretion, for different regimes of trade in goods, trade in financial assets, and international mobility toward inflation. Tetlow and Ironside (2007), although not dealing with globalization, find that for the United States, the slope of the Phillips curve has – largely and continuously – lessened during recent years. Following Razin and Yuen (2002), Razin and Loungani (2007) demonstrate that globalization flattens the Phillips curve and changes the inflation-output weights in the reduced form welfare-based loss function of the central bank in favor of inflation control. These aspects of globalization are also addressed by Clarida (2008) in a model with imported raw materials in addition to imported finished goods.
of labor. Section 6 derives a closed-form solution for the equilibrium inflation and output gap. Section 7 concludes.

2. THE ANALYTICAL FRAMEWORK

The analytical framework draws on the recent New-Keynesian macroeconomics literature (See Woodford (2003)). Main features of the open-economy New-Keynesian model are:
(1) The domestic economy produces a continuum of differentiated goods. Decisions of the representative household are governed by Dixit-Stiglitz preferences (generating fixed elasticities). Purchasing power parity conditions prevail for the flexible price goods and foreign firms’ prices are taken as exogenous.
(2) The representative-household utility is defined over consumption and leisure, as in the standard micro-based welfare analysis.
(3) There is international trade in goods and financial assets.
(4) Labor supply is divided between domestic and foreign destinations. Exported labor receive wage premium over unskilled foreign labor. Imported labor is unskilled and native born labor commands an endogenously determined skill premium.
(5) Price updates are staggered (see Calvo (1983)). Namely, producers update prices upon receiving a signal drawn from a stochastic distribution.
(6) World prices are exogenous (that is, a small open economy assumption).

2.1 The Representative Household

We assume that all goods are tradable. There is a continuum of goods, which is uniformly distributed over the unit interval, so that \( j \in [0,1] \). The utility function of the representative household is:

\[
\text{Max } E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u(C_t, \xi_t) - \frac{1}{1+\phi} \left[ \left( h_t^{\text{hom}}(j) + \delta \cdot h_t^{\text{exp}}(j) \right) \right]^{1+\phi} \cdot dj + \Gamma \left( \frac{M_t}{P_t}, \xi_t \right) \right\},
\]

where \( E \) is the expectations operator The instantaneous utility function consists of a consumption composite, \( C_t \), domestic labor supply, \( h_t^{\text{hom}}(j) \), exported labor supply, \( h_t^{\text{exp}}(j) \), and of real money balances, \( M_t \), \( P_t \) (the ratio of money holdings, \( M_t \), and the price level, \( P_t \)). We denote the discount factor by \( \beta \) and the labor disutility parameter by \( \phi \).

The relative disutility of labor export in terms of domestic labor supply is indicated by the parameter \( \delta > 1 \). The term \( \xi_t \) is a vector of preference shocks. The consumption composite, \( C_t \) is a Dixit-Stiglitz composite of goods produced at home and imported goods:

\[5\] This approach to migration was originally suggested by Engler (2007).
\[
(2) \quad C_i = \left[ \int_0^n \left( c_{i,j}(j) \right) \frac{\partial q_i}{\partial q_j} \cdot dj + \int_0^n \left( c_{W,j}(j) \right) \frac{\partial q_i}{\partial q_j} \cdot dj \right]^{\frac{\theta}{\theta - 1}},
\]

where \( n \) is the number of domestically produced goods, in the consumption basket; and thus, \( 1 - n \) can serve as a trade openness parameter. Subscripts \( H \) and \( W \) indicate home and foreign country variables, respectively. The variable \( c_{i,j}(j) \) is the consumption level of good \( j \), which is produced in country \( i = H, W \). The parameter \( \theta > 1 \) is the elasticity of substitution among different goods in the consumption composite.

The budget constraint is:

\[
P_t C_i = P_t T_i + M_t + B_{H,t} + \epsilon_t B_{W,t} -(1+i_{H,t})B_{H,t-1} -(1+i_{W,t-1})\epsilon_t B_{W,t-1} =
\]

\[
M_{t+1} + \mu_t^H W_t \int_0^1 h_t^{home}(j) \cdot dj + \epsilon_t \mu_t^W W_t \int_0^1 h_t^{export}(j) \cdot dj + \int_0^1 D_t(j) \cdot dj,
\]

where:

- \( B_{H,t} \) = Bond holdings at the beginning of date \( t \) (denominated in the domestic currency).
- \( B_{W,t} \) = Bond holdings at the beginning of date \( t \) (denominated in the foreign currency).
- \( M_t \) = Money holdings in the end of period \( t \).
- \( P_t \) = The Consumer price level.
- \( W_t^H \) = Wage rate of unskilled labor in the domestic market, in domestic currency.
- \( W_t^W \) = Wage rate of unskilled labor in the foreign market, in foreign currency.
- \( \mu_t^H \) = Skill premium, of native-born labor, in the domestic market.
- \( \mu_t^W \) = Skill premium, of domestic native-born labor, in the foreign market.
- \( i_{H,t} \) = The interest rate in the domestic economy.
- \( i_{W,t} \) = The world interest rate.
- \( D_t(j) \) = Profit of the domestic \( j \) firm.
- \( \epsilon_t \) = Exchange rate in period \( t \).
- \( T_t \) = Government lump-sum transfers.

By arbitrage, a free migration of unskilled labor implies that \( W_t^H = \epsilon_t W_t^W \).
2.2 Producers

Domestic firms produce with the aid of a decreasing return-to-scale production function, by using native born labor and immigrants’ labor:

$$y_t(j) = A_t \cdot \left( (1 - \psi) \frac{1}{V} \cdot h_t^{\text{home}}(j) \frac{1}{V} + (\psi) \frac{1}{V} \cdot h_t^{\text{import}}(j) \frac{1}{V} \right)^{1/V},$$

where $y_t(j)$ is the output level of the $j$'th firm and $A_t$ is an exogenous aggregate technology shock, common to all firms. The elasticity of substitution, between imported and native-born labor inputs, is given by $\psi/(\psi - 1)$, where $\psi > 1$, and the degree of returns-to-scale is given by $\chi < 1$. The variable $h_t^{\text{import}}(j)$ is the labor supply by immigrants, employed by domestic firm $j$. We assume that native born are skilled, and immigrant labor is unskilled. (This captures labor market patterns in an industrialized economy). Hence, $\psi \in [0, 1/2]$. It follows that the marginal productivity of domestic labor exceeds that of immigrant labor (for the same amount of labor input). Skill premium in the foreign market $\mu^W$ is exogenous. Skill premium in the domestic market $\mu^H$ is endogenously determined.

2.3 Skill Premium in the Domestic Market

The first order conditions for the domestic household who allocates time between leisure, work in the domestic market, and work in the foreign market are:

$$u_c(C_t, z_t) \cdot \frac{\mu^H W_t^H}{P^H_t} = \left[ h_t^{\text{home}}(j) + \delta h_t^{\text{import}}(j) \right] \varphi,$$

$$u_c(C_t, z_t) \cdot \frac{\mu^W W_t^W}{P^W_t} = \delta \cdot \left[ h_t^{\text{home}}(j) + \delta h_t^{\text{import}}(j) \right] \varphi.$$

Dividing equation (6) by equation (5) yields:

$$\mu^W = \frac{\mu^H}{\delta}.$$

The equilibrium skill premium is determined through outward-migration flows (note the $\delta$ parameter in equation (7)).
2.4 Marginal Cost

Real marginal cost function, in the presence of migration, is given by:

\[ mc_i(j) = z_i \cdot y_i(j)^{\frac{1-\gamma}{\delta}} , \]

where the term \( y_i(j)^{\frac{1-\gamma}{\delta}} \) reflects the diminishing marginal productivity of labor; the exogenous term \( z_i \), is equal to:

\[ z_i = \frac{1}{\hat{\lambda}} \cdot \frac{1}{A_i^{\hat{\lambda}}} \cdot \frac{1}{\delta} \cdot \mu_i^{\frac{w_i}{w_i^W}} \cdot \frac{1}{(1-\psi)^{\frac{\gamma-1}{\delta}}} \cdot \left( \frac{\psi}{1-\psi} \left( \frac{\mu_i^{\frac{w_i}{w_i^W}}} {\frac{1}{\delta}} \right)^{\frac{1}{\gamma-1}} \right) , \]

where the real wage, \( w_i^W \) in the foreign market, is defined by:

\[ w_i^W \equiv \frac{e_i \cdot W_i^W}{P_i} ; \]

Thus, the exogenous term \( z_i \) consists of technology and preferences parameters, the technology shock, the foreign market skill premium, and the labor wage in the foreign market.

If the labor market is open to out-migration but closed to in-migration, the marginal cost function still takes the form of equation (8); in this case however, the exogenous term \( z_i \) will be replaced by

\[ z_i^{out} = \frac{1}{\hat{\lambda}} \cdot \frac{1}{A_i^{\hat{\lambda}}} \cdot \frac{1}{\delta} \cdot \mu_i^{\frac{w_i}{w_i^W}} \cdot \frac{1}{(1-\psi)^{\frac{\gamma-1}{\delta}}} \cdot \left( \frac{\psi}{1-\psi} \left( \frac{\mu_i^{\frac{w_i}{w_i^W}}} {\frac{1}{\delta}} \right)^{\frac{1}{\gamma-1}} \right) . \]

It can be verified that \( z_i^{out} > z_i \). That is, in-migration exerts a lowering cost effect akin to technological progress.

To see the effect of in- and out-migration on the marginal cost, compare equation (8) with the corresponding expression for the marginal cost function with no migration:

\[ mc_i^{closed}(j) = z_i^{closed} \cdot \frac{1}{u_i(Y_i(j))} \cdot y_i(j)^{\frac{1-\gamma}{\delta}} , \]

where,
\[ z_{t}^{\text{closed}} = \frac{1}{\chi} \left( \frac{1}{\frac{1}{\varphi} - \frac{1}{1-\psi \chi}} \right), \]

\[ u_{c} \] is the derivative of \( u \) with respect to \( C \), and

\[ Y_{t} \equiv \left[ \int_{0}^{1} \left( y_{t}(j) \right)^{\frac{\theta-a}{\theta}} \cdot dj \right]^{\frac{\theta}{\theta-a}}. \]

The output elasticity is equal to \( \left( 1 - \chi \right) / \chi \) in equation (8), while the corresponding elasticity is equal to \( \left( 1 + \varphi - \chi \right) / \chi \) in equation (9). This means that in the presence of out-migration, which tends to make the labor supply faced by domestic producers more flexible, the output elasticity of the marginal cost decreases.

When the labor market is closed to outward-migration, wage demands faced by domestic producers are upward sloping, both under in-migration and under a completely closed labor market. However, when the labor market is open to in-migration, domestic producers face an expanded labor supply: additional to the skilled native born labor supply (with upward sloping wage demand), they also face a complementary unskilled foreign labor supply (with exogenously determined wage demand). That means that in-migration acts essentially like a productivity shock, \( A_{t} \).

To summarize, outward-migration reduces the output elasticity of the marginal cost, whereas inward-migration essentially works like a positive domestic productivity shock that lowers marginal costs.

3. THE AGGREGATE SUPPLY

3.1 Perfect International Mobility of Goods, Capital and Labor

When there is perfect mobility of goods, then domestic producers specialize, and \( n < 1 \). That is, the number of domestically produced goods, \( n \), falls short of the number of consumed goods, 1. Perfect mobility of capital implies perfect consumption smoothing; that is \( C_{t} = C_{t}^{N} \). Superscript \( N \) indicates the perfect price flexibility case.

The approximated aggregate supply curve is derived from the log linearization of the aggregate supply equations, around a purely deterministic steady state.

Upper hat denotes proportional deviation from the purely deterministic steady state, and the superscript \( N \) denotes the “natural” value of real variables, that is, the value of a variable that would have prevailed under completely flexible prices.

In the case of perfect mobility of labor, capital, and goods, the approximate aggregate supply curve is given by:
\[ \pi_t = \kappa \left[ \frac{\omega_p \cdot n}{1 + \omega_p \cdot \theta} \cdot x_t + \frac{\omega_p \cdot (1 - n)}{1 + \omega_p \cdot \theta} \cdot \left( \bar{Y}_t^F - \bar{Y}_t^N \right) + \frac{1}{1 + \omega_p \cdot \theta} \cdot \bar{w}_t^H + \frac{(1 - n)}{n} \cdot q_t \right] + \left( \frac{1 - n}{n} \right) \cdot (q_t - q_{t-1}) + \beta \cdot E_t \left[ \pi_{t+1} - \frac{(1 - n)}{n} \cdot (q_{t+1} - q_t) \right] \]

(10)

Hence, \( \hat{\pi}_t \) is the deviation of CPI inflation from its target; \( x_t \) is the domestic output gap; \( \left( \bar{Y}_t^F - \bar{Y}_t^N \right) \) is the difference between foreign output and domestic natural output; the parameter \( \omega_p \), defined in the next section, is the elasticity of the marginal cost with respect to producer’s output.

The term

\[ \kappa = \frac{(1 - \alpha) (1 - \alpha \beta)}{\alpha} \]

captures the price flexibility parameter; and \((1 - \alpha)\) is the probability of receiving a price updating signal. The variable \( \hat{q}_t \) is the real exchange rate, formally defined as:

\[ q_t = \varepsilon_t + P_{F,t} - P_t, \]

where \( P_{F,t} \) is the foreign price index.

The focus of attention of this paper is the slope of the aggregate supply curve. The slope is

\[ \psi_t = \frac{\kappa \cdot \omega_p}{1 + \omega_p \cdot \theta}. \]

The slope of the aggregate supply curve increases with \( n \) ( \( 1 - n \) is the trade openness parameter) and \( \kappa \).

Other terms in the aggregate supply curve capture the effects on the domestic inflation of foreign output shocks, foreign wage shocks, and past, present and future real exchange rate shocks.

3.2 Perfect International Mobility of Goods and Capital, with no Labor Mobility

In the case of perfect international mobility of goods and capital, but with no labor mobility, the aggregate supply curve is given by:

\[ \frac{\kappa \cdot \omega_p}{1 + \omega_p \cdot \theta}. \]

\[ \text{Razin and Yuen (2002) were among the first to extend the closed-economy New Keynesian framework to an open economy with trade in goods and in capital assets. Gali and Monacelli, (2003) analyze the effect of exchange rate movements on inflation.} \]
\[ \hat{\mathcal{R}}_t = \kappa \cdot \left[ \frac{\omega \cdot n}{1 + \omega \theta} \cdot x_t + \frac{\omega \cdot (1 - n)}{1 + \omega \theta} \cdot \left( \hat{Y}_t^F - \hat{Y}_t^N \right) + \frac{(1 - n)}{n} \cdot \hat{q}_t \right] \]

(11)

\[ + \frac{(1 - n)}{n} \cdot \left( \hat{q}_t - \hat{q}_{t-1} \right) + \beta \cdot E_t \left[ \hat{\mathcal{R}}_{t+1} - \frac{(1 - n)}{n} \left( \hat{q}_{t+1} - \hat{q}_t \right) \right], \]

where \( \omega = \omega_p + \omega_w \) is the elasticity of marginal cost with respect to domestic output; It includes the expression

\[ \omega_p = \frac{1 - \chi}{\chi}, \]

which is the elasticity of the desired price with respect to output (for given wages). It is inversely related to the degree of returns to scale. It also includes the expression

\[ \omega_w = \frac{\phi}{\chi}, \]

which is the elasticity of demanded wage with respect to output (consisting of the labor-disutility elasticity and the labor-output elasticity).

Because \( \omega_w > 0 \), we have \( \omega > \omega_p \).

Therefore, shutting off the migration channel (particularly outward migration) raises the slope of the aggregate supply curve.\(^7\)

In this case, the slope of the Phillips curve is:

\[ \psi_2 = \frac{\kappa n \omega}{1 + \omega \theta}. \]

### 3.3 Perfect International Mobility of Goods, with no Capital Mobility and no Labor Mobility

If the domestic economy is not integrated to the international financial market, then there is no possibility of consumption smoothing, and we have that the value of aggregate current spending equals the value of aggregate domestic output:

\[ P_{C,t} \hat{C}_t = \bar{P}_{Y,t} \hat{Y}_t \quad ; \quad P_{C,t} \hat{C}_t^N = \bar{P}_{Y,t} \hat{Y}_t^N, \]

where \( P_{C,t} \) is the CPI-based price level and \( \bar{P}_{Y,t} \) is the GDP deflator.

In this case, the aggregate-supply curve is:

\(^7\) See also Engler (2007).
$$\pi_i = \kappa \cdot \left[ \frac{(\omega n + \sigma)}{1 + \omega \theta} \cdot x_i \right] + \frac{\omega (1 - n)}{1 + \omega \theta} \cdot (\bar{y_i} - \bar{y_i}^n) + \frac{(1 - n)}{n} \cdot q_i$$

(12)

$$+ \frac{(1 - n)}{n} \cdot (q_i - q_{i-1}) + \beta \cdot E \left[ \pi_{i+1} - \frac{(1 - n)}{n} (q_{i+1} - q_i) \right].$$

For the case of perfect mobility of goods with no mobility of capital and labor, the slope of the Philips curve is equal to:

$$\psi_3 = \frac{\kappa (\omega + \sigma)}{1 + \omega \theta}. \quad \text{(13)}$$

The slope of the Phillips curve is steeper than in the previous case.

3.4 The Closed Economy

Because, with the trade account closed, the consumption of each good equal domestic production of the good, production is fully diversified. Namely, \( n = 1 \). If, in addition, the capital account is closed and in- and out-migration is not possible, the aggregate-supply curve becomes:

$$\pi_i = \frac{\kappa}{1 + \omega \theta} \cdot (\omega + \sigma) \cdot x_i + \beta \cdot E \pi_{i+1} \cdot \bar{y_i}.$$

In this case the slope of the Phillips curve is:

$$\psi_4 = \frac{\kappa (\omega + \sigma)}{1 + \omega \theta}, \quad \text{(14)}$$

where, \( \sigma \) is the inter-temporal consumption elasticity of substitution.

The slope of the Phillips curve is steeper than in the previous case.

3.5 Slope of the Aggregate Supply Curve: Regime Comparisons

There is a systematic ranking of the slope of the Philips curve across openness regimes. From subsections 3.1-3.4 one can verify that \( \psi_1 < \psi_2 < \psi_3 < \psi_4 \).

This means that in every successive round of the opening up of the economy, globalization contributes to flatten the aggregate supply curve. The intuition is that when an economy opens up to trade in goods, it tends to specialize in production but to diversify in consumption. This means the number of domestically produced goods (= \( n \)), is less then the number of domestically consumed goods (= 1). Consequently, the commodity composition of the consumption and output baskets, which are identical if the trade account is closed, are different when trade in goods is possible. As a result, the correlation between
fluctuations in output and in consumption (which is equal to unity in the case of a closed trade account) is less than unity if the economy is opened to international trade in goods.

When the capital account is open, then the correlation between fluctuations in consumption and domestic output is farther weakened, this is because with open capital accounts the representative household can smooth consumption through international borrowing and lending and thereby separate current consumption from current output. The inflation effects of shocks to the marginal cost are therefore reduced, because the fluctuations in labor supply are also smoothed, as a consequence of the consumption smoothing.

Out-migration reduces the output elasticity of the marginal cost (compare equation (8) and equation (9)). This implies that, in the presence of out-migration, shocks to domestic output will have smaller effects on inflation, compared to a closed economy.  

When the economy opens up to in-migration, the proportional factor, $z_r$, of the marginal cost curve is lowered. Therefore, the effect of demand shocks on inflation is weakened.

4. UTILITY-BASED LOSS FUNCTION

Distortions in the New-Keynesian equilibrium can be grouped into two types:

(1) Because consumption smoothing is desirable, fluctuations of the output gap, which are correlated with consumption, are welfare-reducing.

(2) Because an efficient allocation of the labor supply implies an equal division of labor across differentiated goods (recall that the disutility of labor is a convex function), any cross-good dispersion in output (the level of output for goods whose prices have been updated is different than the level of output of goods whose prices were not updated) is distortionary. Given that not all the prices are updated simultaneously, inflation generates a distortion.

The utility based-loss function, which captures distortions 1 and 2, is:

\[
L = E_0 \sum_{t=0}^{\infty} \beta^t [\pi_t^2 + \lambda (x_t - x_t^*)],
\]

where, $x_t^*$ is the (log) ratio of the non-distorted aggregate output and the monopolistic-competitive distorted output, under perfect price flexibility; and the parameter $\lambda$ is the weight of the output-gap term relative to the inflation term.

\footnote{If the economy imports intermediate goods there is also a real exchange rate effect. The real exchange rate affects the output inflation tradeoff, even in the absence of other cost push shocks. Clarida Gali and Gertler (2000) discuss this effect.}

\footnote{In appendix A we derive the utility based loss function along the lines of Woodford (2003). We assume that foreign producers use a local currency pricing strategy and they update prices at the same frequency as domestic producers. Note also that we abstract from the money term in the utility function.}
We find (see Appendix A) that:
\[ \lambda_i = \frac{\psi_i}{\Theta}; \quad i = 1, 2, 3, 4, \]
where \( \psi_i \) is the slope of the aggregate supply curve (the inverse of the sacrifice ratio); and \( \Theta \) is the elasticity of substitution across differentiated goods. Recall that in the previous subsection we demonstrated that:
\[ \psi_1 < \psi_2 < \psi_3 < \psi_4. \]
Thus, the ranking of the relative weight of the output-gap term in the loss function is:
\[ \lambda_4 < \lambda_3 < \lambda_2 < \lambda_1. \]
Opening up an economy to trade in goods and capital flows weakens the correlation between the fluctuations in the output gap and the fluctuations in consumption. Recall that the representative household welfare depends on consumption, not on domestic output. Therefore, the output-gap weight in the loss function falls as an economy opens up to trade, and capital assets.
With migration, the representative household’s income and output are separated, one from the other. Because consumption levels are associated with the income levels (not GDP levels), fluctuations of domestic output become less important to the representative household compared to the case of no migration. Thus, the output-gap weight in the loss function declines when migration is allowed.
We thus establish that the output-gap weight in the utility-based loss function decreases with the opening up of the economy, in every successive round of opening up.

5. UTILITY-BASED MONETARY POLICY UNDER DISCRETION

In this section we use the utility-based loss function, along with the aggregate supply and aggregate demand relationships, to formulate an optimal monetary policy rule, under discretion. The approximated aggregate demand equation is:
\[ x_t = E_t x_{t+1} - \sigma^{-1}\left(\tilde{r}_{H,t} - E_t \tilde{r}_{t+1} - \tilde{r}^*\right), \]
where \( \tilde{r}^* \) is the deviation of the natural rate of real interest, from steady state.
The approximated (real) interest-parity equation is:
\[ \tilde{q}_t = E_t \tilde{q}_{t+1} + \left(\tilde{r}_{H,t} - E_t \tilde{r}_{F,t+1}\right)\left(\tilde{r}_{H,t} - E_t \tilde{r}_{t+1}\right). \]
The optimal monetary policy rule is obtained by choosing the path of \( \tilde{\pi}, x_t \) and \( \tilde{q}_t \) so as to minimize the loss function in equation (14), subject to the aggregate supply
equation, aggregate demand equation and the (real) interest parity rule, in every period \( t = 1,2, \ldots \).

The optimal policy rule (under discretion) depends on the degree of openness\(^{10}\) (a step by step derivation is included in Appendix B):

\[
\hat{i}_{n+1} = \hat{i}_n + \gamma_x \cdot E_t \hat{\pi}_{t+1} + \gamma_x \cdot \pi_t + \gamma_x \cdot \hat{u}_t,
\]

where \( \hat{u}_t \) collects terms from the right hand side of the aggregate supply curve (apart from the inflation expectations and the output gap) and where the elasticity of the policy determined interest rate, with respect to the inflation expectations, depends on the degree of openness, as follows:

I. Perfect mobility of Labor, Capital and Goods:

\[
\gamma_x = \frac{\sigma \left[ 1 + \theta \left( \psi_1 + \Pi_x \sigma \right) / \beta \right]}{\left[ 1 + \psi_1 \sigma + \Pi_x \sigma \right] \sigma \theta \beta}.
\]

II. No Labor Mobility; Perfect Mobility of Capital and Goods:

\[
\gamma_x = \frac{\sigma \left[ 1 + \theta \left( \psi_2 + \Pi_x \sigma \right) / \beta \right]}{\left[ 1 + \psi_2 \sigma + \Pi_x \sigma \right] \sigma \theta \beta}.
\]

III. No Labor Mobility; No Capital Mobility; Perfect Goods Mobility:

\[
\gamma_x = \frac{\sigma \left[ 1 + \theta \left( \psi_3 + \Pi_x \sigma \right) / \beta \right]}{\left[ 1 + \psi_3 \sigma + \Pi_x \sigma \right] \sigma \theta \beta}.
\]

IV. Closed economy:

\[
\gamma_x = \frac{\sigma \left[ 1 + \theta \psi_5 \right]}{\beta}.
\]

\(^{10}\) Cecchetti et al (2007) suggest that aggressive monetary policy is the key explanation for the flattening of the trade-off. They argue that the disinflationary impact of globalization is limited, and partly attributable to the fixed exchange rate regime, in some of the East Asian countries. They analyze empirically the change in ex-post Taylor rule, from the high inflation era, to the low inflation era. Our theory can provide an explanation for this change in the Taylor rule.
Where \( \Pi_\pi \equiv \frac{(1-n)}{n} \left(1+\kappa-\beta\right) \) is the aggregate-supply elasticity of inflation, with respect to the real exchange rate. Note that in the closed-economy case \( \Pi_\pi = 0 \).

The expressions for \( \gamma_\pi \) demonstrates that the optimal monetary policy under discretion becomes more aggressive with respect to inflation, when the economy opens up to migration, trade in goods and capital flow. In contrast, the expression for \( \gamma_x \) demonstrates that the monetary policy becomes more benign toward fluctuations in the output-gap when the economy opens up, in every globalization round. \(^{11}\)

6. THE DYNAMIC EQUILIBRIUM

In this section we derive the closed-form solution to the equilibrium levels of inflation and output gap. We use the following procedure in the derivation of the closed form solution. First, we write the system in a matrix notation. Second, we use the method of undetermined coefficients to solve for the state-space equilibrium form.

6.1 Equilibrium Equations in Matrix Notation

Substituting the optimal policy rule (17) into the aggregate demand (15), and then substituting the result in a generic aggregate-supply curve, we can rearrange the system using the following matrix notation:

\[
\begin{bmatrix}
    x_t \\
    \hat{p}_t
\end{bmatrix} = Q \cdot E_t \begin{bmatrix}
    x_{t+1} \\
    \hat{p}_{t+1}
\end{bmatrix} + R \cdot \tilde{u}_t,
\]

where the matrices of parameters are defined as follows:

\[
Q \equiv \begin{bmatrix}
    \frac{1}{1+\gamma_x/\sigma} & \frac{1-\gamma_x}{\sigma+\gamma_x} \\
    \frac{\psi}{1+\gamma_x/\sigma} & \beta - \frac{\psi(\gamma_x-1)}{\left(\sigma+\gamma_x\right)}
\end{bmatrix} \quad ; \quad R \equiv \begin{bmatrix}
    -\frac{\gamma_u}{\sigma+\gamma_x} \\
    1 + \frac{\psi \cdot \gamma_u}{\left(\sigma+\gamma_x\right)}
\end{bmatrix}.
\]

Note that in the writing of the equilibrium system we make a simplification. Although the real exchange rate, \( \hat{q} \), is an endogenous variable in our model, we simplify by assuming that it has an AR(1) representation.

\(^{11}\) Note however, that in the closed economy the real-exchange-rate channel shuts off, decreasing the degree of optimal response to output gap. This point is illustrated by comparing the parameter \( \gamma_x \) for closed economy, equation (21), with those for open economies, equations (18)-(20).
We assume that the generic term that collects variables from the right-hand side of the aggregated-supply curve, \( \hat{\mu}_t \), satisfies the following exogenous AR(1) process:

\[
(23) \quad \hat{\mu}_t = \rho \cdot \hat{\mu}_{t-1} + \tilde{u}_t,
\]

where, the parameter \( \rho \) is smaller than one in absolute value; and the disturbance term, \( \tilde{u}_t \), follows a white-noise process.

### 6.2 The solution

We guess that equation (22) has the following state space representation:

\[
(24) \quad \begin{bmatrix} x_t \\ \hat{\pi}_t \end{bmatrix} = F \cdot \hat{\mu}_t,
\]

where the parameter matrix, \( F \), is a matrix of order 2 by 2. Substituting the guessed solution from equation (24) into equation (22), and using the exogenous process from equation (23), we get:

\[
(25) \quad F \cdot \hat{\mu}_t = \left[ Q \cdot F \cdot \rho + R \right] \cdot \tilde{u}_t.
\]

Since the parameter \( \rho \) is a scalar, we are allowed to rewrite equation (25) with \( \rho \) pre-multiplying the matrix \( F \). Thus, we can use the method of undetermined coefficients to solve for the matrix \( F \):

\[
(26) \quad F = (I_{2 	imes 2} - Q \cdot \rho)^{-1} \cdot R.
\]

Substituting for the matrices \( Q \) and \( R \) we get that:

\[
F \equiv \frac{1}{G} \begin{bmatrix}
-\gamma_s [\sigma + \gamma_s - \rho \beta + \rho \psi (\gamma_r - 1)] + \rho [1 - \gamma_r (\sigma + \gamma_s + \psi \gamma_u)] \\
-\gamma_u \rho \psi + \left(1 + \frac{\gamma_s}{\sigma} - \rho \right) \sigma + \gamma_s + \psi \gamma_u
\end{bmatrix},
\]

\[
G \equiv (\sigma + \gamma_s - \sigma \rho) (\sigma + \gamma_s - \rho [\beta + \psi (1 - \gamma_r)]) + \rho^2 \psi (\gamma_r - 1) \sigma.
\]

### 6.3 Impulse Response

At this stage we can compute the impulse response of the equilibrium inflation and the equilibrium output gap to shocks. We illustrate by computing impulse responses to a cost-push shock. The impulse-response parameter values are presented Table 1.
Table 1
Impulse-Response Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Calvo parameter</td>
<td>$\alpha$</td>
<td>0.35</td>
</tr>
<tr>
<td>Time Discount Factor</td>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>CRRA</td>
<td>$\sigma$</td>
<td>1.00</td>
</tr>
<tr>
<td>CES</td>
<td>$\theta$</td>
<td>5.00</td>
</tr>
<tr>
<td>MC elasticity w.r.t. own output</td>
<td>$\omega_p$</td>
<td>0.25</td>
</tr>
<tr>
<td>Wage demand elasticity w.r.t. domestic output</td>
<td>$\omega_w$</td>
<td>5.00</td>
</tr>
<tr>
<td>Domestically produced goods</td>
<td>$n$</td>
<td>0.75</td>
</tr>
<tr>
<td>Persistence of the cost-push shock</td>
<td>$\rho$</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Figure 1 depicts the impulse response – of the equilibrium inflation and output gap – to a serially-correlated cost-push shock, under different regimes of openness.

**Figure 1**
Impulse response of inflation and output gap, to a serially-correlated cost-push shock

Figure 1 demonstrates that as the economy opens up, the equilibrium inflation response to a cost-push shock would be more moderate, while at the same time, the equilibrium output-gap response to the same shock is more erratic.
6.4 Comparative static

Figure 2 illustrates the solution sensitivity to the share of domestically-produced goods, $n$, and to the substitution elasticity across goods, $\theta$, which is inversely related to the producers-monopolistic power; both structural parameters are related to the degree of openness. Figure 2 shows that as the import share grows (that is, as $n$ falls) equilibrium-output elasticity, with respect to a cost-push shock, becomes more negative. At the same time, equilibrium-inflation elasticity, with respect to the same shock, is not monotonic: it falls so long as the import share grows from 0 to 25 percents, but it increases if import share grows farther. If we assume that monopolistic power falls with openness, we have another channel through which globalization influences equilibrium inflation and output. As monopolistic power falls (that is, as $\theta$ grows), equilibrium-output elasticity, with respect to a cost-push shock, becomes more negative. However, equilibrium-inflation elasticity again responses ambiguously: if import share is smaller than 25 percents, this elasticity falls together with the monopolistic power, but it grows otherwise. Altogether, it implies that our analytical conclusion is limited to the case where economies are open, but only up to a certain degree. However, this does not necessarily weakens our argument, since the degree of openness is restricted anyway; for instance—by the no-Ponzy-game assumption—the import share must be below 50 percents.

Figure 2
Solution sensitivity to structural parameters, under the fully-open case
7. CONCLUSION

The paper provides a unified analysis of the effects of international mobility of goods, labor, and finance, within a unified New-Keynesian open economy framework, on (1) the Phillips curve; (2) the weights of inflation and output gap in the approximated, utility-based, loss function; and (3) the utility-based interest rate rule under discretion, and (4) the equilibrium inflation and output gap. We demonstrate how an endogenously determined monetary policy, which is guided by the welfare criterion of the representative household, becomes more aggressive with regard to inflation fluctuations but more benign with respect to output-gap fluctuations, when the economy opens up to in- and out-migration, trade in goods, and capital flows. To concentrate on the inflation-output trade-off, we treat the real exchange rate as an exogenous-variable unaffected by the globalization regimes. Exploring the effects of globalization on the real exchange rate in our framework is left for future research.

The paper assumes that the flex-price markup is constant, unaffected by globalization forces. But there has been some evidence of greater restraints on domestic prices and wage growth in sectors more exposed to international competition, such as textiles and electronics. Chen, Imbs and Scott (2004) analyzed disaggregated data for EU manufacturing over the period 1988-2000. They find that increased openness lowers prices by reducing markups and by raising productivity. In response to an increase in openness, markups show a steep short run decline, which partly reverse later, while productivity rises in a manner that increases over time. If globalization reduces the markup, our model predicts that this effect, by itself, leads to a more forceful anti-inflation policy, and lessens the attention given by the policy maker to the fluctuations in economic activity.

Finally, as we know, more frequent price updating steepens the tradeoff between inflation and activity. However, to our knowledge, neither theory nor empirical evidence exists in support of any systematic relationship between globalization and frequency of price updating. Interestingly, Gopinath and Rigobon (2007) report, that the time frequency of price adjustment of US imported goods trended downward, on average, over the last decade".
REFERENCES


Appendix A
Derivation of the Utility-Based Loss Function

In this appendix we derive a quadratic approximation for the welfare criterion of the representative household, following Woodford (2003).  

In the full mobility case, a first stage in the log-linear approximation of the utility function, around the purely deterministic steady state, yields:

\[ U_t = \frac{-T U_t}{2} E_j \sum_{x=0}^{\infty} \beta^x \left( \omega_t \cdot n(x_{t+1} - x^*) \right)^2 + \theta (1 + \theta \omega_t) Var_j \rho_{t+1} (j) \]  

where \( Var_j \rho_t (j) \) is the price dispersion of domestically produced goods; and the expression "t.i.p." collects terms that are independent of monetary policy.

Using the Calvo (1983) set up it follows that:

\[ \pi_t = E_j [p_{t+1} (j) - p_{t-1}] \]

\[ = n \alpha E_j [p_{H, t} (j) - p_{t-1}] + n(1 - \alpha) [p_{H, t}^* - p_{t-1}] \]

\[ + (1 - n) \chi^t E_j [p_{F, t} (j) - p_{t-1}] + (1 - n)(1 - \alpha)^* [p_{F, t}^* - p_{t-1}] \]

where \( \alpha^* \) is the foreign country Calvo parameter.

We assume that foreign producers use a local currency pricing strategy and that they update prices at the same frequency as domestic producers, that is, \( \alpha = \alpha^* \). It follows that while optimizing, local and foreign producers set the same price: \( p_{H, t} = p_{F, t} \). Under these assumptions, we get, as in the closed economy, that:

\[ \pi_t = (1 - \alpha)[p_{H, t}^* - p_{t-1}] \]

It follows that:

\[ \Delta_t^n = \alpha \Delta_{t-1}^n + \frac{\alpha}{1 - \alpha} \pi_t^2 \]

where \( \Delta_t^n \equiv Var_j \rho_t (j) \). Equation (A.4) can be employed with (A.1) to get the utility based monetary policy welfare criterion:

\[ U_t = -\frac{1}{2} E_j \sum_{x=0}^{\infty} \beta^x \left( \frac{\psi_t}{\psi_{t-1}} (x_{t+1} - x^*)^2 + \pi_t^2 \right) \]

where \( \psi_t \) is the slope of the relevant aggregate supply curve \( i = 1, 2, 3, 4 \).

\[ ^{12} \text{We abstract from the money term in the utility function.} \]
Appendix B
Optimal Monetary Policy under Discretion

In this appendix we derive the optimal monetary policy under discretion. To do this, we first approximate the aggregate demand, the aggregate supply and the interest parity. Then we minimize the utility-based loss function subject to these relationships.

b.1 Aggregate Demand

Maximizing equation (1) with respect to \( B_{ff} \), subject to budget constraints, equation (3), and log linearizing the first order conditions, yields:

\[
\bar{C}_t = E_ t \bar{C}_{t+1} - \sigma^{-1} \left( \bar{L}_{ff} - E_t \bar{P}_{t+1} \right) + \bar{g}_t - E_t \bar{g}_{t+1},
\]

where \( \sigma = -\frac{u_{cc}}{u_c} \) and \( g_{t} = -\frac{u_{g}}{u_{cc}} \). The upper bar indicate the purely deterministic steady state.

World goods market clearing condition implies that \( \bar{C}_t = \bar{Y}_{t} + \bar{F} - \bar{C} \). Substituting in (B.1), subtracting the term \( \left[ \bar{Y}^N_{t} + \bar{P}^N_{t+1} \right] \) from both sides, and collecting all the exogenous terms, yields:

\[
x_t = E_ t x_{t+1} - \sigma^{-1} \left( \bar{L}_{ff} - E_t \bar{P}_{t+1} \right),
\]

where \( \bar{P}^n_t \equiv \sigma \left( g_t - E_t \bar{g}_{t+1} \right) + \left( \bar{P}^N_t - E_t \bar{P}^N_{t+1} \right) + \left( C^F_t - \bar{P}^F_t \right) - E_t \left( C^F_{t+1} - \bar{P}^F_{t+1} \right) \).

\( \bar{P}^n_t \) is the deviation of the natural real interest rate, from its level at the purely deterministic steady state level.

b.2 Interest Parity

The approximated equation describing interest parity is:

\[
\bar{g}_t = E_t \bar{g}_{t+1} + \bar{F}_{f} - \bar{L}_{H,F}
\]

Subtracting the expression \( E_t \bar{g}_{t+1} + \bar{F}_{f} \) from both sides of equation (B.3), we get the interest parity in real terms, as follows:

\[
\bar{q}_t = E_t \bar{q}_{t+1} + \left( \bar{F}_{f} - E_t \bar{F}_{f+1} \right) - \left( \bar{L}_{H,F} - E_t \bar{L}_{H,F+1} \right).
\]
b.3 Utility-Based Policy Rule

The optimal monetary rule under discretion is obtained upon minimizing the utility-based loss function period by period, subject to the aggregate supply equation, the aggregate demand equation, and the real interest parity.

Formally, optimization is given by:

\[
\min_{\bar{\pi}_t, x_t, \hat{q}_t, \hat{i}_{H,t}} \left\{ \frac{1}{2} \left( \hat{\pi}_t^2 + \frac{\psi'}{\theta} \cdot (x_t - x^*) \right) + F_t \right. \\
- \phi_{1,t} \left( \hat{\pi}_t - \beta E_t \hat{\pi}_{t+1} - \psi' \cdot x_t - \sum_i \Pi_i \cdot i_t \right) \\
- \phi_{2,t} \left( x_t - E_t x_{t+1} + \sigma^{-1} \left( \hat{i}_{H,t} - E_t \hat{\pi}_{t+1} + \hat{\pi}_t^a \right) \right) \\
- \phi_{3,t} \left( \hat{q}_t - E_t \hat{q}_{t+1} - \left( \hat{i}_{F,t} - E_t \hat{\pi}_{F,t+1} + \left( \hat{i}_{H,t} - E_t \hat{\pi}_{t+1} \right) \right) \right),
\]

where \( \Pi_i \) is the aggregate supply elasticity of inflation with respect to variable \( i = (\hat{y}_i, E_t \hat{y}_i) \); \( \hat{\pi}_t, \hat{q}_t, \hat{i}_{H,t}, \hat{i}_{F,t}, \hat{\pi}_{t+1} \); and \( \phi_{1,t}, \phi_{2,t} \) and \( \phi_{3,t} \) denote Lagrange multipliers; and where \( F_t \equiv \frac{1}{2} E_t \sum_{s=1}^n \beta^s \left( \hat{\pi}_s^2 + \frac{\psi'}{\theta} \cdot (x_s - x^*) \right) \)

and private sector expectations are taken as given. The corresponding set of first order conditions with respect to \( \hat{\pi}_t, x_t, \hat{q}_t, \hat{i}_{H,t} \) and \( \hat{i}_{H,t} \) are as follows:

\[
\hat{\pi}_t - \phi_{1,t} = 0 \\
\frac{\psi'}{\theta} \cdot x_t + \psi \cdot \phi_{3,t} - \phi_{2,t} = 0
\]

\[
\Pi_i \phi_{3,t} - \phi_{1,t} = 0 \\
- \phi_{2,t} \sigma^{-1} - \phi_{3,t} = 0.
\]

Assume that the government gives an output subsidy, which fully offsets the distortionary effect of the monopolistic competition market power, so that \( x^* = 0 \). The solution to equation (B.6) is then given by:
(B.7) \[ x_t = -\theta \left( 1 + \frac{\Pi_y}{\psi \sigma} \right) \tau. \]

Define \[ u_t \equiv \sum_i \Pi_i \cdot i_t. \] Substituting equation (B.7) into the aggregate supply equation and solving with forward iterations, we get:

(B.8) \[ x_t = -\Phi E_t \sum_{s=0}^\infty \left[ \tau^s u_{t+s} \right], \]

where \[ \Phi \equiv \frac{\theta \left( 1 + \frac{\Pi_y}{\psi \sigma} \right)}{1 + \theta \left( 1 + \frac{\Pi_y}{\psi \sigma} \right) \psi} \]

and \[ \tau \equiv \frac{\beta}{1 + \theta (\psi + \Pi_y \sigma) \psi}. \]

Substituting for the output-gap from equation (B.7) we get:

(B.9) \[ \overline{x}_t = \frac{1}{1 + \theta \left( 1 + \frac{\Pi_y}{\psi \sigma} \right) \psi} E_t \sum_{s=0}^\infty \left[ \tau^s \hat{u}_{t+s} \right]. \]

Under rational expectations it follows that:

(B.10) \[ E_t \overline{x}_{t+1} = \frac{1}{1 + \theta \left( 1 + \frac{\Pi_y}{\psi \sigma} \right) \psi} E_t \sum_{s=0}^\infty \left[ \tau^s \hat{u}_{t+s+1} \right]. \]

Now, rearranging equation (B.8):

(B.11) \[ x_t = -\Phi \cdot \hat{u}_t - \Phi \cdot \tau \cdot E_t \sum_{s=0}^\infty \left[ \tau^s \hat{u}_{t+s+1} \right]. \]

Substituting (B.10) into equation (B.11):

(B.12) \[ x_t = -\Phi \cdot \hat{u}_t - \frac{\theta \left( 1 + \frac{\Pi_y}{\psi \sigma} \right) \beta}{1 + \theta \left( 1 + \frac{\Pi_y}{\psi \sigma} \right) \psi} E_t \overline{x}_{t+1}. \]
Advancing equation (B.8) by one period, and substituting into equation (B.11):

\[ E_t x_{t+1} = \frac{\Phi}{\tau} \bar{a}_t + \frac{1}{\tau} x_t . \]

Substituting (B.12) and (B.13) into the aggregate-demand equation (B.2), and rearranging, yields the optimal policy rule, as follows:

\[ \hat{i}_{H,t} = r_t^n + \gamma_{\pi} \cdot E_t \hat{r}_{t+1} + \gamma_x x_t + \gamma_u \cdot \hat{u}_t , \]

where:

\[ \gamma_x = 1 + \left( \frac{1 + \Pi_{\Pi}(\sigma)}{1 + \rho(\psi + \Pi_{\Pi}(\sigma))} \right) \sigma \beta > 1 ; \]
\[ \gamma_{\pi} = \left[ \frac{1 + \rho(\psi + \Pi_{\Pi}(\sigma))}{\beta} \right] \sigma \beta > 1 ; \]
\[ \gamma_u = \sigma \cdot \phi \left[ \frac{1 + \rho(\psi + \Pi_{\Pi}(\sigma))}{\beta} + 1 \right] \sigma \beta > 1 . \]