Reputation and Indexation in an Inflation Targeting Framework*

by

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September 2003

* This paper is part of my Ph.D dissertation, under the supervision of Alex Cukierman. I am grateful to Alex Cukierman for his guidance and I thank the seminar participants at the research department for helpful comments.

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Any views expressed in the Discussion Paper Series are those of the authors and do not necessarily reflect those of the Bank of Israel.
Abstract

This paper explores the relationship between a policymaker’s reputation and the optimal wage indexation in an inflation-targeting framework, in which there is uncertainty regarding the policymaker’s ability for commitment. The simulation results suggest that the optimal wage indexation is non–monotonic in the policymaker’s reputation. In particular, at low levels of reputation, a rise in reputation leads to an increase in the wage indexation, while at higher levels of reputation, a rise in reputation leads to a reduction in the wage indexation. This result holds both in the social planner framework and in the case where there is a labor union that determines not only the nominal wage but also its level of indexation.

Key words: Inflation targets, reputation, wage indexation.
1. Introduction

Inflationary bias, which was first introduced by Kydland and Prescott (1977), stems from the time inconsistency problem as the nominal rigidity (along with other factors) increases the policymaker’s incentive to reduce unemployment by exploiting the short-run trade off between inflation and unemployment. One way to avoid the undesirable outcome of inflationary bias is by pre-commitment to rules [Barro and Gordon, (1983)]. Another way is simply to reduce the nominal rigidity by increasing the wage indexation.¹ Although both the effects of wage indexation and the effects of pre-commitment to rules were widely studied in recent years, the link between the two still remains rather ambiguous. The primary purpose of this paper is to explore the appropriate level of wage indexation in an inflation-targeting framework (which explicitly implement the pre-commitment paradigm) and in particular to focus on the evolution of wage indexation as a reaction to the evolution of the policymaker’s reputation for dependability.

The macroeconomic effects of wage indexation have been studied since the early 1970s, starting with the models of Gray (1976) and Fischer (1977) which emphasize the role of indexation in stabilizing or destabilizing output, and later on in frameworks which considered not only the macroeconomic effects of indexation, but also its impact on the optimal behavior of firms and policymakers. It was found that indexation might have two opposing inflationary effects. On the one hand, indexation can be inflationary since the higher the indexation, the lower the cost deriving from inflation, hence policymakers would have lower incentive to reduce inflation. This was explicitly stressed by Fischer and Summers (1989) model.² On the other hand, indexation reduces the nominal rigidity in the economy; hence it also reduces the incentive of the policymakers to inflate. Thus, indexation could be considered as anti-inflationary.

¹ There are also other ways to reduce the inflationary bias; see for example Rogoff (1985).
² Fischer and Summers (1989) found that policies that reduce the cost of inflation, such as indexation, lead policymakers who act in a discretionary manner to choose higher inflation rate.
In these frameworks, the optimal degree of indexation was examined mainly in situations where the policymakers conducted a discretionary policy without taking into account the effects of their reputation for dependability. For instance, Devereux (1989) found that in a discretionary policy there is a positive link between real disturbances and the mean of inflation. This positive relation stems from the fact that a higher variance of real shocks induces wage setters to reduce indexation. Low indexation stimulates the policymakers to inflate, and as a consequence leads to higher inflationary bias. Ball and Cecchetti (1991) found that despite the opposite effects of indexation in total, greater indexation reduces the cost of inflation and also increases welfare. Waller and VanHoose (1992) have demonstrated that the choice of indexation is socially inefficient, since the private wage setters, who are atomistic, do not take into consideration the benefit from the reduced inflation bias. This result suggests that if discretionary policy cannot be eliminated, then government should adopt policies that promote nominal indexation. Milesi-Ferretti (1994) showed that governments that have high incentive to inflate are more likely to “tie their hands” through wage indexation, even if this causes higher output variability. His result is related to that of Rogoff (1985) who demonstrated that instead of increasing indexation, governments that care more about employment would delegate authority to a more conservative central banker in order to reduce the inflationary bias. Diana (2000) explicitly used Rogoff’s model to demonstrate that if the government appointed a more conservative central banker, the labor union that acted as a centralized wage setter would find it optimal to reduce wage indexation in order to increase output stability.

This paper differs from the above works mainly in the monetary regime in which policymakers act. In particular, it implements the inflation target approach, which was first introduced by Barro (1986) and later developed by Cukierman and Liviatan (1991). In this framework the policymakers use pre-commitment to an explicit inflation target at the beginning of the period, before the nominal wage contracts are signed, in order to reduce the inflationary bias. Since there is uncertainty regarding the

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3 In a previous work (1991), Waller and VanHoose found that the interactions between indexation and discretionary policy depend critically upon the informational and timing constraints under which discretionary policy is conducted.
policymaker’s ability to commit, the optimal strategies of both the workers (which embodies their inflationary expectations into wage contracts) and the policymakers are mostly determined by the level of the policymakers’ reputation.

In the inflation target framework, Cukierman and Liviatan (1991) and Cukierman (2000) have shown that when the level of indexation is fixed, there is a clear negative relationship between the inflation target picked by the dependable policymaker and her reputation (i.e., high reputation leads to a low inflation target) since she accommodates to inflationary expectations in order to avoid deep recession. The questions that remain open are how reputation affects indexation, and as a consequence what is the total effect of the former on inflation targets?

The main conclusion of this work is that the optimal proportion of wage indexation does not always decrease as the policymaker’s reputation increases. In particular, at low levels of reputation, a rise in the policymaker’s reputation leads to higher wage indexation, while at high levels of reputation, a rise in reputation leads to smaller proportion of indexation in total wage. This result holds in the social planner framework (section 2) as well as in the case with a labor union that determines not only the nominal wage but also its level of indexation (section 3). In addition, as the reputation and the non-indexed wage proportion increase, both inflationary expectations and inflation target could increase since the impact of the nominal wage rigidity may be higher than that of the policymaker’s reputation. This result is different from that of Cukierman and Liviatan (1991) and other works that assumed a fixed Phillips curve slope and obtained that the higher the policymaker’s reputation, the lower would be both the inflation target and inflationary expectations.

The paper is organized as follows: Section 2 lays down the basic structure and characterizes a strategic monetary policy framework for welfare evaluation under imperfect reputation. This framework is utilized later on to evaluate the relation between reputation and the optimal level of wage indexation from the social planner point of view. Section 3 analyzes the effect of policymaker’s reputation on optimal wage indexation under the existence of a labor union. In this framework the union determines not only the nominal wage but the non-indexed wage proportion as well. Concluding remarks follows this.
2. A Social Planner Framework

The following section analyzes the relationship between the socially optimal level of indexation and the policymaker’s reputation regarding her ability for commitment. The justification for the socially optimal wage indexation analysis can be derived from the fact that in many countries, the public sector is a large employer and therefore it has a large influence on centralized wage bargaining. Such analysis has been done by VanHoose and Waller (1992), Milesi-Ferretti (1994) and others, but unlike their works, which focus on discretionary monetary framework; this paper considers an inflation target regime, in which policymakers can commit to an inflation target at the beginning of the period.

The inflation target framework in this paper has been modeled as in Cukierman and Liviatan (1991) and Cukierman (2000) that assumed two types of policymakers, dependable and weak, denoted (D) and (W) respectively. Although the policymakers share the same objective function, they differ in their ability to commit - the dependable policymaker is truly committed to the target she announces whereas the weak type is not, and chooses her policy actions in a discretionary manner, after expectations have been embedded into wage contracts and supply shocks have been realized. The uncertainty towards the policymaker type is reflected in the probabilities \( p \) and \( (1-p) \), which denote the probability that the policymaker in office is dependable and weak respectively. This probability can be referred as the policymaker’s reputation (as dependable), and it is inherited from the past.

Following Barro and Gordon (1983), the policymakers wish to minimize the variability of unemployment and inflation around a desired rate. For simplicity, the desired rate is normalized to zero. The social loss function of both policymakers is given by:

\[
L^i = \frac{A}{2} u^2 + \frac{1}{2} \pi^2, \quad A > 0 \quad i = D,W, \tag{1}
\]

where \( \pi \) denotes the actual inflation rate and \( u \) is the actual unemployment rate. The parameter \( A \) measures the relative importance that policymakers assign to the objective of low unemployment versus low inflation. The actual unemployment rate is given by a short run Phillips curve,

\[
u = \overline{\nu}_n + \alpha (W - P) - \varepsilon, \quad \overline{\nu}_n, \alpha > 0, \tag{2}
\]
where $\bar{u}_n$ denotes the average natural rate of unemployment, $W$ and $P$ represent the nominal wage level and the price level (in terms of logs) respectively, and $\varepsilon$ denotes a supply shock, which is assumed to have zero mean. Normalizing the log of the real wage of the previous period to zero $[(W_{-1}-P_{-1})=0]$, the Phillips curve can be described as:

$$(2') \quad u = \bar{u}_n + \alpha(\tilde{w} - \pi) - \varepsilon,$$

where $\tilde{w}$ denotes the rate of change of nominal wage. This rate changes as follows:

$$(3) \quad \tilde{w} = \lambda w + (1-\lambda)\pi \equiv \lambda \pi^e + (1-\lambda)\pi.$$

The parameter $\lambda$ can be referred as the proportion of non-indexed wage in total wage, hence it is formed according to inflationary expectations ($w = \pi^e$). This parameter is not only reflect the formal non-indexed wage, but it denotes the "effective" non-indexed wage proportion which include all the non-indexed components of the wage, the frequency at which the nominal wage contracts are opened and adjusted to the current inflation rate [as in Ball (1988)] and etc. The complementary proportion $(1-\lambda)$ denotes the proportion of wage that is fully indexed; therefore, its rate is equal to the actual inflation rate $(\pi)$.

Combining Eq. (3) in Eq. (2') yields:

$$(4) \quad u = \bar{u}_n + \alpha [\lambda \pi^e + (1-\lambda)\pi - \pi] - \varepsilon \equiv \bar{u}_n + \alpha(\lambda \pi^e - \pi) - \varepsilon.$$

After rearranging, one can see that the slope of the Phillips curve depends on the non-indexed wage proportion. In case where the workers’ wage is fully indexed $(\lambda = 0)$, the policymakers could not increase employment by an inflationary surprise, since the real wage cannot be eroded. As a result, the expected unemployment rate will always be equal to its average natural level (the Phillips curve is completely steep). In the opposite case, where there is no indexation $(\lambda =1)$, an inflationary surprise would have a maximum impact on unemployment.

Inserting Eq. (4) into Eq. (1), yields the policymaker welfare loss as a function of actual inflation, expected inflation and the non-indexed wage proportion:
Since policymakers dislike unemployment, one can see that the existence of a positive average natural unemployment rate leads to a time inconsistency problem, since a weak policymaker will always desire to inflate after expectations are formed in order to erode the real wage and reduce unemployment. Eventually, this incentive will lead (in partial reputation) to an inflationary bias.

2.1 Timing of events

The timing of moves is as follows: First, the social planner picks the optimal proportion of wage indexation in order to minimize the expected value of the social loss. This choice is done given the uncertainty regarding the policymaker’s ability to commit. Second, taking the level of wage indexation as given, the policymaker announces the inflation target for the period and inflationary expectations are formed and embedded in nominal wage contracts. Third, supply shocks are realized and the policymaker picks the rate of inflation.

2.2. Characterization of equilibrium policies

In order to obtain the optimal strategies of each player, the model is solved backwards, beginning with the optimal behavior of the policymakers. Starting with the weak type, she always announces the same inflation target, as her dependable counterpart would have, otherwise she would reveal her type at the beginning of the period. Since she is not really committed to a target, she picks her favorite inflation, $\pi^w$, in a discretionary manner, taking inflationary expectations as given. Minimizing Eq. (5) with respect to $\pi$ yields the following reaction function of the weak policymaker:

$$
\pi^w = \frac{\alpha^2 \lambda^2 A}{1 + \alpha^2 \lambda^2 A} \pi^e + \frac{\alpha \lambda A}{1 + \alpha^2 \lambda^2 A} (\bar{u} - \varepsilon).
$$

---

4 The underlying assumption is that although the act of commitment is done after the choice of wage indexation, the social planner knows what would be the announced inflation target for any level of reputation.
As can be seen from Eq. (6), the higher the nominal wage rigidity (λ), the higher the expected inflation rate chosen by the weak type. Higher nominal wage rigidity increases the incentive of the policymaker to obtain higher employment since a given inflationary surprise could erode the real wage at higher rate. In addition, because higher nominal rigidity increases the impact of inflationary surprises on unemployment, it also strengthens the reaction of the policymaker to offset supply shock fluctuations.

Since the probability that the dependable policymaker is in office is p, and since the dependable type always delivers the inflation target (T), the inflationary expectations after the announcement of the target are given by:

\( (7) \quad \pi^e = pT + (1 - p)E(\pi^w) \).

By inserting the weak’s type optimal solution [Eq. (6)] into Eq. (7) and rearranging, one can obtain:

\( (8) \quad \pi^e = \frac{(1 + \alpha^2 \lambda^2 A)p T}{(1 + p \alpha^2 \lambda^2 A)} + \frac{(1 - p)}{(1 + p \alpha^2 \lambda^2 A)} \alpha \lambda A \pi^*_n. \)

Turning to the dependable type optimal policy, the dependable type always picks the same inflation rate, which she announces at the beginning of the period. Taking into account the dependability constraint, \( \pi^d = T \), and the effect of the inflation target announcement on the formation of expectations, the dependable type chooses her target so as to minimize its loss function [Eq. (5)]:

\( (9) \quad L^d = \frac{A}{2} \left[ \pi^*_n + \alpha \lambda \left[ \frac{(1 + \alpha^2 \lambda^2 A)p T}{(1 + p \alpha^2 \lambda^2 A)} + \frac{(1 - p)}{(1 + p \alpha^2 \lambda^2 A)} \alpha \lambda A \pi^*_n - T \right] - \epsilon \right]^2 + \frac{1}{2} T^2. \)

The minimization of Eq. (9) with respect to the inflation target (T) yields:

\( (10) \quad T^* = \frac{(1 - p)}{(1 + \alpha^2 \lambda^2 A p^2)} \alpha \lambda A \pi^*_n. \)

Eq. (10) illustrates how the dependable type partially accommodates the public suspicions concerning her dependability, such that the lower her reputation, the higher the inflation target. In extreme case, in which the policymaker has full reputation (p=1), the target she announces would be equal to zero, since the announcement has a full impact on expectations. At first glance, this
accommodation may look quite similar to the result that appears in Cukierman and Liviatan (1991), but as will be shown later, the policymaker’s reputation has a secondary effect, which runs through the optimal proportion of wage indexation.5

Using Eq. (10), Eq. (8) and Eq. (6), one can explicitly obtain the inflationary expectations and the weak type’s optimal inflation:

\[\pi^e = \frac{(1 - p^2)}{(1 + \alpha^2 \lambda^2 A p^2)} \alpha \lambda A \mu_n \],

\[\pi^w = \frac{1}{(1 + \alpha^2 \lambda^2 A p^2)} \alpha \lambda A \mu_n - \frac{\alpha \lambda A}{(1 + \alpha^2 \lambda^2 A)} \varepsilon \].

The expectation of equilibrium values of each policymaker can be calculated by inserting the appropriate equilibrium strategies [Eq. (10), Eq. (11) and Eq. (12)] into Eq. (5). The resulting expressions are:

\[E(L^w) = \frac{A \mu_n^2 (1 + \alpha^2 \lambda^2 A)}{2(1 + \alpha^2 \lambda^2 A)^2} + \frac{A \sigma^2}{2(1 + \alpha^2 \lambda^2 A)} \],

\[E(L^d) = \frac{A \mu_n^2 (1 + \alpha^2 \lambda^2 A)}{2(1 + \alpha^2 \lambda^2 A)^2} + \frac{A \sigma^2}{2} \].

For any non-zero value of \( \lambda \), the expected social loss when the weak type is in office is always lower than the expected value when the dependable type is in office. This derives from two reasons: First, since inflationary expectations are the weighted average of the policymaker’s strategies, the weak type will always create higher employment than will the dependable type. Second, unlike the dependable type, the weak policymaker is not bound by a strict inflation target; hence she could partially offset supply shocks in order to equalize the marginal cost derived from both inflation and unemployment objectives.

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5 As will be shown later in section 2.4, at certain levels of reputation, an increase in reputation leads to a reduction in the indexation. Hence, although the inflation target announcement has a higher affect on expectations, the reduction of indexation (which increases the weak’s policymaker incentive to inflate), could have higher impact on both inflationary expectations and inflation target.
The impact of $\lambda$ on each policymaker’s expected loss value is as follows: since inflationary expectations are always above the dependable type’s inflation target, the higher the non-indexed wage proportion, the deeper the recession, and therefore the higher the loss when she is in office. As one can see, this proportion does not mitigate the damage from supply shocks, because the dependable type does not offsets them. In contrast to the dependable type’s loss, the impact of $\lambda$ on the weak’s welfare loss is not straightforward and it depends on the size of the other parameters. At low levels of reputation, expectations are high; hence the weak type cannot create high employment. In this case an increase in $\lambda$ increases the loss from inflationary bias, which is higher than the benefit from employment. Since the efficiency of shock neutralization rises with $\lambda$, its total impact depends on the ratio between $\sigma^2$ and $\bar{u}_n^2$. The higher that ratio, the lower the damage derived from an increase in $\lambda$. At high levels of reputation, an increase in $\lambda$ reduces the total loss of weak policymaker, since the benefit from employment and shock neutralization is always higher than the cost derived from inflationary bias.

### 2.3. The socially optimal non-indexed wage proportion

This section endogenizes the degree of wage indexation, and analyzes its reaction to a change in the policymaker’s reputation. In particular, the level of wage indexation is determined by a social planner at the beginning of the period, before the policymaker commit to an inflation target, so as to minimize the expected social loss. This expected loss is:

\[
E(L) = p \cdot E(L^d) + (1 - p) \cdot E(L^w) = 
\]

\[
= \frac{A\bar{u}_n^2(1 + 2\alpha^2\lambda^2Ap^2 + \alpha^4\lambda^4A^2p^3)}{2(1 + \alpha^2\lambda^2Ap^2)^2} + \frac{A^2\bar{u}_n^2\alpha^2\lambda^2(1 - 2p^2 + p^3)}{2(1 + \alpha^2\lambda^2Ap^2)^2} + \frac{A\sigma^2(1 + \alpha^2\lambda^2A)}{2(1 + \alpha^2\lambda^2A)}. 
\]

The expected loss value can be divided into three separate arguments, [as in Eq. (15)]. The first argument reflects the expected loss from unemployment; the second argument reflects the expected loss from inflation, and the third argument reflects the expected loss from supply shock fluctuations. As one can see, the policymaker’s reputation has a different effect on each argument. While an increase in reputation increases the loss from supply shock fluctuations and reduces the loss from
inflation, it has a non-monotonic impact on the loss from unemployment (see Figures 1.A-3.A in Appendix). This non-monotonic shape stems from the dependable policymaker’s unemployment loss: at low levels of reputation, an increase in reputation increases the loss from unemployment since expectations are decreasing slower than the inflation target. As a result, the recession under the dependable type becomes deeper. At high levels of reputation, an increase in reputation reduces the loss from unemployment since expectations are decreasing faster than the inflation target. As a result the recession under the dependable type is less severe. Figure 1.A also illustrates how a reduction in wage indexation increases the expected loss from unemployment, since it intensifies the impact of unexpected inflation.

The optimal non-indexed wage proportion can be derived from the minimization of the average expected loss [Eq. (15)]. In case of "pure discretion" \((p = 0)\) the optimal non-indexed wage proportion \((\hat{\lambda})\) is:

\[
\hat{\lambda} = \sqrt{\frac{1}{\alpha^2 A} \left( \frac{\sigma}{\bar{u}_n} - 1 \right)}.
\]

**Proposition 1:** In the case of "pure discretion" \((p = 0)\), the optimal non-indexed wage proportion is higher - 

i. the higher is the supply shock variance \((\sigma)\).

ii. the lower are \(\alpha, A\) and \(\bar{u}_n\).

**Proof.** In part 2.A of the Appendix.

The underlying intuition, which presented in proposition 1 and explicitly described in Eq. (17) is as follows: On the one hand, the higher the damage from supply shocks \((\sigma)\), the higher will be the optimal non-indexed wage proportion \((\hat{\lambda})\), as high nominal wage rigidity increases the efficiency of weak policymakers to offset supply shocks, i.e., they could achieve their preferred unemployment rate by a relatively small inflation variability. On the other hand, the greater the inflation bias motives
As for the case of partial reputation \((0 < p < 1)\), the optimal non-indexed wage proportion is 8:

\[
\frac{\partial E(L)}{\partial \hat{\lambda}} = 0 \Rightarrow \frac{\sigma^2}{\bar{u}^2} = \frac{(1 + \alpha^2 \hat{\lambda}^2 A)^2 ((1 - p^2) (1 - \alpha^2 \hat{\lambda}^2 A p^2) + p(1 + \alpha^2 \hat{\lambda}^2 A p^2))}{(1 + \alpha^2 \hat{\lambda}^2 A p^2)^3}
\]

where the second order condition (see in part 3.A at the Appendix) implies that \(\frac{\sigma^2}{\bar{u}^2} \) must be higher than \(\left(\frac{\sigma^2}{\bar{u}^2}\right)\). 

**Proposition 2:** For any ratio of \(\left(\frac{\sigma^2}{\bar{u}^2}\right)\) above the threshold - \(\left(\frac{\sigma^2}{\bar{u}^2}\right)_c\), the optimal non-indexed wage proportion is non-monotonic in the policymaker’s reputation. In particular, at low values of reputation, \(\hat{\lambda}\) is decreasing in \(p\), while at high levels of reputation \(\hat{\lambda}\) is increasing in \(p\).

(The explicit derivative is presented in part 4.A of the Appendix.).

The examination of \(\hat{\lambda}\) in the case of partial reputation is based on numeric simulation, since the solution for \(\hat{\lambda}\) contains a high-order polynomial. These simulation solutions were constrained to positive values between zero and one in order to interpret the results as the non-indexed wage fraction and obtain an economic meaning. The simulation was solved for \(p\) between zero and one,

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6 For internal solution to the minimization of Eq. (15), the ratio \(\sigma^2/\bar{u}^2\) must be higher than 1.

7 In order to reduce the inflationary bias, Roggof (1985) suggested to appoint a central banker, who is conservative than does society as a whole. In his work the optimal degree of conservatism also depends positively on the supply shock variance and negatively on inflation bias motives.

8 Note that when the policymaker’s reputation is equal to one \((p=1)\), the expected welfare loss is not a function of \(\hat{\lambda}\), since a dependable type would pick a zero inflation target, and by definition she does not offset supply shocks. As a result the optimal non-indexed wage proportion [in Eq. (17)] is not defined.
while the other exogenous parameters values were arbitrarily chosen as: $\bar{u}_n = 2\ , \ \alpha = 1\ , \ A = 1$ and $\sigma^2 = 5$ (the optimal non-indexed wage proportion for wider range of parameters value is described at the figures in the Appendix).

From the simulation results illustrated in Figure 1 below, one can notice that the optimal non-indexed proportion is non-monotonic in $p$. At low values of reputation, $\hat{\lambda}$ is decreasing in $p$, since all other things being equal, a marginal reduction in the non-indexed wage fraction reduces the cost derived from unemployment and inflationary bias at higher magnitude than it increases the cost derived from supply shock fluctuations. This downward slope of the optimal non-indexed wage proportion is compatible with the claim of Friedman (1974) that in order to reduce “side effects” (recession) resulting from an unanticipated disinflation process, firms should create wage contracts in real and not in nominal terms. In other words, at low levels of reputation, in which the inflation target decreases faster than inflationary expectations, the social planner should increase the wage indexation ($\text{reduce } \hat{\lambda}$) in order to mitigate the recession. At high levels of reputation this trend changes and the optimal non-indexed wage proportion ($\hat{\lambda}$) increases in reputation ($p$), since the loss from unemployment and the inflation bias is relatively low (at high levels of reputation expectation are decreasing faster than the inflation target), while the loss from supply shock fluctuations is relatively high (hence an increase in $\hat{\lambda}$ reduces the loss from fluctuations. See figures 1.A- 3.A).

![Figure 1 - The optimal non indexed wage proportion](image)

* The continuous line and the striped line in Figure 1 stands for $\sigma^2/\bar{u}_n^2 = 2$ and $\sigma^2/\bar{u}_n^2 = 1.5$ respectively.
Another interpretation of this result is as follows: indexation is a secondary tool to eliminate inflationary bias. At low levels of reputation, reputation by itself cannot reduce the inflationary bias, therefore the social planner reduces the nominal rigidity by increasing the level of wage indexation, and as a result decreases the policymaker’s incentive to inflate. At high levels of reputation a rise in reputation is sufficient to reduce inflationary bias, hence, the social planner can reduce indexation in order to help policymakers to reduce the cost derive from supply shock fluctuations. Note that the greater the ratio of $\sigma^2$ to $\bar{u}_n^2$, the higher is $\hat{\lambda}$, since the damage from supply shock fluctuations exceeds that from inflationary bias. As the average natural unemployment rate effect, a decrease in the relative importance that the policymaker ascribe to unemployment ($A$) and a decrease in the flexibility of labor demand with respect to the real wage ($\alpha$), would increase the optimal non-indexed wage proportion for any level of reputation (see Figures 4.A-5.A).

2.4 The optimal non-indexed wage proportion and its impact on inflation target and inflationary expectations

Proposition 3: Taking the total effect of reputation into account, both inflationary expectations and inflation target could increase in reputation. (For sufficient conditions, see part 5.A in Appendix.).

The non-monotonic behavior of $\hat{\lambda}$ is reflected also in the evolution of inflationary expectations and the inflation target (Figures 6.A and 7.A). At low levels of reputation, both the increase in $p$ and the reduction of $\hat{\lambda}$ pull inflationary expectations and the inflation target downwards (an increase in $p$ contributes to a reduction in $\hat{\lambda}$). From a certain point, the effect of $p$ and $\hat{\lambda}$ run in opposite directions as an increase in the policymaker’s reputation leads to a reduction in the proportion of wage indexation (an increase in $\hat{\lambda}$). The opposing effects moderate the decrease in the inflation target and expectations, and at higher levels of reputation, the total effect can lead to an increase in both the target and expectations. This movement lasts until the optimal proportion of wage indexation reaches zero [at this stage, all wage contracts are formed according to inflationary expectations ($\hat{\lambda} = 1$)]. From this point on, both expectations and the inflation target decrease monotonically since the only effect that exists is the increasing reputation.
3. A Labor Union as a Wage Setter

This section analyzes the relation between the policymaker’s reputation and the optimal non-indexed wage proportion under the existence of a single labor union that operates under an inflation target regime. The main difference between the two frameworks is in the determination of the real wage and as an outcome – the determination of the actual unemployment rate. While in the previous framework the unemployment rate was eventually determined by the social planner, who takes the policies of both policymakers type as given, in this framework the real wage (which directly affects unemployment) is determined by a labor union that plays strategically as a leading player. The real wage is determined according to the union’s preferences regarding inflation, employment and real wage objectives.

Following Cukierman and Lippi (1999), the union loss function is:

\[ L^{un} = \frac{\delta}{2} u^2 + \frac{\gamma}{2} \pi^2 - W^r, \quad \delta, \gamma > 0. \]

As the policymakers, the union desires to reduce unemployment and minimize inflation fluctuations around zero rate. In addition, the union is also motivated to increase the level of real wage \((W^r = W - P)\) among its members. Although the union cares about the real wage, it directly sets the nominal wage and the level of indexation. The parameters \(\delta\) and \(\gamma\) denote the weight that the union ascribes to unemployment and inflation relative to its real wage objective.

The demand for labor \((L^d)\) that the union faces is:

\[ L^d = [\alpha(d - W^r) + \varepsilon]L, \quad d, \alpha > 0. \]

where \(L\) is the labor supply in the economy (all labor is unionized) and \(\varepsilon\) is a supply shock, which assumed to have zero mean.

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9 This is motivated by the observation that in many cases the pensions of the union workers are not indexed and that union members, like other individuals, generally dislike inflation.
Reformulating the labor demand [Eq. (19)] in terms of real wage premium leads to the following unemployment equation:\(^{10}\):

\[
(20) \quad u = \frac{L - L^d}{L} = [W' - (EW^{rc} + \epsilon)] \equiv [W' - W^{rc}]
\]

where \(W^{rc} = (d - 1 + \epsilon)\) is the market clearing real wage at which \(u = 0\) (\(EW^{rc}\) is the expected market clearing real wage). This equation shows that unemployment is increasing in the real wage premium, such the higher is the gap between the real wage that is chosen by the union to the competitive real wage, the higher is the unemployment rate. By normalizing the log of the real wage of the previous period to zero \([W'_{-1} = (W_{-1} - P_{-1}) = 0]\), the real wage can be expressed in terms of a change in the real wage [as in Eq. (3)]:

\[
(20') \quad u = [W' - (EW^{rc} + \epsilon)] \equiv [W'_{-1} + (\bar{w} - \pi) - (EW^{rc} + \epsilon)] \equiv [\lambda(w - \pi) - (EW^{rc} + \epsilon)].
\]

By inserting Eq. (20') into Eq. (18), the union loss function can be expressed as:

\[
(21) \quad L^{un} = \frac{\delta}{2} \left[ \lambda(w - \pi) - (EW^{rc} + \epsilon) \right]^2 + \frac{\gamma}{2} \pi^2 - \lambda(w - \pi),
\]

where \(w\) is the rate of change of the nominal wage and \(\lambda\) is the non-indexed wage proportion. Those variables are the union's instruments in determining its optimal real wage. Note that the union faces a tradeoff between the real wage and employment considerations since a higher real wage among the union members would increase the rate of unemployment.

### 3.1 Timing of events

**Figure 2: The timing of events during the period**

| The determination of the wage indexation by the union | Policymaker commits to an inflation target | Union chooses the nominal wage | Supply shock realization | The policymaker chooses the inflation rate |

\(^{10}\) For analytic convenience and without loss of any generality, the parameter \(\alpha\) is normalized to unity.
As shown in Figure 2 above, at the beginning of the period the union determines the level of wage indexation, i.e., this determination enables the union to control the extent in which the nominal wage is exposed to erosion resulting from actual inflation. This choice is done given the policymaker’s reputation. Second, the policymaker announces the inflation target. Third, the union chooses the nominal wage and with its favorite level of wage indexation, it obtains its optimal real wage. This determination is done under two sources of uncertainty: the uncertainty regarding the magnitude of the supply shock realization and the uncertainty regarding the exact type of the policymaker. Again, the prior $p$, denotes the probability that the union ascribes to the possibility that the dependable policymaker is in office, and with the complementary probability $(1-p)$ that the weak policymaker is in office. At the end of the period the policymaker observe the supply shock and chooses inflation rate. Note that when the dependable policymaker is in office, the union acts after the policymaker (in the determination of nominal wage), while in the case that the weak policymaker is in office, the union acts before she chooses her favorite inflation; hence it must consider the weak policymaker’s backlash to its pre-determined nominal wage.

3.2 The weak policymaker’s optimal strategy

As stated in the previous section, a weak type always announces the same inflation target as her counterpart would have, otherwise she would reveal her type at the beginning of the period. Since the weak policymaker is not really committed to the inflation target, she would pick her optimal inflation rate in a discretionary manner, after observing the supply shock realization, and after the union has determined the nominal wage and the level of wage indexation.

By inserting Eq. (20') in Eq. (1), one can obtain the reaction function of the weak policymaker:

\[
\frac{\partial E(L^w)}{\partial \pi_w} = 0 \Rightarrow \pi_w = \frac{\lambda A}{(1 + \lambda^2 A)} [\lambda w - (EW^r + \varepsilon)].
\]

Since the weak type has an incentive to reduce unemployment, she would increase inflation rate as the real wage premium is higher ($\lambda w$ reflects the nominal change of the wage besides the change resulting from indexation). It can be seen that this incentive depends positively on the non-indexed
wage proportion \((\lambda)\), as the erosion of real wage is higher when the proportion of indexation is lower \((\lambda\) is higher). Higher \(A\) also increases the incentive to inflate, as the unemployment objective becomes more important.

3.3 The choice of nominal wage by the union

Since the union does not know for certain what type of policymaker it is facing, its expected loss is:

\[
E(L^{un}) = p \mathbb{E} \left\{ \frac{\delta}{2} \left[ \lambda (w-T) - (EW^{rc} + \varepsilon) \right]^2 + \frac{\gamma}{2} T^2 - \lambda (w-T) \right\} + \\
(1-p) \mathbb{E} \left\{ \frac{\delta}{2} \left[ \lambda (w-\pi_w) - (EW^{rc} + \varepsilon) \right]^2 + \frac{\gamma}{2} (\pi_w)^2 - \lambda (w-\pi_w) \right\}.
\]

Inserting the reaction function of the weak policymaker [Eq. (22)] into Eq. (23) and minimizing it with respect to the rate of nominal wage change \(w\), yields the reaction function of the union:

\[
\frac{\partial E(L^{un})}{\partial w} = 0 \Rightarrow \lambda w(T) = \left[ EW^{rc} + \frac{\lambda \delta (1 + \lambda^2 A)^2 pT + (1 + p \lambda^2 A) (1 + \lambda^2 A)}{p\delta (1 + \lambda^2 A)^2 + (1-p)(\delta + \lambda^2 A^2 \gamma)} \right].
\]

From the union’s reaction function, [Eq. (24)], one can notice that the union would like to obtain higher real wage than the expected competitive real wage, such that the expected unemployment rate would be positive. The real wage premium is higher the higher is the inflation target announced at the beginning of the period, and the lower is the union's inflation aversion \(\gamma\). The inflation aversion parameter moderates the nominal wage because of the union’s recognition that a higher rate of nominal wage would lead to higher inflation rate under the weak type. Since the union wants to avoid high rates of inflation (according to \(\gamma\)), it would reduce its nominal wage.

3.4 The dependable policymaker’s strategy

The dependable policymaker chooses the inflation target at the beginning of the period, taking the reaction function of the labor union, Eq. (24), as given. Since this reaction function embodies the preferences of the weak type, the latter has indirect impact on the inflation target announced at the beginning of the period. The expected loss function of the dependable policymaker is:
Inserting the reaction function of the union [Eq. (24)] into Eq. (25) and minimizing it with respect to inflation target \((T)\) yields:

\[
(26) \quad T^* = \frac{(1 - p)A(1 + pA^2)(1 + A^2)(\delta + A^2)}{p\delta(1 + A^2)^2 + (1 - p)(\delta + A^2)^2} + A^2(1 - p)^2(\delta + A^2)^2.
\]

The inflation target in Eq. (26) illustrates the accommodation of the D type to its level of reputation. When she has full reputation \((p=1)\), the policymaker’s declaration has full impact on the nominal wage setting process; hence she would prefer to announce a zero inflation target in order to reduce the inflationary bias \((T=0)\). Since the union reacts after the dependable policymaker, it would obtain its preferred real wage with the cost of low inflation. In the opposite extreme case in which the D type has no reputation at all \((p=0)\), she would accommodate and announced the same inflation as W type is expected to deliver:

\[
(27) \quad T^* (p=0) = E(\pi_w / p=0) = \frac{A}{(\delta + A^2)}.
\]

From Eq. (27) it is clear that the higher the weight that the union ascribes to unemployment and inflation in its loss function \((\delta \text{ and } \gamma \text{ respectively})\), the lower would be the chosen inflation rate, since the union would prefer to obtain lower real wage at lower inflation rate. Thus, a reduction of the nominal wage rate would lead to a reduction in the discretionary inflation rate produced by the W type. However, the impact of the non-indexed wage proportion is not straightforward and it depends on the value of the parameters. If the inequality \(\delta > A^2 \gamma \) \((\delta < A^2 \gamma \text { holds})\) holds, the greater the non-indexed wage proportion, the higher (lower) the inflation that would be picked by the W type. If the union is highly inflation averse \((\text{higher } \gamma)\), an increase in the non-indexed wage proportion would also reduce its nominal wage rate such that, although the W type has a higher incentive to inflate, the optimal inflation rate that she chooses will be lower.
Using Eq. (22), Eq. (23) and Eq. (24), one can obtain the optimal real wage of the union for any level of reputation. If the policymaker in office has been revealed as dependable \((p=1)\), the union’s optimal real wage would be:

\[
W^r_d(p=1) = \lambda(w-T) = EW^{rc} + \frac{1}{\delta},
\]

and the expected real wage if the policymaker in office has been revealed as weak \((p=0)\) is:

\[
E[W^r_w(p=0)] = \lambda[w-E(\pi_w)] = EW^{rc} + \frac{1}{(\delta + \lambda^2 A^2 \gamma)}.
\]

It is easy to see that in both cases \((p=0\) and \(p=1\)) the union's favorite real wage will be higher than the expected clearing market real wage. Note that as long as the union is inflation averse \((\gamma > 0)\), the real wage under a weak policymaker would be always lower than the real wage under the dependable type, since the union will take into consideration the backlash of the policymaker once the nominal wage contracts are closed. The inflation aversion parameter is also related to the proportion of non-indexed wage and the importance of unemployment, \(A\), since the weak policymaker would increase inflation as these parameters increase. The parameter \(\delta\) also moderates the real wage since the union’s incentive to reduce unemployment is higher.

For the partial reputation \((0 < p < 1)\), the expected real wage under each policymaker is:

\[
E(W^r_d) = EW^{rc} + \frac{(1+p\lambda^2 A)(1+\lambda^2 A)}{Z}\left[1 + \frac{(1-p)\lambda^2 A(\delta + \lambda^2 A^2 \gamma)}{Z^2 + \lambda^2 A(1-p)^2(\delta + \lambda^2 A^2 \gamma)^2}\right],
\]

\[
E(W^r_w) = EW^{rc} + \frac{(1+p\lambda^2 A)}{Z}\left[1 + \frac{p(1-p)\lambda^2 A\delta(1+\lambda^2 A^2 \gamma)}{Z^2 + \lambda^2 A(1-p)^2(\delta + \lambda^2 A^2 \gamma)^2}\right],
\]

where \(Z = p\delta(1+\lambda^2 A)^2 + (1-p)(\delta + \lambda^2 A^2 \gamma)\) .
3.4 The optimal non-indexed wage proportion

Previous to the determination of the rate of nominal wage, the labor union chooses the non-indexed wage proportion \( \hat{\lambda} \) in order to minimize its expected loss value (the subscript * denotes the equilibrium strategies of each policymaker):

\[
E(L^\text{un}) = pE\left\{ \frac{\delta}{2} \left[ \lambda(w^* - T^*) - (EW^{rc} + \epsilon) \right]^2 + \frac{\gamma}{2} (T^*)^2 - \lambda(w^* - T^*) \right\} + (1 - p)E\left\{ \frac{\delta}{2} \left[ \lambda(w^* - \pi_w^*) - (EW^{rc} + \epsilon) \right]^2 + \frac{\gamma}{2} (\pi_w^*)^2 - \lambda(w^* - \pi_w^*) \right\}.
\]

(32)

This expected loss could be also represented as follows:

\[
E(L^\text{un}) = \frac{\delta}{2} \left\{ p\left[ E(W_d^r) - EW^{rc} \right]^2 + (1 - p)\left[ E(W_w^r) - EW^{rc} \right]^2 \right\} + \frac{\gamma}{2} \left\{ p(T^*)^2 + (1 - p)\left[ E(\pi_w^*) \right]^2 \right\} + \frac{\sigma^2}{2} \left\{ (1 - p)\left[ \frac{\delta^2 A^2 + \delta}{(1 + \lambda^2 A)^2} \right] + p\delta \right\} - \left[ pE(W_d^r) + (1 - p)E(W_w^r) \right].
\]

(32’)

where Eq. (32’) consists of four arguments: the expected loss from unemployment, the expected loss from inflation, the expected loss from supply shock fluctuations and the benefit from the expected real wage.

As reflected in Figure (8.A), the expected real wage is increasing with the policymaker’s reputation, since as the reputation rises, the probability that the policymaker will react after the union determines its favorite nominal wage is decreasing. Hence the union has an incentive to increase the nominal wage in order to increase its real value. In addition, an increase of the non-indexed wage proportion \( \hat{\lambda} \) will reduce the expected real wage because a higher non-indexed wage proportion will induce the weak type to create higher inflation in order to erode the real wage and gain lower unemployment. Since the union dislikes inflation, and it knows the weak policymaker’s incentive to raise inflation, it reduces the nominal wage, and as a consequence its real value. The level of reputation and the non-indexed wage proportion has the same effect on the loss from unemployment, since this loss increases with real wage [the loss from unemployment is described in Figure (11.A)].
Figure (9.A) describes the loss from supply shock fluctuations. As expected, since the dependable policymaker does not offset the supply shocks, the higher the policymaker’s reputation, the greater is the expected loss. An increase in $\hat{\lambda}$ reduces this expected loss for any level of reputation since the shock neutralization becomes more efficiency (the weak policymakers could stabilize unemployment through changes in real wage with less inflation variability).

The loss from inflation is presented in Figure (10.A). As one can see, this loss does not always decreases with reputation (as in the social planner framework), since at low levels of reputation, a rise in reputation leads to a rise in the nominal wage and as a consequence it raises also W’s inflation rate. At low levels of reputation the inflation that is chosen by W type has a dominant impact on the total expected loss from inflation, hence this loss increase in reputation. At higher levels of reputation, the dependable type has a higher impact on the wage setting process and as a consequence both nominal wage and W’s inflation rate are decreasing in reputation. As expected, the reduction of the non-indexed wage proportion (a rise in indexation) reduces the loss from inflation since the incentive of the weak policymaker to implement inflationary policy in order to create employment is reduced.

The minimization of Eq. (32) yields the optimal non-indexed wage proportion, $\hat{\lambda}$, as a function of the policymaker’s reputation ($p$), its unemployment relative importance ($A$), supply shocks variance ($\sigma^2$), and the relative weight that the union ascribes to unemployment and inflation in its loss function ($\delta$ and $\gamma$):\(^{11}\)  

$$\frac{\partial E(L^{un})}{\partial \lambda} = 0 \Rightarrow \hat{\lambda} = g(p, A, \sigma^2, \delta, \gamma).$$

As in the previous section, the optimal values of the non-indexed wage proportion were obtained by using a numeric simulation, in which the exogenous parameters were arbitrarily chosen as $A = 1$, $\sigma^2 = 5$, $\delta = 0.4$ and $\gamma = 0.3$ (the optimal non-indexed wage proportion for a wider range of

\(^{11}\) Note that $Ew^{rec}$ does not affect the optimal non-indexed wage proportion, since the union considers its full effect in the determination of real wage.
parameters value is described at the figures in the Appendix). Again, these simulation results were constrained to positive values between zero and one in order to interpret them as the non-indexed wage proportion and to obtain an economic meaning. As can be seen from Figure (12.A), the non-monotonic behavior of \( \hat{\lambda} \) characterizes also the labor union framework. At low levels of reputation, an increase in \( p \) reduces \( \hat{\lambda} \), since in this range a reduction of the non-indexed wage proportion reduces the incentive of the weak type to inflate, and hence produces a higher real wage and lower inflation bias. The benefit from a higher real wage and lower inflation bias is higher than the loss deriving from unemployment and supply shocks fluctuations (see Figures 8.A-11.A). At higher levels of reputation this trend changes, since once the inflation bias is relatively low and the real wage is relatively high (as a result of the reputation effect), the union would prefer to increase the non-indexed wage proportion in order to reduce the loss from supply shock fluctuations. This non-monotonic shape of the optimal indexation could also characterize a case of a labor union that does not have inflation aversion (\( \gamma = 0 \)). In this case, as a result of partial information regarding the policymaker, the expected real wage would decrease beneath its full information level (which is identical to both weak and dependable policymakers). Since higher indexation reduces the weak’s incentive to inflate, it would increase the real wage for any level of reputation. The real wage considerations will be dominant at low levels of reputation in which the real wage erosion is high. As a result, the optimal indexation would increase with reputation. In the range of high reputation, the union would prefer to reduce indexation in order to increase unemployment stability (in this range the erosion of real wage is less severe).

12 Figures 8.A-11.A also show that a marginal increase in reputation reduces the cost deriving from a marginal increase in the non-indexed wage proportion.

13 Since the damage from supply shock fluctuations increase with \( \gamma \); in the case where the union does not have inflation aversion the u-shaped optimal indexation would derive for higher levels of supply shock variance. In very low levels of the latter, the optimal indexation would increase in reputation since the marginal cost that derives from real wage erosion will always be higher than the marginal cost derives from unemployment instability.
3.5 Comparative statics

Comparative statics are essential in order to examine the differential impact of the exogenous parameters on the optimal non-indexed wage proportion. From this exercise, one can see that other things being equal, a higher variance of supply shock yields a higher level of $\hat{\lambda}$. This result derives from the fact that an increase in the nominal wage rigidity enables the weak policymaker to offset supply shocks more efficiently, i.e., with low cost of inflation variability [Figure (13.A)].

The negative impact of $\gamma$ on the optimal non-indexed wage proportion is shown in Figure (14.A). If the union is more inflation averse (has a higher $\gamma$), it would moderate the W policymaker’s incentive to inflate by reducing the nominal wage rigidity (i.e., lowering $\hat{\lambda}$). The same impact can be seen from a change in $A$ - the parameter which measures the relative importance of unemployment (Figure 16.A). Given closed nominal wage contracts, a higher $A$ increases the incentive of the W type to inflate. Since the union knows the reaction function of the policymaker, it would choose to prevent the high inflation by reducing the nominal wage rigidity ($\hat{\lambda}$).

As can be seen from Figure (15.A), the higher the relative weight that the union ascribes to unemployment relative to real wage objective ($\delta$), the higher the non-indexed wage proportion ($\hat{\lambda}$). That is since higher $\delta$ indicates that the union is willing to achieve lower real wage in order to obtain higher employment. By increasing $\hat{\lambda}$, the union intensifies the weak policymaker’s incentive to raise inflation and create lower unemployment. The impact of $\hat{\lambda}$ on the real wage is demonstrated in Figure (8.A).

4. Concluding remarks

This paper explores the relationship between the policymaker’s reputation and the optimal wage indexation in an inflation-targeting framework, in which there is uncertainty regarding the policymaker’s ability for commitment. The simulation results suggest that the optimal non-indexed wage proportion is non-monotonic in the policymaker’s reputation. In particular, at low levels of reputation, a rise in reputation leads to an increase in the wage indexation, while at higher levels of
reputation, a rise in reputation leads to a reduction in the wage indexation. This result holds in both the social planner framework, which is introduced in Section 2 and in the existence of a labor union that plays strategically as a leading player (in Section 3).

Although the two frameworks yield the same result, the explanation for it is not exactly similar. In the social planner framework, the non-indexed wage proportion decreases in reputation (at low levels of reputation) since the social planner wants to avoid the high recession resulting from the dependable policymaker, as the inflation target is decreasing faster than inflationary expectations. The reduction of the non-indexed wage proportion also reduces the inflationary bias as the weak policymaker’s incentive to inflate goes down. At higher levels of reputation, once the inflationary bias is smaller and the recession is less severe (since the inflation target decreases more slowly than expectations), the non-indexed wage proportion increase in reputation in order to reduce the undesirable unemployment and inflation fluctuations.

In the labor union framework the union determines both the nominal wage and the level of indexation so as to achieve its preferred real wage. At lower levels of reputation, in which the probability that the weak policymaker is in office is high, the union would tend to reduce the non-indexed wage proportion in order to reduce the weak type’s incentive to inflate. As a consequence it will lead to a higher real wage and a lower inflationary bias. At higher levels of reputation, once the inflationary bias and the probability that the policymaker will deviate from the inflation target are both small, the union would increase the non-indexed wage proportion as reputation increase, in order to reduce the cost deriving from unemployment and inflation fluctuations.

The simulation results also show that at high levels of reputation, as the reputation and the optimal non-indexed wage proportion increase, it is possible that both inflationary expectations and the inflation target would increase since the impact of nominal wage rigidity could be greater than the impact of the policymaker’s reputation. This result contrasts to Cukierman and Liviatan (1991) and other works that assumed a fixed Phillips curve slope and obtained that the higher the policymaker’s reputation, the lower both the inflation target and inflationary expectations.
Appendix - Social Planner Framework

For analytical convenience the following definitions are being used:

\[ \alpha^2 A = K, X = \left(1 - p^2 \right) \left(1 - K\lambda^2 p^2 \right) + p \left(1 + K\lambda^2 p^2 \right) \] and Eq. (16) as

\[ F : \frac{\sigma^2}{u_n^2} = \frac{(1 + K\lambda^2)^2 X}{(1 + K\lambda^2 p^2)^3}. \]

1.A. Under uncertainty \((0 < p < 1)\), the expected loss from unemployment is always higher than the expected loss in case there is full information regarding the policymaker.

\[
\frac{A\bar{U}_n^2}{2} \left[ \frac{(1 + 2K\lambda^2 p^2 + K^2\lambda^4 p^2 \sigma^4)}{(1 + K\lambda^2 p^2)^2} \right] > \frac{A\bar{U}_n^2}{2} \left[ (1 + 2K\lambda^2 p^2 + K^2\lambda^4 p^2 \sigma^4) > (1 + K\lambda^2 p^2)^2 \right] = (1 + 2K\lambda^2 p^2 + K^2\lambda^4 p^4)
\]

This inequality exists for any \( p \) lower than one.

2.A. In case that the policymaker is revealed as weak \((p = 0)\), the optimal non-indexed wage proportion is higher the higher is the supply shock variance and the lower are the inflation bias motives \((\bar{u}_n, K)\).

Proof (proposition 1):

\[
\frac{\partial \hat{\lambda}}{\partial \sigma} = \frac{1}{2K\bar{u}_n} \left[ \frac{1}{K} \left( \frac{\sigma}{\bar{u}_n} - 1 \right) \right] \frac{1}{2} > 0;
\]

\[
\frac{\partial \hat{\lambda}}{\partial \bar{u}_n} = - \frac{\sigma}{2K\bar{u}_n^2} \left[ \frac{1}{K} \left( \frac{\sigma}{\bar{u}_n} - 1 \right) \right] \frac{1}{2} < 0; \quad \frac{\partial \hat{\lambda}}{\partial K} = - \frac{\sigma}{2K^2\bar{u}_n} \left[ \frac{1}{K} \left( \frac{\sigma}{\bar{u}_n} - 1 \right) \right] \frac{1}{2} < 0
\]

3.A. The second order condition (S.O.C) for the minimization of Eq. (15) is:

\[
\frac{\sigma^2}{\bar{U}_n^2} > \frac{(1 + K\lambda^2)^3}{2(1 + K\lambda^2 p^2)^4} \left( p^2 + p - 1 \right) \left(1 + K\lambda^2 p^2 \right) - 3X = \left( \frac{\sigma^2}{\bar{U}_n^2} \right)_c
\]

4.A. The derivative of \( \hat{\lambda} \) with respect of the policymaker’s reputation \( p \) is:

\[
\frac{\partial \hat{\lambda}}{\partial p} = \frac{F_p}{F_\lambda} = \frac{(1 + K\lambda^2) \left[ \frac{\partial X}{\partial p} (1 + K\lambda^2 p^2) - 6K\lambda pX \right]}{\left(1 + K\lambda^2 p^2 \right) \left[ 4K\lambda X + (1 + K\lambda^2 \frac{\partial X}{\partial \lambda}) \right] - 6K\lambda^2 X (1 + K\lambda^2)}
\]

Where \( \frac{\partial X}{\partial p} = K\lambda^2 p \left(4p^2 + 3p - 2\right) + 1; \quad \frac{\partial X}{\partial \lambda} = 2K\lambda^2 \left(p^2 + p - 1\right). \)
Derivative values

\[
\frac{\sigma^2}{\mu_n} = 1.5 :
\]

\[
P & \quad 0 & \quad 0.1 & \quad 0.2 & \quad 0.3 & \quad 0.4 & \quad 0.5 & \quad 0.6 & \quad 0.7 & \quad 0.8 \\
K=0.4 & -1.021 & -0.888 & -0.840 & -0.828 & -0.809 & -0.708 & -0.294 & 0.640 & 1.372 \\
K=0.6 & -0.833 & -0.725 & -0.685 & -0.675 & -0.659 & -0.577 & -0.235 & 2.106 & 2.896 \\
K=0.8 & -0.721 & -0.628 & -0.593 & -0.584 & -0.570 & -0.499 & -0.206 & 3.611 & 6.317 \\
K=1 & -0.645 & -0.562 & -0.531 & -0.524 & -0.511 & -0.444 & -0.185 & 5.566 & 11.999 \\
K=1.4 & -0.545 & -0.474 & -0.448 & -0.442 & -0.431 & -0.377 & -0.154 & 14.274 & 147.69 \\
K=2 & -0.456 & -0.397 & -0.375 & -0.369 & -0.361 & -0.316 & -0.128 & 30.629 & 201.54 \\
\]

\[
\frac{\sigma^2}{\mu_n} = 2 : 
\]

\[
P & \quad 0 & \quad 0.1 & \quad 0.2 & \quad 0.3 & \quad 0.4 & \quad 0.5 & \quad 0.6 & \quad 0.7 & \quad 0.8 \\
K=0.4 & -0.875 & -0.593 & -0.385 & -0.165 & 0.133 & 0.346 & 0.548 & 0.640 & 1.236 \\
K=0.6 & -0.709 & -0.484 & -0.314 & -0.134 & 0.145 & 0.819 & 1.431 & 2.106 & 2.896 \\
K=0.8 & -0.614 & -0.419 & -0.272 & -0.116 & 0.126 & 0.709 & 2.246 & 3.611 & 6.317 \\
K=1 & -0.549 & -0.375 & -0.243 & -0.104 & 0.112 & 0.634 & 3.152 & 5.566 & 11.999 \\
K=1.4 & -0.464 & -0.317 & -0.206 & -0.088 & 0.095 & 0.537 & 3.596 & 14.274 & 147.691 \\
K=2 & -0.388 & -0.265 & -0.172 & -0.073 & 0.079 & 0.450 & 19.834 & 32.564 & 201.666 \\
\]

\[
\frac{\sigma^2}{\mu_n} = 2.5 :
\]

\[
P & \quad 0 & \quad 0.1 & \quad 0.2 & \quad 0.3 & \quad 0.4 & \quad 0.5 & \quad 0.6 & \quad 0.7 & \quad 0.8 \\
K=0.4 & -0.881 & -0.605 & -0.380 & -0.122 & 0.133 & 0.346 & 0.548 & 0.640 & 1.236 \\
K=0.6 & -0.669 & -0.366 & -0.109 & 0.206 & 0.565 & 0.940 & 1.431 & 2.106 & 2.896 \\
K=0.8 & -0.579 & -0.317 & -0.094 & 0.178 & 0.688 & 1.442 & 2.246 & 3.611 & 6.317 \\
K=1 & -0.518 & -0.283 & -0.084 & 0.160 & 0.614 & 1.939 & 3.152 & 5.566 & 11.999 \\
K=1.4 & -0.438 & -0.239 & -0.077 & 0.134 & 0.519 & 1.897 & 5.791 & 14.274 & 147.691 \\
K=2 & -0.366 & -0.200 & -0.059 & 0.113 & 0.434 & 1.588 & 19.834 & 32.564 & 201.666 \\
\]

5.A. Sufficient conditions for proposition 3 are:

\[
\frac{d\pi^e}{dp} = \frac{\partial \pi^e}{\partial p} + \frac{\partial \pi^e}{\partial \lambda } \frac{\partial \lambda }{\partial p} > 0 \quad \Rightarrow \quad (1 - p^2)(1 - \alpha^2 \lambda^2 Ap^2) \cdot \left( \frac{F_p}{F_\lambda} \right) > 2 \lambda p(1 + \alpha^2 \lambda^2 A) \\
\frac{dT}{dp} = \frac{\partial T}{\partial p} + \frac{\partial T}{\partial \lambda } \frac{\partial \lambda }{\partial p} > 0 \quad \Rightarrow \quad (1 - p)(1 - \alpha^2 \lambda^2 Ap^2) \cdot \left( \frac{F_p}{F_\lambda} \right) > \lambda \left[ 1 + \alpha^2 \lambda^2 Ap(2 - p) \right]
\]
Appendix – Figures – Social planner’s framework

Figure 1.A - Expected loss from unemployment

Figure 2.A - The expected loss from supply shock Fluctuations
Figure 3.A - The expected loss from inflation

Figure 4.A - The optimal non-indexed wage proportion

Figure 5.A - The optimal non-indexed wage proportion
Figures – Union Framework

Figure 8.A - Expected real wage

Figure 9.A - The loss from supply shock fluctuations

Figure 10.A - The loss from inflation
Figure 11.A - The loss from unemployment

Figure 12.A - The optimal non-indexed wage proportion

Figure 13.A - The optimal non-indexed wage proportion
References


