Wage Distribution and Economic Growth

by

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Abstract

This paper presents the following question: what is the long-run effect of the minimum wage on economic growth? In order to deal with this question, a model that creates a synthesis between labor search theory and endogenous growth theory is constructed.

In the model, the wage distribution, investment in human capital, active production technologies and long-run growth are all determined endogenously. The analysis implies that policies that affect directly the wage distribution such as minimum wage laws, have a non-monotonic effect on economic growth. The positive effect is due to the change in production technologies that creates an incentive to increase investment in human capital. The negative effect is the result of a disproportional reduction of monopsonistic power of firms. This affects negatively the skill premium, causing a reduction in investment in human capital. This negative effect of the minimum wage is the novel result of integrating labor market frictions in an endogenous growth framework. The aggregate effect on growth depends on the structural parameters of the model. The model is flexible enough to analyze also other policies that affect the reservation wage - such as unemployment benefits and negative income taxation.
1 Introduction

What is the long-run macro-economic effect of the minimum wage legislation? The minimum wage is a legal constraint that forces the supply side of the economy to adjust itself. The "text-book" adjustment, at least in competitive markets, is simply to lay off workers with low labor productivity, which results in an immediate reduction in employment and output.

In contradiction with standard economic wisdom, minimum wages are observed in many economies\(^1\), and these economies do not seem to be less efficient or to grow more slowly than others (Cahuc and Michel 1996); Moreover, recent theoretical and empirical studies imply that under some conditions, these economies can grow faster (Askenazy, 2003).

Minimum wages change the relative price of cheap, mostly unskilled, labor. In the long-run, one can think of several ways in which the economy can adjust to this constraint. For example, the reaction can be to increase investment, both in physical and human capital. Firms might invest in new technologies and increase the speed of modernization. In open economies, raising the minimum wage can shut down production in low productivity industries and cause a structural change\(^2\). But what happens to the unemployed workers? Are they left unemployed? Does this affect somehow the next generation of workers? In the long run, workers may increase their schooling level in order to avoid unemployment, and the economy may shift to sectors with higher output-per worker. These adjustments, by both firms and workers, take time; for this reason minimum wages can effect the economy's growth path in the long-run.

In spite of all these relatively trivial hypotheses, almost all empirical research on the minimum wage has focused on its short run impact\(^3\). The standard methodology in this literature is to

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\(^1\) And there is no tendency to abolish them: for example in the UK a minimum wage was reintroduced by the Blair administration at 1998, after it was abolished by the Thatcher administration.

\(^2\) An example for this process is given by the changes in the Israeli industry during the 90's. The rise in the real minimum wage above labor productivity growth, on the top of liberalization policy that reduced trade barriers unilaterally, and opened the economy to imported goods, caused many "traditional" industries (textile, wood, apparel) to stop production.

\(^3\) There are, however, some exceptions to this rule. See the literature review for references.
estimate the minimum wage "treatment effect" by following the wage and employment status of workers that earned that wage or near it immediately before and after a minimum wage rise. This methodology suffers from several disadvantages: it does not test for possible long run effects both on firms and workers, and it rules out "externalities" - the effect of minimum wage policies on workers (or potential workers) who were not exposed to the direct treatment - for example, the effect on high school graduates who have to decide about going to college or entering the job market.

In order to overcome these problems, a labor search model with endogenous growth is developed. The modeling strategy, and the motivation are derived from the following insights:

- **Minimum wages exert an asymmetric pressure on the wage distribution, affecting mostly "weak" workers in the lower part of the wage distribution. Thus, taking the minimum wage question to models with a single wage in equilibrium, is of limited value.**

- **In order to analyze the long-run effects of minimum wage on economic growth one needs to have a joint theory of wage determination, especially (non degenerate) wage distribution, and economic growth.**

In this paper I construct a growth model with imperfections in the labor market. The labor market is based on the search model by Burdett-Mortensen (1998, BM hereafter), and is extended for heterogeneous workers with respect to their level of human capital, and heterogeneous firms with respect to their production technologies. Investment in human capital is endogenous, and it is assumed that the level of investment in human capital in the economy, which equals the share of skilled workers in the workforce, affects positively the growth rate of both the skilled and the unskilled workers sector.

The model suggests that the effect of minimum wage on economic growth is non-monotonic, and depends on the initial labor market equilibrium, which is a function of the model’s structural parameters.
In economies with substantial labor market frictions the effect of imposing a minimum wage is positive; in more competitive economies it might hamper economic growth.

The reason for the positive effect of the minimum wage is that in less competitive economies, inefficient firms that use unskilled labor can survive in equilibrium, due to labor market frictions. Raising the minimum wage can therefore speed modernization in the economy. In the new economy unskilled workers face a higher unemployment rate, so in order to avoid unemployment, workers increase their investment in human capital. Thus, both workers and firms make adjustments to the minimum wage rise.

In more competitive economies, however, the information structure is such that production is already efficient, so raising the minimum wage (up to a point) reduces the monopsonistic power of firms, and raises wages throughout the wage distribution, but disproportionately in the lower part of it. This causes a decrease in the skill premium, so workers reduce their investment in human capital, and therefore reduces long run growth.

This negative effect of the minimum wage is the novel result of integrating labor market frictions in an endogenous growth framework, while in the existing endogenous growth literature the effect of minimum wages on economic growth is unambiguously positive\(^4\).

The model predictions are in-line with the recent empirical investigation of Cukierman, Rama and van Ours (2001) - who found, using "Barro type" growth regressions in a country panel, that the minimum wage increases growth in low-income economies, while in developed economies the effect is negative\(^5\).

In the next section a short literature review relates the model to previous work. The theory section includes three parts: first a static model with exogenous "2x2" worker and firm heterogene-

\(^4\) This result depends on the production function (the elasticity of substitution between skilled and unskilled labor), and holds for low and high substitutability. In case of perfect substitutability, however, minimum wage has a negative effect on growth (see Askenazy (2003)).

\(^5\) These regressions, as all the literature on which they are based, suffer from possible endogeneity. The reason is that single growth equations are estimated, while in fact some of the LHS variables may be a function of current and expected economic growth. The solution to this problem is usually to use lagged variables; This solution might be insufficient in case the determination of these variable (especially, in case it is a policy variable) is base on forward-looking rules.
ity is presented, with its possible equilibria. Next, the BM (1998) model is extended to include exogenous growth. The result, summarized in proposition 1, is that there is a simple wage contract (though not unique) that preserves the BM equilibrium at every point in time.

In the last subsection, an endogenous growth model where the decision on human capital accumulation is a function of the equilibrium in the labor market is presented. Using this model, I examine the long run effects of the minimum wage on economic growth.

2 Literature Review

The review is divided into two parts. The first part surveys the literature which is related directly to my research question. The second part reviews relevant labor search theory, which contains the building blocks of the model, and in addition, relevant growth literature.

2.1 Literature on the Long-Run Effects of the Minimum Wage

Retrospection of the minimum wage literature that has been written during the last two decades reveals a striking finding: while there is an abundant literature on the short-run impacts of minimum wage, which includes hundreds (!) of papers, there are only a handful of papers that question, theoretically, or try to estimate, empirically, the long-run effects of minimum wages.

These exceptions include: Flug and Galor (1986), who analyzed the long run effects of the minimum wage on international trade; Three theoretical endogenous growth papers that are based on competitive labor markets- Cahuc and Michel (1996), Weiss (1996) and Askenazy (2003), where some empirical evidence is reported too; And two "pure" empirical papers - Neumark and Nizalova (2004), and Cukierman, Rama & van Ours (2001). Below, these papers are reviewed in some detail.

Endogenous growth models: The connection between minimum wage and growth has been addressed recently in three endogenous growth papers. The first, Cahuc and Michel (1996), which is more related to this work, deals with the human capital adjustment to a change in the minimum

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6 The bulk of the literature was stimulated by the initial findings of Card and Krueger (1995). Their results, that the minimum wage increase did not have a negative effect on employment were re-examined in many follow-up papers, and the debate on this issue is still ongoing.
wage. They show, using an overlapping generations model, that in a competitive labor market raising the minimum wage might cause workers to invest more in education, in order to avoid unemployment. In their model the minimum wage is set above the marginal product of unskilled labor, so this sector disappears in the long run. The model is based on Flug and Galor (1986), who showed that minimum wage can increase the long-run ratio of the skilled to unskilled in a general equilibrium model of international trade.

A different approach is to model the reaction of production to a minimum wage change leaving labor supply unchanged. Askenazy (2003) shows that raising the minimum wage reduces the relative price of research and development, so skilled workers leave the production sector for the R&D sector\(^7\), which leads to a higher growth path, as this sector has positive externalities.

As argued before, there is a common factor to these models: a perfect, competitive labor market, with a degenerated wage distribution: workers with the same skill/human capital level earn identical wages, and these wages equal their marginal product of labor\(^8\).

The third paper listed above - Weiss (1996) - is an endogenous growth paper that models the human capital (positive) spillover mechanism which is assumed to be at work in many endogenous growth models (including this one). In the model, this spillover is a function of labor mobility, as workers can learn from their co-workers. As a result, policies which reduce labor mobility, such as the minimum wage, can impede economic growth. This conclusion, however, holds in case of a differential minimum wage (especially, when the minimum wage in the advanced sector is higher), a policy which is conducted in some developing economies\(^9\).

**Empirical evidence:** As argued previously, almost all empirical work on the minimum wage issue has focused on contemporaneous effects. The standard methodology in this literature is to

\(^7\) In case it is possible to trade the "excess R&D" for goods via international trade, and under some assumptions on the production function.

\(^8\) In Aghion (2002) there is a random element in wages. Yet, similar workers with respect to their histories, will earn identical wages.

\(^9\) A sectorial minimum wage policy is conducted also in Australia, where sectors on which the relative wage is high are exempted.
estimate the "treatment effect" of the minimum wage on workers who were exposed to it, using partial equilibrium models. This strand includes for example Card and Krueger (1995) and many follow-up papers that were motivated by their findings.

There are, however, two exceptions. Neumark and Nizalova (2004), question whether the minimum wage effect lasts, by following workers for long periods of time. Their results are that workers who were "treated" with a minimum wage have a lower lifetime earnings path; They work less and earn less the longer they were exposed to a higher minimum wage, especially as teenagers. They speculate that the reason might be that higher minimum wages might affect negatively school enrolment, an effect that was tested directly and was found to be significant in previous work by Neumark (1995). However, one might suspect that minimum wage earners are simply low-ability workers, and hence, there is no causal relation between minimum wage and lifetime earnings.

The minimum wage effect in the long run was estimated also based on macro-data. Cukierman, Rama and van Ours (2001), who ran "Barro-type" growth regressions on a country panel\textsuperscript{10}, found some evidence for non-monotonic effects of the minimum wage on growth rates, where in poor countries high minimum wage raises increases growth, while in richer countries the effect on long run growth is negative.

\subsection{2.2 Relevant Labor Search Theory}

Labor search theory is a "natural" candidate for analyzing policies such as the minimum wage, since it is a general equilibrium framework that allows for policy evaluation that is not subject to the Lucas critique (see Eckstein and Van Den Berg (2003)).

The aim of this section is not to give a wide, general review on labor search theory, but to cover only relevant literature on which the model is based: the basic BM (1998) model, labor search with heterogeneous workers (Bowls and Eckstein (2002)), labor search theory with heterogeneous firms

\textsuperscript{10} Analyzing the effect of policy variables using this framework might be problematic. See the detailed drawback in the introduction.
(Van Den Berg (2003)), and labor search theory or other non-competitive labor market theory that is related to growth.

Economic growth theory and non-competitive labor market theory are generally two strands in economic literature that evolved independently without intersection. On the one hand, almost all existing endogenous growth theory has been based on competitive labor markets\(^{11}\), and on the other hand non-competitive labor theory has been based on fixed productivity over time\(^{12}\).

The exception to this rule, and the most related paper to my work from this perspective, is the model by Laing, Palivos and Wang (1995, LPW hereafter) that introduces labor market frictions in the lines of Pissarides (2000) into an endogenous growth model in the lines of Romer (1986). LPW show that with endogenous schooling effort multiple equilibria may exist: an economy with low investment in human capital, "thin" labor market (relatively low number of vacancies) and low growth, and an economy with high investment in human capital, "thick" labor market and a high growth rate. This happens because both employers and employees have to invest, in opening vacancies and in schooling, respectively, in order to be in the "good" equilibrium. Thus, the economy is exposed to what is generally known in the game theory literature as a "coordination failure". This result was also demonstrated by Manning (2003) in a static "ghetto" model\(^{13}\).

However, the LPW model uses the matching/bargaining framework of Pissarides (2000) that generates a single wage in equilibrium, and for this reason it is not built to analyze asymmetric intervention in the wage distribution. Thus using it as is without substantial modifications is of limited value\(^{14}\).

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\(^{11}\) Although some of endogenous growth theory is based on a non-competitive goods market.

\(^{12}\) This is not to say that search models do not have dynamic implications, on the contrary: search models generate a dynamic selection process of work, and deal with the durations of unemployment and employment spells. However, production technology in these models is static so there is no long-run growth. For an updated survey on empirical search models see Eckstein and Van Den Berg (2003).

\(^{13}\) That is, sections in the city where there are no jobs and investment in human capital is low. See section 3.6 pp. 66 there.

\(^{14}\) The minimum wage changes the threat point in a bargaining model so it raises the wage in equilibrium, causing firms to open fewer vacancies and therefore increases unemployment. Yet, the rational of using a single wage model to analyze a question which has implications on the wage distribution is problematic. Shimer (2004) shows how a bargaining model with on the job transitions can generate a non-degenerate wage distribution.
Three other papers which are directly related to this work are the models by Bunzel, Christensen, Kiefer and Korsholm (2000), Mortensen (2000) and Flinn (2004). In the first model, the basic BM (1998) model is extended to allow for on-the-job (exogenous) productivity growth, as a result of accumulation of experience. Thus, the model generates endogenous productivity differences between firms, but still, in a static production environment in the long run, as productivity gains are lost once the employer-employee match is dissolved.

Mortensen (2000) extends the basic BM model by allowing firms to complement each employer-employee match with investment in physical capital. Again, this results in endogenous productivity differences between firms\(^\text{15}\). An interesting result in this case is that a minimum wage can increase output, as it reduces the job turnover in the market which is inefficiently large\(^\text{16}\).

Flinn (2004) analyses the minimum wage effect on labor market outcomes and welfare using a bargaining model. His results are that although minimum wage increases unemployment, it can be welfare improving because it increases the bargaining power of workers. His estimates, using CPS data, indicate that the 1996 minimum wage increase was probably welfare improving.

The main building block of the model presented in this paper is the BM (1998) model. This model generates an endogenous, non-degenerate wage distribution, and equilibrium unemployment, even when workers and firms are homogeneous. This model was extended by Van Den Berg (2003) who solved the BM model with firm heterogeneity, and by Bowlus and Eckstein (2002) who solved the model for different (observed) worker types. Altogether, theses three models form the basis for the static model presented here.

In basic search models (Albrecht and Axell (1984), BM (1998)) a peculiar result emerges: there is a “free lunch” element in raising the minimum wage, as acknowledged by Eckstein and Wolpin (1990), Manning (2003) and others. Generally, this result is due to the dichotomy between

\(^{15}\) These models correct for the basic problem in the BM (1998) model: an increasing wage density function, unlike real life wage densities. The solution to this problem in many empirical papers is to assume exogenous differences in firm productivity.

\(^{16}\) This result is somehow related to the “efficiency wage” literature, where paying above the market wage may be optimal since it reduces the quit rate. However, this result holds for the firm level.
wages and unemployment, where raising the minimum wage changes only labor share, leaving unemployment unchanged. This led Manning to the conclusion that the basic BM (1998) model can not provide an adequate answer on the issue of optimal minimum wage, and in order to use it, one needs to extend this model by endogenizing decisions of firms or workers so that they can be affected by incentives. Manning (2003) also presents a comprehensive analysis of minimum wage efficiency in search models. His analysis implies that although in partial equilibrium models of monopsonistic competition a minimum wage can be optimal, in general equilibrium search theory models a "strong" conclusion is usually the result of a model which is not rich enough, so it can be reversed easily by changing assumptions about worker heterogeneity, entry costs of firms and workers, or endogenous recruitment activity. Although the unconstrained equilibrium is almost always inefficient, the direction of this inefficiency is not clear so a minimum wage intervention may not be desirable. Thus, as claimed by Manning, "theory alone can be of little use in evaluating policy".

Following Manning's (2003) general conclusion, in this model, workers' decisions on human capital accumulation are endogenized.

3 Theory

In the section below an extended wage posting model based on the BM (1998) model is presented. The extension allows for heterogeneity of both workers and firms, and for endogenous information frictions. First, a static model with exogenous worker and firm heterogeneity is presented. In the next subsection, exogenous growth is introduced into the BM model. Next, these two building blocks are combined to form a joint search model with endogenous human capital and economic growth. In the last subsection, the conclusions from the theoretical model are summarized.

3.1 A 2*2 Static Model

The model is a wage posting model in the lines of BM. Firms offer wages, and workers, both unemployed and on-the-job, get wage offers randomly at the rate which is a function of the
number of active firms in the market. The firms make “take it or leave it” offers to workers; there is no bargaining. The workers human capital/skill/education level is fully observed by firms, so firm offers are conditional on worker type. This basic framework is extended for exogenous worker and firm (minimal) heterogeneity. The result is a 2*2 model with high/low productivity firms and skilled/unskilled workers. The model combines elements from Van Den Berg (2003) who solved the BM model with firm heterogeneity, and Bowlus and Eckstein (2002) who solved the model for different (observed) worker types.

The wage offer distribution in equilibrium is described in a set of candidate equilibria.

3.1.1 Workers

There are skilled and unskilled workers denoted by $e_h$ and $e_l$ respectively. The the labor supply, that is, the proportion of skilled and unskilled workers in the labor force is fixed and exogenous $s_h + s_l = 1$. The skill level of the workers is fully observed by firms, and in addition, labor productivity of skilled workers is higher. As a result, firms wage offers are conditioned on the skill level of workers (Mortensen (1990)).

Skilled and unskilled workers, when unemployed, receive job offers that are drawn randomly from a conditional wage distribution $F_i(w)$, at rates $\lambda_{i0}^h$ and $\lambda_{i0}^l$, respectively, and at rates $\lambda_{i1}^h$ and $\lambda_{i1}^l$ when employed. These rates are a function of the number of active firms in the market. A common, constant, exogenous separation rate $\delta$ returns the workers to unemployment. In this setting, the optimal behavior of workers is given by a reservation wage when unemployed, and a rule for on-the-job transition. The reservation wage is solved by equating the asset value of unemployment with the asset value of employment at the reservation wage:

$$V_i^{U} = V_i^E(\phi_i)$$  \hspace{1cm} (1)

where
\[ rV_t^{U} = b_t + \lambda_t^0 \max(0, V_t^E(w_t) - V_t^{U}) \] (2)

\[ rV_t^E(w_t) = w + \lambda_t^1 \max(V_t^E(w_t), V_t^E(u_t^P/w_t^P > w_t) - V_t^E(w_t)) + \delta(V_t^{U} - V_t^E(w_t)) \] (3)

The reservation wage for unemployed workers is given by:

\[ \phi_t = b_t + (\lambda_t^0 - \lambda_t^1) \int_{\phi_t}^{w_t^{max}} \frac{1 - F_t(w)}{\delta + \lambda_t^1 (1 - F_t(w))} dw_t \] (4)

\( F_t(w) \) is the conditional wage offer distribution in equilibrium and \( b_t \) is the opportunity cost of employment.

Employed workers take the higher wage - they move to a new job when the wage offer is higher than their current wage.

### 3.1.2 Firms

There are high and low technology firms denoted by \( p_h \) and \( p_l \), where there measure is normalized to one\(^{17}\). Assume that a fraction \( q \) of these firms are of type \( p_h \). Both types can employ skilled (\( \epsilon_h \)) and unskilled workers (\( \epsilon_l \)). The value of the employer-employee match is a function of the skill level of the worker and the technological level of the firm (Postel-Vinay and Robin, 2002), and is given by:

\[ m_{i,j} = f(\epsilon_i, p_j) = \epsilon_i \ast p_j \] (5)

The production function is linear in labor and separable for skilled and unskilled workers. It is assumed that \( m_{i,h} > m_{h,t} > m_{t,h} > m_{t,l} \) ("worker dominance").

\(^{17}\) The number of firms in this version is fixed. Manning (2003) extends the basic BM (1998) for endogenous firm entry, where there are fixed entry costs. For simplification, the model presented here ignores these issues and is based on a simple CRS (Ricardian) production function.
The profit flow of high productivity firms is given by:

\[ \Pi_h = (m_{h,h} - w_{h,h})l(w_h/F_h) + (m_{l,h} - w_{l,h})l(w_l/F_l) \]  

(6)

and profit flow of low productivity firms is given by:

\[ \Pi_l = (m_{h,l} - w_{h,l})l(w_l/F_h) + (m_{l,l} - w_{l,l})l(w_l/F_l) \]  

(7)

### 3.1.3 Information

The arrival rates of the job offers are a function of the measure of active firms in the market for skill level \( \iota \). Consider a contact function\(^{18}\) \( M(l_{\iota}, n_{\iota}) \) that takes the measure of active firms and the measure of workers as arguments\(^{19}\) and generates a flow of worker-employee contacts. Assume that \( M(., .) \) is satisfies \( \frac{\partial m}{\partial u_i} > 0, \frac{\partial m}{\partial n_i} > 0, \frac{\partial^2 m}{\partial u_i^2} < 0, \frac{\partial^2 m}{\partial n_i^2} < 0. \)

\[ \lambda^0_{\iota} = \frac{M(u_i, n_{\iota})}{u_i} \]  

(8)

\[ \lambda^1_{\iota} = \frac{M(1 - u_i, n_{\iota})}{1 - u_i} \]  

(9)

Where the measure of active firms \( n_{\iota} \), is normalized to 1 in case production is carried out by low productivity and high productivity firms.

This information structure differs from Van Den Berg (2003), where the ratio \( \lambda^0_{\iota}/\lambda^1_{\iota} \) is fixed and exogenous. Here this ratio is determined in equilibrium.

Note that the endogenous arrival rates are an indirect function of the measure of workers in each sector as this affects the equilibrium unemployment rate (equation(10)). This will become important later on when the decision on human capital level is endogenous.

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\(^{18}\) Usually the used term is "matching function", however, as mentioned by Van Den Berg (2003) in search models, unlike bargaining models "contact function" is a more accurate term as some of the job offers are rejected.

\(^{19}\) Usually, matching functions take the number of unemployed workers and the number of vacancies as inputs. In the case here all the firms have vacancies.

\(^{20}\) This assumption is not trivial. When the number of active firms decline, they are easier to locate. We assume here that this positive effect is weaker than the "vacancy" effect.
3.1.4 Equilibria in the Skilled and Unskilled Worker Sectors

The model above can have several equilibria. In order to see that, note first that production functions are additive in worker types and that firms set type specific wage offers in separate markets. This implies that equilibrium can be solved as if the workers were in separate markets (Bowlus and Eckstein (2002)). This means that each of the sectors - skilled and unskilled workers is a separate market that has high and low productivity firms. Since the separability of the markets reduces the problem of firm heterogeneity with homogenous workers, which was solved by Van Den Berg (2003), we use his results to describe possible equilibria in each market. The equilibrium in the whole economy (for both markets) is derived by combinations of the separate markets equilibria.

The equilibrium definition is based on BM (1998), and is extended for firm heterogeneity and for endogenous job offer arrival rates.

Equilibrium: for each sector (skilled and unskilled), equilibrium is a triplet \(( F_i^s(w^s), \phi_s, \Pi^s) \) and arrival rates \( (\lambda_i^0, \lambda_i^1) \) that satisfies:

1. The system is at steady state; and,
2. The reservation wage is optimal (utility maximizing); and,
3. The profit flows are identical for firms with the same productivity level.

From the first condition we get:

\[
  u_i = \frac{\delta S_i}{\delta + \lambda_i^1}
\]

which implies the inflow and outflow from employment are equal and,

\[
  G_i(w_i) = \frac{\delta F_i(w_i)}{\delta + \lambda_i^1 (1 - F_i(w_i))}
\]

that is, the inflow and outflow from every cross section of workers with a wage up to \( w \) are equal where \( G_i(w_i) \) is the actual wage distribution.
From the second condition we get equation (4). From the third condition we get the equilibrium wage offer distribution $F_i(w_i)$ in equilibrium. Next, we describe several candidate equilibria and the conditions for existence for each.

**Only High Productivity Firms**  In this equilibrium, low productivity firms can not survive, that is, they make a negative profit. Thus, the condition for the existence of this equilibrium is simply given by a reservation wage that is higher than the value of the employer-employee match: $\phi_i > s_i \cdot b_i = m_{ii}$.

If there are only homogeneous firms in equilibrium, the wage offer distribution is identical to BM (1998) solution:

$$F_i^h(w_i) = \frac{\delta + \lambda_i^1}{\lambda_i^1} \left( 1 - \sqrt{\frac{m_i^0 - w_i}{m_i^0 - \phi_i}} \right)$$  \hspace{1cm} (12)

and by substituting (12) in (4) we get the reservation wage in this case,

$$\phi_i = \frac{(\delta + \lambda_i^1)^2 b + (\lambda_i^0 - \lambda_i^1) \lambda_i^1 m_{ih}}{(\delta + \lambda_i^1)^2 + (\lambda_i^0 - \lambda_i^1) \lambda_i^1} > m_{ii}$$  \hspace{1cm} (13)

and the explicit condition for equilibrium is given by:

$$\left( \lambda_i^0 - \lambda_i^1 \right) \lambda_i^1 (m_{ih} - m_{ii}) > (\delta + \lambda_i^1)^2 (m_{ii} - b)$$  \hspace{1cm} (14)

The intuition is simple: if $m_{ii} > b$ low productivity firms will survive unless $\lambda_0 > \lambda_1$ and the technological difference between the firms is big enough. Note that if $\lambda_i^0 \leq \lambda_i^1$ (and $m_{ii} > b$ as before) this equilibrium can not exist since low productivity firms will have an incentive to enter the market. The reason is that if employed workers are indifferent, or have an "information advantage" when employed they will take low-paying jobs - lower even than the opportunity cost of employment.
**High and Low Productivity Firms** In equilibrium, the support of the wage offer distribution consists of two adjacent parts where the low productivity firms offer lower wages. The wage offer distribution for low productivity firms is

\[ F^l_i(w) = \frac{\delta + \lambda_i^1}{\lambda_i^1} \left( 1 - \sqrt{m_i^l - w_i} \right) \quad w_i \in [\phi_i, \bar{w}] \]  

and for high tech firms,

\[ F^h_i(w) = \frac{\delta + \lambda_i^1}{\lambda_i^1} \left( 1 - \frac{\delta + \lambda_i^1 q}{\delta + \lambda_i^1} \sqrt{m_i^h - w_i} \right) \quad w_i \in [\bar{w}_i, \hat{w}_i] \]

where \( q \) is the fraction of high productivity firms.

Again, the reservation wage is derived by substituting (15) and (16) in the reservation wage equation (4):

\[ \phi_i = \frac{(\delta + \lambda_i^1)^2 b + (\lambda_i^0 - \lambda_i^1) \lambda_i^1 (cm_{ih} + (1-c) m_{il})}{(\delta + \lambda_i^1)^2 + (\lambda_i^0 - \lambda_i^1) \lambda_i^1} > m_{il} \]  

The value of the match is replaced by a weighted average where

\[ c_i = \left( \frac{\delta + \lambda_i^1 q}{\delta + \lambda_i^1} \right)^2 \]

and by imposing the condition \( \phi_i < e_i * p_i = m_{il} \) we get

\[ (\lambda_i^0 - \lambda_i^1) \lambda_i^1 c (m_{ih} - m_{il}) \leq (\delta + \lambda_i^1)^2 (m_{il} - b) \]

Note that the RHS and LHS in this condition are different because the endogenous arrival rates are different.

**Multiplicity** As shown by Van Den Berg (2003), in case \( \lambda_i^0 > \lambda_i^1 \), \( m_{il} > b \) and the productivity difference between the firms is not "big enough" there is a range of parameters for which the
intersection of (14) and (19) is not empty, thus multiple equilibria may occur - the model supports both types of equilibria simultaneously. For the time being, we ignore this case.

3.1.5 Equilibria in the Economy

Due to the assumption that labor markets are completely separable, the set of equilibria in the whole economy is composed of all combinations of equilibria in each sector ($E_k, E_h$).

**Equilibrium A**  Low productivity and high productivity firms produce with skilled and unskilled labor: in this case condition (19) is fulfilled for both the skilled and unskilled sectors. Note that the reservation wage for the skilled workers is not necessary higher than the one for unskilled workers, however, if the arrival rates are identical in both sectors, and if $\lambda^0_i > \lambda^1_i$ this is the case. If $\lambda^0_i = \lambda^1_i$ the reservation of both types is equal to the common opportunity cost $b$. It is also possible that the reservation wage of skilled workers will be lower than for unskilled.

**Equilibrium B**  Low productivity firms produce only with skilled labor, high productivity firms with both. The condition for this equilibrium is $\phi_l > m_H$, and condition (19) for the skilled

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21 Since the information structure differs from Van Den Berg (2003), proposition 1 there does not hold.
workers sector.

**Equilibrium C** Only high productivity firms survive, low productivity firms are inactive. Condition (14) is met for both sectors. This happens if the information frictions in the market are not big enough to "support" the productivity gap.

**Equilibrium D** Only high productivity firms for skilled workers. Low and high productivity firms in the unskilled sector.

**Equilibrium E** No trade equilibrium in the unskilled sector. This happens if the opportunity cost of employment is higher than the value of the match between high productivity firms and unskilled workers. As a result, $\lambda^0_q = 0$ (there are no job offers for unemployed, unskilled workers) so the unskilled are all unemployed. Again, the skilled workers sector can be heterogeneous (E.1) or homogeneous (E.2).
Figure 3: Wage offer densities for equilibrium C

Figure 4: Wage offer densities for equilibrium E.1
3.2 Exogenous Growth

The BM (1998) model describes the wage distribution in equilibrium when labor productivity is fixed. We now introduce exogenous growth to the basic version of the model, with homogenous firms and workers, and exogenous job offer arrival rates. The aim is to find a wage contract that maintains the "BM equilibrium" properties.

Assume that the economy is in a "static" BM equilibrium, where the value of the employer employee match is denoted by $p_0$, then as a result of a technological shock, productivity starts to grow in a constant rate $\dot{p} = \gamma$. In addition, assume that $\dot{h} = \gamma$, which can be justified if unemployment benefits or other transfer payments for unemployed are indexed to productivity growth, which is, for example, the case in Israel.

The proposition below describes a wage contract that ties the level of productivity to wages. This wage contract ensures that the economy is in a "BM type" equilibrium during the growth process.

**Proposition 1** The triplet $[\phi_t, F_t(w_t), \Pi_t]$, where $\dot{w} = \gamma = \dot{p}$, and $w_t = w_0 \exp(\gamma t)$ constructs a "BM equilibrium" at every point in time.

**Proof.** In order to prove the proposition we need to show that the wage contract above matches the BM equilibrium for any time interval. Recall that conditions are: 1. Steady state; 2. Optimal reservation wage; 3. Equal profits of firms. As to the first condition, it does not depend on productivity or wages (see equation(10)). The optimal reservation wage is simply given by (4).

We have to prove that in this case firms, which are different with respect to the wage they pay and their size, make equal profits. The discounted flow of profits of firms up to time $T$ is given by

$$\int_0^T \Pi_t \exp(-\rho t) dt = \int_0^T (p_t - w_t) h_t(w_t) \exp(-\rho t) dt$$

Where $h_t(w_t)$ is the labor supply for firms that pay a wage $w_t$. Assume that all wages are indexed to productivity growth as proposed. First, since all firms follow the wage indexation rule, note that for all $w_0$ and time interval $T$, $F_0(w_0) = F_T(w_T)$. 19
Next, note that $\gamma = \gamma$ since the reservation wage is linear with respect to productivity (equation (13) and (17)). The labor supply to an employer setting a wage $w$ is:

$$l_t(w/F) = \frac{(1 - u)\delta(\delta + \gamma^1)}{(\delta + \gamma^1(1 - F^1_t(w_t)))^2}$$

since the CDF for any indexed wage is constant $l_0(w_0) = l_t(w_t)$. Now, we can express the present value of profits, in terms of the original BM equilibrium at time $t_0$:

$$(p_0 - w_0)l_0(w_0) \int_0^T \exp(\gamma t - \rho t) dt = (p_0 - w_0)l_0(w_0) * cons$$

for each $(w^i, w^j)$ that are in the support of the distribution of actual wages $G_0(w_0)$

$$\forall w^i, w^j \ni [G_0(w_0)] \int_0^T \Pi_t(w_t^i) dt = \int_0^T \Pi_t(w_t^j) dt$$

Which is the exactly the condition for equilibrium. ■

The implication of indexing the to productivity growth is that the labor share $w_t/p_t$ is constant over time in each firm, and in the economy as a whole. This result is intuitive, because the labor share reflects the monopsonistic power of firms, which is a function of the information frictions in the market ($\lambda$) and is time invariant.

A note on multiplicity: The wage contract presented here, where each firm indexes wages to productivity growth, is not the unique equilibrium outcome. In fact, as a direct result of indeterminacy in the firm level in the BM model, there are infinite number of equilibria.

3.3 Endogenous Human Capital and Growth

In this section the decision weather to invest in human capital and become skilled or to remain unskilled is endogenized. Workers make an optimal decision based on their lifetime earnings and the cost of investment in human capital, which is a negative function of their ability.

The endogenous growth of the economy is based on the following notion: the level of investment in human capital in the economy, measured by the share of skilled workers in the workforce, affects positively the (common) growth rate in both sectors of the economy. The justification to this
assumption is the existence of positive externalities of human capital on economic growth. In this context, one can think of the high skilled sector as an R&D sector, which creates also technologies that are used by the unskilled workers. Thus, the whole economy benefits from the development of knowledge. This assumption is relatively standard in endogenous growth theory (see Romer (1986), Cahuc and Michel (1996)).

The analysis is based on the result in the previous section: indexing wages and wage offers to labor productivity keeps the economy in a BM equilibrium.

The fact that both sectors grow at the same rate implies that in steady state, the skill premium (the relative wage of skilled workers) is constant. Additional assumptions on the cost of education that are specified below ensure that in steady state the share of skilled workers in the workforce is also fixed over time.

Below, a set of assumptions on which the analysis is based on are detailed.

- Workers: assume that workers are infinitely lived, and are heterogeneous with respect to their ability \( q_i \), which is distributed continuously in the population according to some distribution function in the interval \( (q_l, q_h) \).

- Growth: assume that the growth rate of labor productivity, which is common for skilled and unskilled is given by \( \gamma(s_h) \), where \( s_h \) is the proportion of skilled workers in the workforce, and \( \frac{d\gamma(s_h)}{s_h} > 0, \frac{d^2\gamma(s_h)}{s_h^2} < 0 \)

- Time preferences: assume that workers have a discount rate \( \delta \), and that \( \delta > cons > \gamma(1) \), that is, even if all workers invest in education, the present discounted value of workers is bounded away from infinity.

- Opportunity cost of employment: assume that \( \hat{b} = \gamma \).

- Wages: assume that \( \hat{w} = \gamma \) - wages are indexed to productivity growth, a contract which is an equilibrium contract as shown in the previous section.
• Cost function: The cost of investment in human capital for worker \( a_t \) at time \( t \) is given by:

\[
C_t(a_t) = f(a_t) + w_t^e
\]  

(20)

Where \( f(a) \) is decreasing in \( a \), \( \frac{df(a)}{da} < 0 \), and is a time invariant function that reflects the negative relation between ability and the cost of education, and \( w_t^e \) is the average wage of skilled workers at \( t \)\(^{22}\).

• Let the present discounted values for skilled and unskilled workers be \( V_t(e_h) \) and \( V_t(e_u) \), respectively. Assume that the in the interval \((a_u, a_h)\), there is a "cut-off type" worker, denoted by \( a^* \), who is just indifferent between investing and not investing in human capital,\(^{23}\) so that: \( V_t(e_u) < V_t(e_h) - C_t(a_h) \) and \( V_t(e_u) > V_t(e_h) - C_t(a_u) \).

• Multiplicity: assume that the relative labor productivity of skilled workers is big enough so that multiple equilibria can not exist (see note on multiplicity).

The original BM equilibrium definition is valid for a static economy with respect to labor productivity. In an endogenous growth economy, workers maximize their utility not only with respect to their reservation wage, but also with respect to their investment in human capital. The formal definition of the equilibrium in an economy with endogenous growth is given by:

**Definition:** Equilibrium with endogenous growth is a wage offer distribution, reservation wage and profit flow \((F_t(w^h_t), \phi, \Pi)\), arrival rates \((\lambda^0_t, \lambda^1_t)\), and a growth rate \( \gamma(s_h) \) such that the following conditions hold:

1. The system is at steady state;
2. The reservation wage is optimal (utility maximizing);

\(^{22}\) The cost function can be thought as a multiplication of the quantity of education and its price. The quantity is time invariant, and the price is indexed to the wage of skilled workers. This indexation is a short cut to a model with an education sector, where the "teachers" are skilled, so they have to be paid like skilled workers.

\(^{23}\) The assumption here is that \( a^* \) exists, so there are no corner solutions where all individuals choose to invest in human capital or remain unskilled.
3. The profit flows are equal for identical firms;

4. Investment in human capital is optimal (utility maximizing);

A worker at time $t$ will choose to become skilled only if the present discounted value of being skilled, denoted by $V_t(e_h)$, minus the cost of investment, is higher than the present discounted value of remaining skilled, denoted by $V(e_l)$. By assumption, ability is distributed in an interval around $a^*$, the indifferent worker. For this worker, the following condition holds:

$$V_t(e_l) = V_t(e_h) - C_t(a^*)$$  \hspace{1cm} (21)

Note that all the terms in (21) grow at the same rate $\gamma(s_h)$, hence, $a^*$ is time-invariant. The present discounted values $V_t(e_l)$ and $V_t(e_h)$ are given by:

$$V_t(e_l) = \int_{t}^{\infty} \left[ u^t b^t \gamma^t + (1 - u^t) E_t(u^t) \right] \exp(-\rho\tau) d\tau =$$

$$= \int_{t}^{\infty} \left[ u^t b^t \gamma^t + (1 - u^t) E_t(u^t) \right] \exp(\gamma(s_h)\tau - \rho\tau) d\tau = \frac{u^t b^t \gamma^t + (1 - u^t) E_t(u^t)}{\rho - \gamma(s_h)}$$  \hspace{1cm} (22)

The worker is able to make an optimal decision on $t$ based on the expected value at $t$, which is a function of the probability of employment $(1 - u^t)$, the expected wage given the worker is employed $E_t(u^t)$, and the parameters - the opportunity cost of employment $b^t$, the discount rate $\rho$ and the growth rate which the workers takes as given. From the workers point of view, all the factors that determine the wage except education are exogenous. These include the firm’s productivity and the dynamic wage contract.

Intuitively, in equilibrium, low ability workers who face high cost of investment in human capital, will remain unskilled. Substituting the present discounted values (22) of each state in equation (21) we can rewrite it in present terms:

$$\frac{u^t b^t \gamma^t + (1 - u^t) E_t(u^t)}{\rho - \gamma(s_h)} - C_t(a^*) = \frac{u^t b^t \gamma^t + (1 - u^t) E_t(u^t)}{\rho - \gamma(s_h)}$$  \hspace{1cm} (23)
where $s_h$, the share of unskilled workers, is the sum of workers for whom $a^* \leq a_i$, divided by the total number of workers in the workforce.

**Proposition 2** In equilibrium, there is a single steady state growth rate.

**Proof.** The additional equilibrium condition requires optimal investment in human capital. By assumption, there is a "marginal" skilled worker, $a^* = \min(a_i), \forall(a_i) \subset s_h$ who is just indifferent between investing and not investing in human capital. For this worker, the following condition holds: $V(e_h) = V(e_{h}) - C_t(a^*)$. This worker divides the labor force and determines the share of skilled workers $s_h$ and the growth rate $\gamma(s_h)$. Note that investment in human capital by workers with $a < a^*$, or remaining unskilled for workers with $a > a^*$ is not optimal, for any value of $\gamma(s_h)$, so there is no other equilibrium which can be supported by the optimal investment rule, given $V(e_h)$ and $V(e_{q})$.

The reason for this result is that wage growth is common for skilled and unskilled workers, so the skill premium - the relative wage of skilled workers - is fixed over time.

### 3.3.1 Minimum Wage and Steady State Growth

We now turn to check the implications of imposing a minimum wage on the economy. The main result is summarized by the following proposition:

**Proposition 3** the effect of minimum wage on economic growth is ambiguous. The imposition of a minimum wage on the economy might increase or decrease steady state growth. The effect depends on the equilibrium type in the non-constrained economy, and in the minimum wage level with respect to other model parameters.

**Proof.** To prove the proposition, three examples are given. The implications for steady state growth are negative in example 1 and positive in example 2. In example 3 the implications are ambiguous.

**Example 1:** assume that the unconstrained economy is in "type C" equilibrium at $t_0$. In this equilibrium, only high productivity firms survive. Next, a minimum wage such that, $\phi_h < MW < \phi_h$, and that $MW < m_wh$ is imposed. First, note that such a minimum wage is effective, as it is higher than the reservation wage of low skilled workers, but it is lower than the reservation wage of high skilled workers. Next, note that the second condition implies that production and equilibrium unemployment in the economy are unchanged.

Since the minimum wage is higher than the reservation wage of unskilled, it raises all the wages of these workers in equilibrium, because firms make equal profit. Consequently, the LHS of equation (23) increases while the RHS is unchanged. This change decreases the skill premium, so the new indifferent worker $a^*$ has higher ability than before. As a result the share of skilled workers decreases, and by assumption, the steady state growth decreases as well.

**Example 2:** now, assume that the unconstrained economy is in "type A" equilibrium at $t_0$, so both low productivity and high productivity firms are active. A minimum wage $m_{wh} < MW < \gamma_h$, is imposed. As a result, firms stop production with low skilled workers (even high productivity firms), and the economy shifts to equilibrium "type E". This causes the skill premium to increase, as all unskilled workers are unemployed, so the new $a^*$ is lower, and therefore, the share of skilled workers and steady state growth are higher.
Example 3: assume that the unconstrained economy is in “type A” equilibrium at \( t_0 \). A minimum wage, \( \bar{w}_A < \bar{w}_V \), is imposed. As a result low productivity firms in the unskilled sector stop production, but, the wage of the unskilled workers in high productivity firms increases. Thus, the total effect on the expected wage of the unskilled is ambiguous.

The intuition for the non-monotonic effect of the minimum wage is simple. Generally, in a labor market with frictions employers have monopsonistic power. Imposing an effective minimum wage reduces this power so labor share increases, but only in the unskilled workers sector. Other things equal, this reduces skill premium, and thus, the incentive of workers to invest in human capital. The result is lower growth in equilibrium. However, if the minimum wage changes the supply side (production) in the economy, it can have positive effect on long-run growth. This can happen without eliminating the low-skilled sector, which is the standard result in competitive job market analysis (Cahuc and Michel (1996)). The reason is that unlike competitive markets, where inefficient firms can not survive in equilibrium, frictions allow inefficient firms to survive. The minimum wage is effective if it is higher then the labor productivity in these firms, but still lower than labor productivity of unskilled in advanced firms.

3.4 Discussion

This paper questions the long-run effect of minimum wage on economic growth. To this end, a model that creates a synthesis between labor search theory and endogenous growth theory is constructed.

The model implies that the effect of minimum wages on growth works through two opposite channels, thus, the net effect depends on the model’s structural parameters.

The positive channel works through the negative effect of the minimum wage on the demand for workers at the lower end of the wage distribution. Since these are mostly unskilled workers, low demand for unskilled labor may create an incentive for workers to invest in human capital\(^\text{24}\). This channel is well known in the endogenous growth literature which is generally based on

\(^\text{24}\) In addition to the higher long run growth path, there is an also short run gain, as the low productivity firms are eliminated and workers reallocate to high productivity firms. This reallocation effect (from low to high productivity firms) was studies recently by Lentz and Mortensen (2004), and was found to be an important source of aggregate
competitive labor markets (for example see Cahuc and Michel (1996)).

The negative channel is at work due to labor market frictions. The building block of the labor market is the BM (1998) model, on which firms have some monopsonistic power over workers in the sense that labor supply curve they face is not infinitely elastic (not in the sense of a single buyer of labor, see Manning (2003) for this interpretation). As a result, the wage distribution is non-degenerate, and wages are set below the marginal product of labor. An effective minimum wage in this model reduces the monopsonistic power of firms in the "unskilled" sector, causing the relative wages of unskilled workers to rise. Since investment in human capital in the model is determined endogenously, the erosion of the skill premium reduces investment in human capital in equilibrium.

While the positive effect is well known, the negative effect of the minimum wage is the novel result of integrating labor market frictions in an endogenous growth framework. In the current endogenous growth literature, the introduction of the minimum wage, as long as it is below the marginal product of labor, increases steady state growth

The predictions of the model are supported indirectly by some empirical studies. Cukierman, Rama and van Ours (2001) found, by running "Barro type" growth regressions using a country panel, that the minimum wage increases growth in low-income economies, while in developed economies the effect is negative. However, in order to test the hypotheses of the model directly, a structural estimation of the model parameters based on micro data on workers and firms is needed. This is left to future work.

Although the framework presented here is used to analyze the minimum wage question only, the structural growth model with labor market frictions which is presented here may be used, with some extensions, to analyze other long-run policy implications. Natural candidates are policies

\[25\] The exception is Weiss (1996, see literature review). As to search literature, in a model with heterogeneous workers (with respect to their opportunity cost) raising the minimum wage causes unemployed workers to increase their search efforts and reduces unemployment. All this holds up to a point where the minimum wage is "too high" and there is no production at all. See Manning (2003) table 3.1 pp. 57 for a discussion on model structure and the minimum wage effect.
that affect directly the reservation wage and the wage distribution, such as unemployment benefits or negative income taxation.
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