Estimating the NAIRU for Israel, 1992–2011

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Discussion Paper No. 2013.04
April 2013

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Professor Alex Cukierman of the Bank of Israel Monetary Committee encouraged us to carry out this research. We thank Professor Cukierman for many valuable suggestions and comments on this project. We alone are responsible for any errors and shortcomings of this paper.

Any views expressed in the Discussion Paper Series are those of the authors and do not necessarily reflect those of the Bank of Israel

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David Elkayam and Alex Ilek

Abstract
We use a state space model to estimate the time-varying NAIRU for Israel for the period 1992–2011. We specify a forward looking Phillips curve, and use data on inflation expectations derived from the bond market in Israel ("breakeven inflation") as a proxy for inflation expectations. This enables us to avoid making an assumption regarding the formation of expectations, especially avoiding the usual practice of assuming adaptive expectations and using lags of inflation as proxies for inflation expectations. We find that the estimated NAIRU is fairly variable and explains a great deal of the low frequency dynamics of the actual unemployment rate. For example, from 2004 to 2011, actual unemployment declined by 6.5 percentage points. Our estimates suggest that 5.5 percentage points (most of the reduction) were due to a decline in the NAIRU. We also found that the estimated NAIRU fits very well in a Beveridge curve, and thus helps to identify the close relationship between job vacancies and unemployment.
1. Introduction

Between 2004 and 2011, the unemployment rate in Israel declined from 13.5 percent to 7 percent without any noticeable effect on inflation. One obvious reason for this is the prolonged decline in the relative price of imports, which moderated imported inflation. Another possible explanation for the lack of inflationary pressures is a decline in the Non-Accelerating Inflation Rate of Unemployment (the NAIRU—also frequently referred to as the natural rate of unemployment). To assess this possibility we estimate a time-varying NAIRU for Israel over the period 1992–2011.

The time-varying NAIRU is, of course, an unobserved variable. We therefore specify it as a latent variable and use the Kalman filter to evaluate it. As a usual practice in the literature\(^1\), the NAIRU is inferred by exploiting the information available in an expectations augmented Phillips curve. That is, by estimating the relationship between the inflation rate, its lags, supply shock variables and the gap between the actual and the natural rates of unemployment (henceforth, the unemployment gap). An additional important unobservable variable that appears in the Phillips curve relationship is the public’s inflation expectations. Usually, inflation expectations are proxied by distributed lags of inflation. Here we instead use an estimate for one year inflation expectations that is derived from the capital market. Specifically, we use the difference between the yields to maturity on two identical one-year bonds that differ from each other only in that one is un-indexed while the other is fully indexed to the CPI (the resulting directly measured expectation is also known as “breakeven inflation”). As a check for robustness we estimated the model under the assumption of adaptive expectations as well.

The estimates of the inflation equation show that a change in the relative price of imported goods have a sizable and prolonged—up to four lags—effect on inflation. An increase of one percent in the relative price of imported goods raises inflation by a quarter of a percent (cumulative effect during a year). A negative unemployment gap of one percent (that is, the actual unemployment rate is below the NAIRU) increases inflation by a quarter of percent, with a one

\(^1\) See, for example, Gordon (1997), Laubach (2001) and Turner et al. (2001).
year lag. Such a change also temporarily raises inflation, for one quarter after the change, by a third of a percent.

We find that the estimated NAIRU is fairly variable and explains a great deal of the low frequency dynamics of the actual unemployment rate. For example, from 2004 to 2011, actual unemployment declined by 6.5 percentage points. Our estimates suggest that 5.5 percentage points (most of the reduction) were due to a decline in the NAIRU. This decline seems to be partly a result of a policy of reduction in social security payments and in unemployment benefits that took place in 2002–2003.²

In order to assess the usefulness of the estimated NAIRU, we checked its ability to explain movements in a Beveridge curve (as specified by Dickens (2009)). The results suggest that the estimated NAIRU (from the Phillips curve) makes a sizable and significant contribution to the identification of a relationship between vacancies and unemployment.

The order of the paper is as follows. In Section 2 we present the model and the estimates obtained using "breakeven inflation" as a proxy for inflation expectations in the Phillips curve. In Section 3 we check the usefulness of the estimated NAIRU as a "driver" of the Beveridge curve. In Section 4 we present and compare the estimates obtained by using distributed lags of inflation as proxies for inflation expectations. In Section 5 we present the impulse response of inflation to a shock in the unemployment gap under both rational and adaptive expectations. Section 6 concludes.

2. The Phillips Curve and the NAIRU

2.1 The model

Following the relevant literature we start with a Phillips curve of the form:

\[ \pi_t = \pi^e_{t+1} - \sum_{h=1}^{k_h} \beta_h (u_t - u^o_t) + \sum_{z=1}^{k_z} \gamma \Delta z_{t-z} + \epsilon^\pi_t, \]

where: \( \pi_t \) is the inflation rate in quarter \( t \), \( \pi^e_{t+1} \) is the expected rate of inflation in the following quarter, \( u_t \) is the unemployment rate, \( u^o_t \) is the NAIRU, \( \Delta z_t \) represents supply shocks and \( \epsilon^\pi_t \) is an i.i.d. shock with a mean of zero and

² See the chapters on the labor market in the Bank of Israel Annul Reports for 2002 and 2003.
standard deviation $\sigma_{\pi}$. The preceding specification contains two unobserved variables, $\pi^e_{t+1}$ and $u^n_t$.

It is commonly assumed in the related literature that expected inflation can be represented by several lags of inflation, that is:

$$ (2) \quad \pi^e_{t+1} = \sum_{i=1}^{k} \alpha_i \pi_{t-i} $$

If we assume that the weights of the inflation lags sum to one (that is $\sum_{i=1}^{k} \alpha_i = 1$) then the Phillips equation in (1) can be written as:

$$ (3) \quad \Delta \pi_t = (\alpha_1 - 1) \Delta \pi_{t-1} + (\alpha_1 + \alpha_2 - 1) \Delta \pi_{t-2} + \ldots + (\alpha_1 + \alpha_2 + \ldots + \alpha_{k-1} - 1) \Delta \pi_{t-k+1} $$

$$ + \sum_{i=1}^{k_u} \beta_i (u_{t-i} - u^n_{t-i}) + \sum_{i=1}^{k_z} \gamma_i \Delta z_{t-i} + \epsilon^n_{t} $$

The above specification has been used in most existing studies, including Friedman and Suhoy (2005) who estimate a time-varying NAIRU for Israel for the period 1987 to 2001.3

Instead of assuming adaptive expectations, as in (2), we shall use the data on one year ahead “breakeven inflation” from the capital market, as a proxy for inflation expectations. That is, we shall estimate equations of the form:

$$ (4) \quad \pi_t - \hat{\pi}(\text{market})_{t,t+4} = \sum_{i=1}^{k_u} \beta_i (u_{t-i} - u^n_{t-i}) + \sum_{i=1}^{k_z} \gamma_i \Delta z_{t-i} + \epsilon^n_{t}, $$

where $\hat{\pi}(\text{market})_{t,t+4}$ is one year ahead breakeven inflation derived from the government bond market4.

To distinguish empirically between the natural rate of unemployment and the unemployment gap we assume that the unemployment rate ($u_t$) consists

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3 This paper differs from Freidman and Suhoy (2005) in several respects: (1) Different periods of estimation; (2) As proxy for inflation expectations we use data from the capital market; (3) We use different specification for the effect of the real exchange rate changes; (4) We include drift in the NAIRU dynamics which captures a potential stochastic but stationary trend in the NAIRU; (5) We abstain from including data on output, which they use in order to improve the identification of the NAIRU (through Okun’s rule). This is because such a procedure requires estimation of potential output which is itself an unobservable variable and subject to uncertainty. It should be noted that the estimated NAIRU in our model is fairly variable, while the NAIRU estimated by them is nearly constant.

4 The expectations in equation (1) are for the next quarter. The data that we derive from the bond market represent expectation for one year ahead.
of two unobserved components, the NAIRU \((u^*_t)\) and the unemployment gap \((ugap_t)\):  

\( u_t = u^*_t + ugap_t. \)

We model the data generating process of the NAIRU as a random walk with a stationary drift \((g_t)\) as follows:  

\( u^*_t = u^*_{t-1} + g_{t-1} \)

(7) \( g_t = \phi g_{t-1} + e^g_t. \)

Here \(-1 \leq \phi \leq 1\) measures the degree of the persistence of the change in the NAIRU, and \(e^g_t\) is an i.i.d. shock with mean zero and standard deviation \(\sigma^g\).

The inclusion of the drift captures a potential stochastic but stationary trend in the NAIRU. This assumption is consistent with the approach of Turner, et al. (2001) and implies that changes in the NAIRU are stationary and that in the long run the NAIRU is constant. Laubach (2001) and others also included a stochastic drift in the natural rate, but assumed that the drift follows a random walk—implying that only the second difference of the NAIRU is stationary (an assumption that does not seem reasonable for Israel's unemployment rate).

As for the unemployment gap, following Laubach (2001), we assume that it follows an autoregressive process of the form:  

(8) \( ugap_t = \delta_1 ugap_{t-1} + \delta_2 ugap_{t-2} + e^{ugap}_t, \)

where \(-1 < \delta_1 + \delta_2 < 1\) and \(e^{ugap}_t\) is an i.i.d shock with mean zero and standard deviation \(\sigma_{ugap}\).

We assume that all shocks are mutually uncorrelated and normally distributed. The system of five equations (4)--(8) can be estimated by means of the Kalman filter in order to obtain maximum likelihood estimates of the relevant parameters and time series estimates of the unobserved NAIRU.

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\(^5\) This specification was largely motivated by sample of European countries in which unemployment was trending up over sample period used by Laubach (2001).
2.2 Empirical considerations and results using "breakeven inflation" as a proxy for inflation expectations

Let $\Delta e_t$ represent the rate of change in the shekel/dollar exchange rate, and let $\Delta pim_t$ stand for the rate of change in the world’s price (in dollar terms) of Israeli imports. Let $\Delta epim_t$ stand for the sum: $\Delta e_t + \Delta pim_t$. $\Delta epim_t$ represents the rate of change of Israeli import prices in shekel terms.

In the estimation of the system we specified a Phillips curve in the form of equation (4)\(^6\) where on the right hand side we allowed four lags of each of the following variables: \{ $ugap_t$, $\Delta pim_t$, $\pi_t$ \}. (We also included the current value of $\Delta pim_t$, assuming it is exogenous to domestic inflation.) At first, we did not impose any restrictions on the lags. After a few trials we imposed several restrictions that seemed to characterize the data\(^7\), and settled on the following empirical specification of the Phillips equation:

\[
(4a) \quad \pi_t - \hat{\pi}(market)_{t,1,4} = \beta_1 ugap_{4-t} + \beta_2 ugap_{3-t} + \gamma_1 \Delta 53_t + \gamma_2 \Delta pim_t + \gamma_3 \Delta \pi_{t-1} + e_t^\pi,
\]

where $\Delta 53_t = (1/5) \sum_{i=0}^{4} \Delta pim_{t-i} - (1/3) \sum_{i=1}^{3} \pi_{t-i}$,

$\Delta 53_t$ is a sort of moving average of the change in import prices in "real" terms. This variable represents the effects of supply shocks from abroad. The term $\Delta \Delta pim$ represents the acceleration (or deceleration) in the rate of change in the shekel price of imports, and it is retained due to its statistical significance. The

\(^6\) For inflation ($\pi$) we use the change in the CPI excluding housing services, fruit and vegetables. We exclude these components due to empirical considerations (better fit, order of magnitude and significance of the coefficients). The fruit and vegetable component is known for high volatility and irregularity. As for housing services, in 1999 the CBS changed the method of deriving this data (until 1999 they used house prices as a proxy, from 1999 they started to use data on rents). Furthermore, until 2007 the housing component was almost fully linked to the shekel-dollar exchange rate. From 2007 onwards we observe a noticeable disruption in this linkage. The fact that the proxy for inflation expectations that we use (from the capital market) include housing, fruit and vegetables, but the inflation data doesn’t, reveals that in fact the inflation expectations included in the model have measurement errors. This increases the variance of the residuals in the inflation equation but does not bias the parameter estimates (note that inflation expectations appear on the left hand side of the estimated equation). In the appendix we present the results when we use the CPI including housing and fruit and vegetables. As can be seen the results are quite similar.

\(^7\) Note that in the long run (that is when inflation is constant) all the variables on the right hand side of equation (4) should be zero, and this restriction should be imposed on the parameters. In particular, in the long run we should have: $\pi = \Delta e + \Delta pim$. 

4
last term, $\Delta \pi_{t-1}$, captures inflation persistence possibly due to staggered price and wage adjustments.

A maximum likelihood estimation of the system in (4a) and (5)–(8), for the period 1992–2011, yields significant estimates, with the correct signs for all the parameters apart from the standard deviation $\sigma_g$. This is a well-known problem with the application of the Kalman filter to such a system: the estimate of $\sigma_g$ might tend to zero even when its "true" value is positive.\(^8\) To identify the variance of $\sigma_g$ the following procedure was implemented: we estimated the system for various values of the variance ratio $\sigma_g^2/\sigma_\pi^2$ in the range \{0.001 to 0.1\} (in the unconstrained estimation the estimated value was 0.0187 but insignificant). As can be seen in Figure 1, in the range \{0.005 to 0.1\} the likelihood function is rather flat. So we added another criterion: the maximum t-value of the unemployment gap in the Phillips curve (the parameter $\beta_2$). In Figure 2 we can see that the t-value is rather constant in the range \{0.01 to 0.1\}. The "preferred" ratio under the above criterion is 0.019.

The resulting parameter estimates are presented in Table 1 and their implications for the time-varying NAIRU in Figure 3. We note first that all parameters are statistically significant. The estimates of equation (4a) imply that, other things the same, an unemployment gap of one percent (below the natural rate) raises inflation (with a one year lag) by about a quarter of a percent. Such a change also temporarily raises inflation for one quarter after the change by about a third of a percent. One percent increase in the relative price of imported goods raises inflation by about a quarter of a percent and the coefficient of the change in lagged inflation on its current value is 0.4.

The stochastic drift in the NAIRU is fairly persistent with a first order autoregressive coefficient of 0.96. The coefficients of the unemployment gap in equation (5) imply that the unemployment rate converges monotonically to the NAIRU.

\(^8\) This is known as the "pile-up" problem, see Gordon (1997) and Laubach (2001) and the references there for discussions on that issue.
Table 1
The parameter estimates of the model of equations (4a)-(8), period 1992-2011

The Phillips curve (4a)
\[
\begin{align*}
\beta_1 & = -0.277 \ (4.7) & \beta_2 & = -0.240 \ (4.4) \\
\gamma_1 & = 0.245 \ (4.9) & \gamma_2 & = 0.040 \ (3.0) & \gamma_3 & = 0.386 \ (6.0)
\end{align*}
\]

The stochastic drift in the NAIRU (4)
\[
\varphi \quad 0.955 \ (24.7)
\]

The unemployment gap (5)
\[
\delta_1 \quad 1.168 \ (32.9) & \delta_2 \quad -0.311 \ (4.0)
\]

The standard deviations
\[
\sigma_g \quad 0.0046 \ (10.3) & \frac{\sigma^2_g}{\sigma^2_\pi} = 0.019 & \sigma_{ugov} \quad 0.0044 \ (11.2)
\]

Figure 1 — The relationship between the values of the log-likelihood function and the variance ratio \(\frac{\sigma^2_g}{\sigma^2_\pi}\)
Figure 2 — The relationship between the t-statistic of the parameter $\beta_2$ and the variance ratio $\sigma^2 / \sigma^2_\pi$.

![Graph showing the relationship between $T_{value}$ and VRatio.]

Figure 3 — Actual unemployment and estimated NAIRU of the model of equations (4a)–(5).

![Graph showing actual unemployment (u) and estimated NAIRU (u_n1) from 1992 to 2010.]

7
Figure 3 indicates that the Israeli NAIRU is fairly variable. At the beginning of the period (1992) the actual unemployment and the NAIRU were both very high (14% and 13%, respectively). This reflects the massive immigration from countries of the former Soviet Union which took place during the years 1989 to 1992 (and continued afterward as well, though at a lower rate). Then they both declined to a local minimum of 9% and 8%, respectively, in 1997. This seems to reflect, at least partly, absorption of the immigrants (of 1989 to 1992) in employment. From 1997 the NAIRU increased gradually to a sample maximum of around 12% in 2003. Since then, the NAIRU has been on a course of steep descent, to around 6.8% at the beginning of 2012. Some of this decline of the NAIRU may reflect a government policy of reducing structural unemployment, a policy which began in 2002 and continued in the following years.

2.3 The estimated NAIRU and the Beveridge curve

The decline in the NAIRU, at least since 2006, might reflect improved efficiency of the labor market. This possibility was noted and discussed in the Bank of Israel Annual Report for 2011. In Figure 4 we present a scatter plot of the unemployment rate and the Job vacancies rate from 1998 to 2011. As can be seen, since the middle of 2006 we can note a shift to the left of the "Beveridge curve", which can be interpreted as improvement in the matching process between vacancies and unemployed workers.

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9 From 1998 to 2003, the economy suffered from massive shrinkage of the construction industry. This may explain an increase in structural unemployment (mismatch). Another possible reason is a rise in the participation rate that took place during the above period.
10 That policy included a reduction in social security payments and in unemployment benefits, as well as steps to reduce the number of foreign workers.
11 It is a similar diagram to the one which appears in the Bank of Israel 2011 Annual Report.
12 For additional indicators of improved efficiency of the matching process see Box 5.1 in the Bank of Israel 2011 Annual Report.
Another way to assess the usefulness of the estimated NAIRU is to check its ability to explain movements in the Beveridge curve. Following Blanchard (2009) the relationship between the flow into unemployment and the flow out of unemployment (and back to employment) can be written as follows:

\[
(9) \quad s(1-u) = mF(u, v),
\]

The left hand side of equation (9) represents the flow of separations from employment (s is the separation rate) and the right hand side represents the flow of new hires, which is assumed to be captured by a matching function \( F(u, v) \) of unemployed workers with vacancies.\(^{13}\) m is a scale variable that represents the efficiency of the matching process. In equilibrium those two flows should be equal. Equation (9) is the Beveridge curve. Along the curve (holding s and m constant) it represents inverse relationship between unemployment and vacancies. In recessions, unemployment is high and vacancies are low, and in boom periods, unemployment is low and vacancies are high. The factors that shift the curve are the changes in the separation rate and changes in the efficiency of the matching process.

\(^{13}\) This function increases with u and v.
Following Dickens (2009) we assume that the matching function has a Cobb-Douglas form:

\[ F(u,v) = ku^{1-b}v^b \]  

Substituting (10) into (9) and arranging terms we get the following form of the Beveridge curve:

\[ (1-u)/u = (km/s)(v/u)^b \]  

Following Blanchard (2009) and Dickens (2009) it is reasonable to assume that the natural rate of unemployment is a function of the separation rate and the efficiency of the matching process. Specifically, Dickens assumes that the NAIRU is the main driver of the relationship between \((1-u)/u\) and \(v/u\). In terms of equation (11), he actually assumes that the NAIRU is a function of the separation rate \(s\) and the efficiency of the matching process \(m\) and that this function has the following form:

\[ NAIRU = k_0 + k_1 \log(km/s) \]  

Taking logs on the two sides of (11) and substituting (12) into (11) we get the following relation between unemployment, the NAIRU and job vacancies:

\[ \ln[(1-u)/u] = k_0 + k_1 NAIRU + b \ln(v/u) \]  

Using the estimated NAIRU from the Phillips curve we get the following Ordinary Least Squares estimates of the parameters of equation (11a)\(^1\):  

\(^1\) Blanchard claims that the NAIRU is also a function of the bargaining power of workers.  

\(^1\) Dickens (2009) considered a potential endogeneity of \(\ln(v_i/u_i)\) in the Beveridge curve and used instrumental variables (IV) to identify the parameter \(b\). He found, however, that the results from IV estimation are very similar to those from the OLS estimation indicating that the endogeneity bias is negligible. We assert that estimation of the Beveridge curve by OLS should provide consistent estimates even if \(\ln(v_i/u_i)\) is endogenous. This is because we actually estimate a cointegration relationship where the unemployment rate and the NAIRU are both integrated of order one, Thus all parameters should be consistent irrespectively whether \(\ln(v_i/u_i)\) is stationary or not.
Table 2
The parameter estimates of the Beveridge curve (equation (11a)) for the period 1998–2011

\[ k_0 \quad 3.55 \ (36.2) \quad k_1 \quad -10.52 \ (12.2) \quad b \quad 0.22 \ (5.4) \]

DW \ 0.65 \quad R^2 \ = \ 0.91

(The values in brackets are t-values based on Newey-West adjustment.)

In Table 2 we can see that all the parameters are significant and have the correct sign. The fit is high and we can conclude that the NAIRU that was estimated from the Phillips curve functions quite well as a shifter of the Beveridge curve as was specified by Dickens (2009). However, the low Durbin-Watson (DW) statistics may indicate that the estimated NAIRU does not capture all factors that may have shifted the Beveridge curve. We should also note that excluding the NAIRU from the equation (or equivalently, assuming that the NAIRU is constant), we get \( R^2 \) of 0.47 and DW of 0.27, a result that clarifies the usefulness of the estimated NAIRU to the identification of the Beveridge curve. Figure 5 displays the fit and the residuals of the equation that includes the estimated NAIRU.

Figure 5—the fit of the Beveridge curve (11a)
2.4 Results using distributed lags of inflation as a proxy for inflation expectations and comparison with the results in Section 2.2

In the previous section we utilized the data on breakeven inflation derived from the capital market as a proxy for the inflation expectations. In this section we estimate the model with the more conventional assumption where distributed lags of inflation are used as a proxy for inflation expectations. That is, we replace equation (4) with equation (3). The empirical specification of (3) is represented by eq. (3a) as:

\[ \Delta \pi_t = (\alpha_t - 1) \Delta \pi_{t-1} + \ldots + (\alpha_t + \ldots + \alpha_n - 1) \Delta \pi_{t-n} + \beta_1 \Delta \text{gap}_{1,t} + \beta_2 \Delta \text{gap}_{2,t} + \gamma_1 \Delta \pi^{53} + \gamma_2 \Delta \text{epim}_t + \epsilon_t \]

Table 3 presents the estimation results of the model using equation (3a). To derive the variance ratio \[ \frac{\sigma^2_{e}}{\sigma^2_{\pi}} \], we apply the same criteria as in Section 2.2 which is based on finding maximum t-values for the unemployment gap in the Phillips curve.

The estimation results of the model are presented in Table 3. As can be seen, 7 lags of inflation enter in the specification (6 lags of \( \Delta \pi_t \)). The results for the rest of the variables in Table 3 are quite similar to the results shown in Section 1 (Table 1).

Figure 6 shows the evolution of the estimated NAIRU under the two alternative specifications of the inflation equation. The blue line (Nairu1) is based on the specification where break-even inflation expectations were included in the model. The red line (Nairu2) is based on the specification where the expectations are represented by distributed lags of inflation. The green line represents the actual unemployment rate. Generally, the evolution of the NAIRU estimated from the two alternative models is similar over the whole sample. There are two periods where there is a small systematic gap between them: in period 1994–97, the NAIRU from the first model was lower than the NAIRU from the second model and closer to the actual unemployment rate. In period 1997–2004, the NAIRU from the first model was higher. Note that since 2004 the two estimates of the NAIRU almost coincided. This strengthens the evidence that the NAIRU decreased during the last seven years. Moreover, in
the last three years, both estimators were close to the actual unemployment rate (zero unemployment gap), implying absence of inflation pressures from the labor market.

Table 3
The parameter estimates of the model of equations (3a),(5)-(8), period 1992-2011

The Phillips curve (3a)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
<td>-0.304</td>
<td>(1.7)</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>-0.304</td>
<td>(4.2)</td>
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<tr>
<td>$\alpha_1$</td>
<td>0.71</td>
<td>(9.8)</td>
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<tr>
<td>$\alpha_2$</td>
<td>-0.26</td>
<td>(3.4)</td>
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<tr>
<td>$\alpha_3$</td>
<td>0.05</td>
<td>(0.6)</td>
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<tr>
<td>$\gamma_1$</td>
<td>0.367</td>
<td>(4.9)</td>
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<tr>
<td>$\gamma_2$</td>
<td>0.071</td>
<td>(3.7)</td>
</tr>
<tr>
<td>$\alpha_4$</td>
<td>0.16</td>
<td>(2.5)</td>
</tr>
<tr>
<td>$\alpha_5$</td>
<td>0.09</td>
<td>(1.1)</td>
</tr>
<tr>
<td>$\alpha_6$</td>
<td>0.02</td>
<td>(0.3)</td>
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The stochastic drift in the NAIRU (7)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
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<tr>
<td>$\varphi$</td>
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The unemployment gap (8)

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<td>$\delta_1$</td>
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<td>(14.3)</td>
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<td>$\delta_2$</td>
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<td>(3.8)</td>
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The standard deviations

<table>
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<th>Parameter</th>
<th>Estimate</th>
</tr>
</thead>
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<tr>
<td>$\sigma_\pi$</td>
<td>0.0052</td>
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<td></td>
<td>(11.0)</td>
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<tr>
<td>$\sigma_{\pi}^2 / \sigma_{\pi}^2$</td>
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</tr>
<tr>
<td>$\sigma_{ugap}$</td>
<td>0.0044</td>
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<tr>
<td></td>
<td>(11.2)</td>
</tr>
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</table>
2.5 Impulse response of inflation to a 1 percentage point shock to the unemployment gap (under rational and adaptive expectations)

In this section we present the response of inflation to a 1 percentage point shock to the unemployment gap. We start with the model that includes the Phillips curve equation (4a). We simulate the model under two alternative assumptions with respect to inflation expectations formation. Under the first alternative, we assume that inflation expectations are forward looking and that the relevant expectations that enter the Phillips curve are expectations to four quarters ahead, that is: \( \pi^e_t = 0.99 \times 0.25 \times \sum_{i=1}^{4} E_i \pi_{t+i} \). Under the second alternative, we assume that expectations are backward looking and that the relevant expectations that enter in the Phillips curve are the actual inflation in the previous four quarters, namely:

\[ \pi^e_t = 0.99 \times 0.25 \times \sum_{i=1}^{4} E_i \pi_{t-i} \].

The results are presented in Figure 7 (and in Table 4). The blue and the red lines in Figure 7 represent the impulses under alternatives 1 and 2, respectively. As can be seen, under forward looking expectations, the initial response is quicker and much stronger than that under adaptive expectations, but
inflation converges back much faster. In both cases the unemployment gap shock directly affects inflation with a lag, and thus inflation is expected to decline in the future. In the forward looking case this has an immediate affect on inflation. In the backward looking case, however, inflation declines only when the effect of the shock materialized. In both cases the unemployment gap shock has persistence. In the forward looking case the expected rise in the unemployment gap after the shock occurs (due to its persistence) dampens inflation expectations and thus amplifies the negative effect of the shock on inflation. As time passes, the convergence of the unemployment gap leads to convergence of inflation expectations and thus of inflation. In the backward looking case, however, the convergence of the inflation is much longer due to its strong dependence on past inflation.

In the green line, we present the impulse of inflation from the estimated model which includes the backward looking expectations Phillips curve equation (3a). As can be seen, the green and the red lines are quite similar.

Table 4 summarizes the responses of inflation to the unemployment gap shock under the three alternatives. The values are in quarterly terms (in percent). For example, a 1 percentage point increase in the unemployment gap (say from 10 to 11 percent) reduces inflation in the same quarter by 0.56 percents, under the forward looking case. Under the backward looking case the same shock reduces inflation, in a lag of one quarter, by only 0.28 percent.

We can conclude that the assumption regarding the formation of inflation expectation has relatively moderate effect on the estimated parameters but a very large effect on the impulse responses of inflation with regard to a shock to the unemployment gap.
Figure 7 — Impulse response of inflation to unemployment gap shock of 1 percentage point

![Graph showing impulse response of inflation to unemployment gap shock of 1 percentage point.]

Table 4 — The responses of inflation to unemployment gap shock of 1 percentage point

<table>
<thead>
<tr>
<th>Periods</th>
<th>Forward looking (Case 1)</th>
<th>Backward looking (Case 2)</th>
<th>Backward looking (Case 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>t = 0</td>
<td>-0.56</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>t = 1</td>
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<td>-0.30</td>
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<td>-0.25</td>
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<td>t = 8</td>
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<td>-0.60</td>
<td>-0.60</td>
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<td>t = 12</td>
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<tr>
<td>t = 100</td>
<td>0</td>
<td>-0.51</td>
<td>-0.56</td>
</tr>
</tbody>
</table>
3. Conclusions

We estimated a time-varying NAIRU for Israel for the period 1992-2011. We specify a forward looking Phillips curve and use data on "breakeven inflation" as proxy for inflation expectations. This enabled us to avoid assuming how inflation expectations are formed, especially avoiding the usual practice of assuming adaptive expectations and using lags of inflation as proxy for inflation expectations. As a sensitivity check we estimated the model also under adaptive expectations. The estimated parameters and the estimated NAIRU are similar under the two alternatives, although not identical. However, the assumption regarding the process of the formation of expectations has a far reaching effect on the impulse response functions of inflation with regard to a shock to the unemployment gap.

The estimates of the inflation equation show that a change in the relative price of imported goods have a sizable and prolonged, up to four lags, effect on inflation. We found that the unemployment gap affects inflation with a lag of four quarters. Changes in that gap also have an effect (speed limit effect) with one quarter lag.

We find that the estimated NAIRU is fairly variable and matches the low frequency dynamics of the actual unemployment rate. From 2004 to 2011 actual unemployment declined by 6.5 percentage points. Our estimates suggest that most of that decline (5.5 percentage points) paralleled a decline in the NAIRU. We found that the estimated NAIRU (from the Phillips curve) has a sizable and significant contribution to the identification of the relationship between vacancies and unemployment in the Beveridge curve.
Appendix

Estimation of the NAIRU using total CPI (including housing, fruits and vegetables)

The parameter estimates of the model of equations (4a)-(8), period 1992-2011

The Phillips curve (4a)

\[ \beta_1 = -0.114 \ (0.6) \quad \beta_2 = -0.20 \ (2.4) \]

\[ \gamma_1 = 0.150 \ (2.2) \quad \gamma_2 = 0.096 \ (4.7) \quad \gamma_3 = 0.396 \ (5.7) \]

The stochastic drift in the NAIRU (4)

\[ \varphi = 0.897 \ (12.8) \]

The unemployment gap (5)

\[ \delta _1 = 1.191 \ (14.6) \quad \delta _2 = -0.2847 \ (3.7) \]

The standard deviations

\[ \sigma _x = 0.0062 \ (9.4) \quad \frac{\sigma _e^2}{\sigma _\pi^2} = 0.019 \quad \sigma _{ugop} = 0.0045 \ (10.6) \]

Figure 8 — Estimated NAIRU from the CPI excluding housing, fruits and vegetables (blue line), the estimated NAIRU from the total CPI (red line) and the actual unemployment rate (green line), 1992-2011
References


