Cyclicality of Uncertainty and Disagreement

Osnat Zohar*

Discussion Paper 2021.09
May 2021

Bank of Israel - http://www.boi.org.il

* Bank of Israel and the Hebrew University of Jerusalem, osnat.zohar@boi.org.il. Phone: 972-2-6552652
I thank Ilan Kremer, Edouard Schaal, Marina Halac, Eran Hoffman, Moti Perry, Jaroslav Borovička, Nimrod Segev, Alon Eizenberg, and participants of the Bank of Israel's Research Department seminar for their helpful comments.

חטיבת המחקר, בנק ישראל ת״ד 780 ירושלים 91007
Research Department, Bank of Israel. POB 780, 91007 Jerusalem, Israel

Cyclicality of Uncertainty and Disagreement

Osnat Zohar

Abstract

The empirical literature often uses dispersion in forecasts (disagreement) as a proxy for uncertainty, yet these variables behave differently throughout the business cycle. The difference is especially salient in non-crisis periods, in which disagreement among professional forecasters in the US is positively correlated with growth, while measures of uncertainty are negatively correlated with it. This finding is explained using a noisy information model with endogenous learning. In the model, agents observe noisy private information, but only when they are active. Holding uncertainty fixed, a rise in activity introduces noisy information to the market, and agents' beliefs draw apart, i.e., disagreement rises.

Keywords: endogenous learning, business cycles, private information.

JEL Classification: D81, D83, D84, E32, E37.

מחזוריות של חוסר וודאות וחוסר הסכמה

אסנת זהר

תקציר

הספרות האמפירית משתמשת לעתים קרובות בפיזור של תחזיות (חוסר הסכמה) כאומדן לחוסר וודאות. אולם משתנים אלה מתנהגים אחרת לאורך מחזור העסקים. ההבדל ביניהם בולט במיוחד בתקופות שאינן משבריות, בהן חוסר ההסכמה בין החזאים המקצועיים בארה"ב מתואם חיובית עם הצמיחה, ואילו מדדי חוסר וודאות מתואמים אתה שלילית. ממצא זה מוסבר באמצעות מודל של מידע רועש עם למידה אנדוגנית. הפרטים במודל צופים במידע פרטי רועש, אך רק כאשר הם פעילים. עלייה בפעילות מכניסה מידע רועש לשוק ולכן מרחיקה בין תחזיות הפרטים. כך, עלייה בפעילות מעצימה את חוסר ההסכמה.

Cyclicality of Uncertainty and Disagreement

Osnat Zohar*

May 2021

Abstract

The empirical literature often uses dispersion in forecasts (disagreement) as a proxy for uncertainty, yet these variables behave differently throughout the business cycle. The difference is especially salient in non-crisis periods, in which disagreement among professional forecasters in the US is positively correlated with growth, while measures of uncertainty are negatively correlated with it. This finding is explained using a noisy information model with endogenous learning. In the model, agents observe noisy private information, but only when they are active. Holding uncertainty fixed, a rise in activity introduces noisy information to the market, and agents' beliefs draw apart, i.e., disagreement rises.

Keywords: endogenous learning, business cycles, private information.
JEL Classification: D81, D83, D84, E32, E37.

1 Introduction

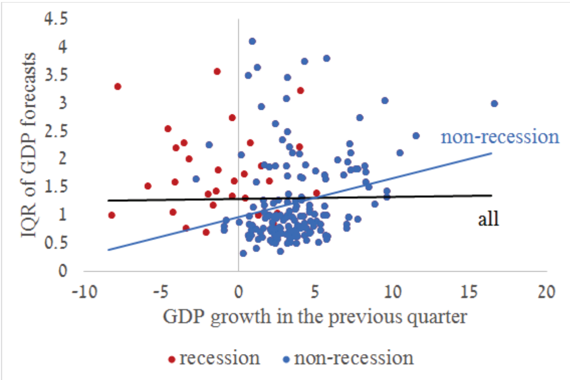
Dispersion of forecasts, namely, disagreement, is a common proxy for uncertainty. However, these concepts are distinct as agents may disagree about the future for reasons unrelated to uncertainty. In fact, the definitions of the two concepts do not immediately imply that there should be any connection between them. Consider several agents that form forecasts about future GDP growth. Disagreement among these agents is the dispersion of their forecasts, while uncertainty is the variance of their forecast errors (e.g., Jurado et al., 2015).

Uncertainty and disagreement also differ in empirical aspects, especially regarding their behavior along the business cycle. There is a vast literature that discusses the adverse effects of uncertainty on aggregate activity (Bloom, 2014; Jurado et al., 2015; Baker

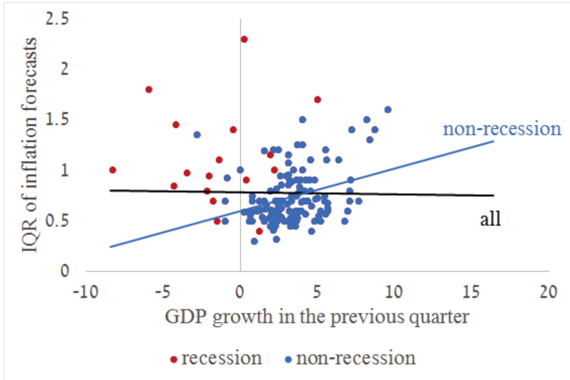
*Bank of Israel and the Hebrew University of Jerusalem, osnat.zohar@boi.org.il. I would like to thank Ilan Kremer, Edouard Schaal, Marina Halac, Eran Hoffman, Moti Perry, Jaroslav Borovička, Nimrod Segev, Alon Eizenberg, and participants of the Bank of Israel's Research Department seminar for their helpful comments. The views in this paper do not necessarily reflect the views of the Bank of Israel.

Figure 1: Measures of Disagreement and Uncertainty Against Lagged GDP Growth (1960-2019)

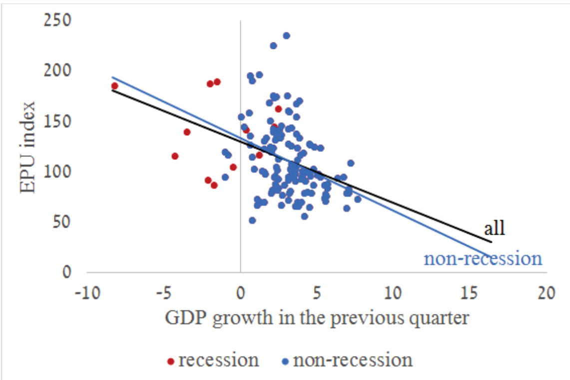
A. Disagreement about Future GDP Growth



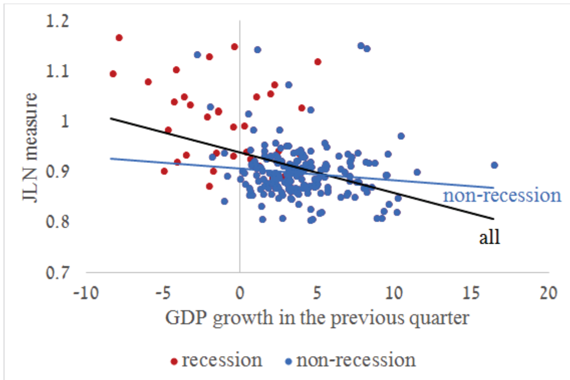
B. Disagreement about Future Inflation



C. Economic Policy Uncertainty



D. Macro Uncertainty



Notes: The figure shows scatter plots of disagreement measures (Panels A and B) and uncertainty measures (Panels C and D) against GDP growth in the previous quarter. Panels A and B show the interquartile range (IQR) of one-year-ahead forecasts of GDP growth and inflation, respectively, from the Survey of Professional Forecasters. Panel C shows a quarterly average of the newspaper-based Economic Policy Uncertainty (EPU) index of Baker et al. (2016). Panel D shows a quarterly average of the one-year ahead macro uncertainty measure of Jurado et al. (2015). In each panel, red dots mark periods of NBER recessions. Solid lines depict regressions on all observations (in black) and non-recession periods (in blue).

et al., 2016; Ludvigson et al., 2015). If disagreement is a good proxy for uncertainty, it should also be negatively correlated with activity. However, unlike uncertainty measures, disagreement shows no clear relationship with aggregate activity (Mankiw et al., 2003; Jurado et al., 2015). Figure 1 demonstrates the different cyclical behavior of uncertainty and disagreement. Panels A and B show that disagreement among professional forecasters in the US is not correlated with lagged GDP growth. In contrast, Panels C and D show that two common measures of uncertainty (Baker et al. (2016) and Jurado et al. (2015), respectively) are negatively correlated with growth. The difference is especially salient in non-crisis periods, in which disagreement is *positively* correlated with growth, while the measures of uncertainty are still negatively correlated with it.

This paper aims to improve the understanding of the forces that drive disagreement, focusing on its connection with uncertainty and aggregate activity. The contribution of the paper is twofold. First, motivated by the empirical evidence of Figure 1, I propose a stylized model that generates a positive effect of activity on disagreement. Second, I provide empirical evidence of this link, which so far has not been discussed in the literature.

The model is a novel variation of the noisy information model, which features *endogenous learning*: agents observe noisy private information regarding the state of the world, but only when they are active. The variation is motivated by the observation that while rational agents rely on available information to choose optimal actions, their actions may also generate information. For example, a firm learns more about aggregate demand when it expands its business; a venture capital fund learns more about the economy's technological frontier as it invests more in startups. While activity generates monetary gains in these settings, it also provides information about the economy's fundamentals. Endogenous learning also appears in Fajgelbaum et al. (2017), who study its effects on subjective uncertainty and the propagation of recessions. In their model economic activity generates *public* information, while I incorporate *private* signals to study disagreement.

In the model, agents face an investment opportunity each period, which yields a stochastic payoff. This payoff is privately observed and provides noisy information about the state of the world.[1] Past realizations of the state are public information, and agents who choose not to invest observe only these signals. Disagreement originates from the use of noisy private signals to form expectations about the future state and from the fact that active agents use timely information, while inactive agents use stale information.

The departure from the standard noisy information model to include endogenous learning introduces two direct channels through which activity affects disagreement. The first channel generates a positive linear effect of activity on disagreement, and I refer to it as the "noisy information channel". When activity is depressed, agents rely mostly on public information, and so disagreement is low. As activity rises, more agents receive noisy private signals, and their expectations spread further apart. The second channel is the "synchronization channel". It captures the effect that activity has on the heterogeneity of information that agents use. When agents use different information sources, they tend to disagree more than when they use similar sources. In this setting, this issue amounts to what share of agents uses information about the current state. The channel's effect is maximal when half the agents are active and therefore use timely information, and half are inactive and use only stale information. It decreases as agents' information sources become more coordinated, either as more agents become active or as more agents become inactive. Overall, this channel generates an inverted U-shaped effect of activity

[1]In the model, there is a dichotomous difference between active agents who receive information and inactive agents who do not. In reality, however, we would not see such dichotomous differences but rather a continuous spectrum of activity levels. Nonetheless, the simplistic modeling captures the idea that for the average agent, more activity generates more information.

on disagreement.

Up to a certain level of activity, both channels operate in the same direction. As activity increases, disagreement rises because noisy information enters the market and because the agents' sources of information differ. Above this level, the two channels have conflicting effects. While a rise in activity introduces more noise to the market, it also means that more agents consider information about the current state, bringing their beliefs together. The synchronization channel weakens the noisy information channel's positive effect at these activity levels and may even reverse it (depending on the model parameters). However, I show that around the mean level of activity, both channels generate a positive effect on disagreement.

The model also generates an effect of uncertainty on disagreement, similarly to standard noisy information models. Higher uncertainty means that shocks to the state of the world are more volatile. In these cases, when agents form their expectations regarding the future state, they put less weight on the state's past realization (the public signal) and more weight on their noisy private signals. Consequently, their beliefs become more dispersed, i.e., disagreement rises with uncertainty.

Endogenous learning also generates a mitigating effect of uncertainty on aggregate activity. This effect holds even though standard mechanisms through which uncertainty affects activity are muted in this model. The agents are risk-neutral, so they are oblivious to the variability of the potential payoffs. They are also myopic, so they place no value on the real option of future investment (in contrast to Bernanke (1983), Fajgelbaum et al. (2017) and Zohar (2019)). Nonetheless, uncertainty does play a role in determining the level of activity, as it affects the weight active agents place on information about the lagged state of the world versus their private signal. As uncertainty rises, agents place less weight on the former and more weight on the latter. This effect is enhanced when there are more active agents, which occurs when the fundamental state is higher. In these cases, agents that put less weight on the lagged state (due to the rise in uncertainty) become more pessimistic, so the level of consequent activity falls. Thus, the combination of noisy information with endogenous learning generates a novel mechanism through which uncertainty depresses activity.

I provide empirical evidence for the model's main predictions using disagreement among professional forecasters in the US regarding GDP growth and inflation. Taking the model to the data is not trivial as it is highly stylized and cannot be directly estimated. Furthermore, it features some complex non-linearities. To tackle these challenges, I consider several empirical approaches. First, I examine the simple correlations predicted by the model. I show that after controlling for various measures of uncertainty, disagreement is positively correlated with activity. This result is robust to the use of various proxies of uncertainty and various measures of activity.

Second, I address the model's non-linearities by considering transitions between two states of uncertainty. The theoretical model predicts that the positive effect of activity on disagreement is more substantial at the higher state of uncertainty. High uncertainty incentivizes agents to put less weight on the public signal and more on their private signals. Thus, when uncertainty is high, a rise in activity induces a larger response of disagreement than when uncertainty is low. I test this prediction using two methods. First, I differentiate uncertainty states according to observed measures of uncertainty. Second, I estimate a linear switching model that endogenously identifies uncertainty states. Using both methods, I find strong support for the model's predictions using GDP expectations but weaker support using inflation expectations.

Finally, abstracting from the model's static nature, I estimate the dynamic effects of uncertainty and activity on disagreement. Using the linear projection method proposed by Jordà (2005), I show that both uncertainty and activity have a positive effect on disagreement, as the theoretical model predicts.

In line with previous results in the literature (e.g., Lahiri and Sheng, 2010; Jurado et al., 2015), my analysis indicates that disagreement may not be a good proxy for uncertainty. Nonetheless, in the last part of the paper, I argue that it is still a macroeconomic indicator worth tracking.[2] In the context of the linear projection model, I show that exogenous shocks to disagreement increase activity, in contrast to the moderating effect that uncertainty shocks have. According to the theoretical model, exogenous changes in disagreement result from changes in the precision of private signals, although the precision has been held constant in the basic model.[3] I show that an increase in the precision is similar to an increase in uncertainty since it causes agents to put more weight on their private signals than on the public signal. As mentioned, on average, such a change decreases activity.

The rest of the paper is organized as follows. Section 2 surveys related literature; Section 3 describes the model and presents the main theoretical results; Section 4 tests it main predictions regarding disagreement. Section 5 discusses the role of disagreement as a macroeconomic indicator.

2 Related Literature

The existing literature points to uncertainty as a common factor that drives both disagreement and activity. Several papers use disagreement as a proxy for uncertainty based on the perception that the latter significantly affects the former (Bloom, 2014; Bachmann

[2]While the macroeconomic literature has yet to engage in the effects of disagreement, there is a vast literature that discusses its implications to financial markets (e.g., Hong and Stein, 2007; Banerjee and Kremer, 2010; Banerjee, 2011; Atmaz and Basak, 2018, and references therein).

[3]One might regard the precision of signals as another type of uncertainty since it affects the variance of individuals' forecast errors. However, in this paper, I focus on a more fundamental notion of uncertainty related to public information.

et al., 2013; Bloom et al., 2018). Additionally, there is a vast literature that shows that uncertainty lowers aggregate activity.[4] Bloom (2009) and Bloom et al. (2018) show that uncertainty shocks cause downturns in activity. This effect can be explained either by agents' risk aversion or by a "real option" mechanism under which the value of delaying activity increases when uncertainty is high (Bernanke, 1983).[5]

The fact that uncertainty raises disagreement and lowers activity implies a negative correlation between activity and disagreement. However, several papers show that this is not the case empirically. Mankiw et al. (2003) study dispersion in inflation expectations from several sources and find no clear relationship with real activity measures. As for expectations of growth, Jurado et al. (2015) show that while disagreement is counter-cyclical, after controlling for real and financial variables in a structural VAR, disagreement is positively related to production and employment (while other measures of uncertainty remain counter-cyclical in that setting). These findings are consistent with mine since I show that disagreement between professional forecasters is pro-cyclical after controlling for uncertainty (which is correlated with the variables in the SVAR).

The fact that disagreement shows no clear relationship with activity implies that in addition to the indirect connection through uncertainty, there is a direct, positive link between activity and disagreement. However, existing theoretical models do not generate such a link. The most straightforward explanation for disagreement is noisy private signals (e.g., Lucas, 1973). More recent work showed that the signal extraction conducted by agents who face noisy information could also represent the inference of agents with limited attention (Sims, 2003; Mackowiak and Wiederholt, 2009). In any case, agents in these models disagree because their private signals include idiosyncratic noise, so their signal extraction process yields different inferences. However, unless time-variation in the signal-to-noise ratio is assumed, this mechanism cannot explain cyclical patterns in disagreement (Coibion and Gorodnichenko, 2012).

Disagreement that is time-varying arises in models with sticky information (Mankiw and Reis, 2002). In these models, agents observe full and accurate information but at different times. Thus, in each period, some agents hold stale information, while others hold timely information. Disagreement arises when unexpected changes to the unobserved fundamental state occur. Following such changes, agents who hold stale information produce different forecasts from those who hold timely information. If the fundamental evolves as expected (no shocks hit the economy), all agents form similar forecasts, regardless of when their information was last updated, and disagreement is low. This mechanism is

[4] Segal et al. (2015) find evidence that some increases in uncertainty are associated with positive innovations to macroeconomic growth. In these cases, uncertainty is associated with possible growth opportunities, while downside risks are limited.

[5] Fajgelbaum et al. (2017) discuss an endogenous component of uncertainty, which increases following a slowdown in economic activity due to the decline in information acquisition. Ludvigson et al. (2015) show that macroeconomic uncertainty often rises endogenously following real shocks, while financial uncertainty is a source of downturns in output.

independent of the level of the fundamental or activity. Thus, sticky information cannot explain the correlation between disagreement and activity (Mankiw et al., 2003).

Empirical tests of the noisy information and sticky information predictions provide inconclusive results and depend on the survey used in the analysis. They are inconclusive even when only the predictions about disagreement are considered. Andrade and Le Bihan (2013) show that disagreement among European professional forecasters is correlated with shocks to the relevant fundamental (inflation, growth, or unemployment), which is consistent with the sticky information model. In contrast, using various US surveys, Coibion and Gorodnichenko (2012) find no such evidence and conclude that noisy information better captures the expectation formation process. In an agnostic approach to the source of information rigidity, Coibion and Gorodnichenko (2015) find evidence that rigidity exists and that it depends on economic conditions.[6] Although not tested directly, these findings may also imply that disagreement is state-dependent.

3 Model

There is a fundamental state of the world y_t that follows an AR(1) process:[7]

$$y_t = \rho y_{t-1} + \epsilon_t \,, \quad \epsilon_t \overset{iid}{\sim} N(0, u_{t-1}) \,,$$

where $\rho \in (0, 1)$ is the auto-correlation parameter of the state, and ϵ_t is the innovation. I assume that the variance of innovations to the state, u_{t-1}, and the state's realization, y_{t-1}, are public information at the end of period $t-1$. It follows that the one-step-ahead forecast for y_t given the public information is $E(y_t|u_{t-1}, y_{t-1}) = \rho y_{t-1}$, and its variance is $Var(y_t|u_{t-1}, y_{t-1}) = u_{t-1}$. According to the common convention (e.g., Jurado et al., 2015), I refer to u_{t-1} as *uncertainty*.

I assume that uncertainty is a Markov chain with two possible states, u^H and u^L, where $u^H > u^L > 0$, and transition probabilities $p^H = \Pr(u_t = u^H|u_{t-1} = u^H)$ and $p^L = \Pr(u_t = u^L|u_{t-1} = u^L)$.

There is a continuum of agents $i \in [0, 1]$, each holding a single investment opportunity. An investment taken in period $t-1$ pays a stochastic payoff at the beginning of period

[6] Coibion and Gorodnichenko (2015) find that information rigidity increased during the Great Moderation and decreased in recessions. They argue that these findings are consistent with rational inattention theories, in which agents choose to devote more attention to new information when shocks are more volatile. However, one might also interpret these results in the context of rational Bayesian agents who face changes in shock volatility (i.e., changes in uncertainty). Time-variation in the volatility of shocks should lead rational agents to change the weight they assign to new information.

[7] The setting of the model resembles that of Fajgelbaum et al. (2017) but with private information.

t, which is private information:[8]

$$\pi_{i,t} = y_t + \eta_{i,t}\,, \quad \eta_{i,t} \overset{iid}{\sim} N(0, \gamma^{-1}).$$

If the agent does not invest in period $t-1$, he earns nothing in the following period. The timing of the model in each period t is as follows.

1. ϵ_t and u_t are drawn but not observed.
2. Agent i receives his payoff ($\pi_{i,t}$ if he invested in period $t-1$, and zero otherwise).
3. He updates his belief about y_t.
4. He decides whether to take the investment opportunity at period t or not; namely, he chooses an action $a_{i,t} \in \{0, 1\}$.
5. y_t and u_t are publicly revealed.

Note that all agents enter period t knowing the realization of y_{t-1}. Thus, active agents who receive a payoff $\pi_{i,t}$ infer a noisy signal about ϵ_t. Consequently, when agents form their beliefs for period t, they rely on the public signal y_{t-1} and the private signal $(\epsilon_t + \eta_{i,t})$, if they observe one. Specifically, agent i uses Bayesian updating to form his t-period belief. Since all variables are normally distributed, the belief is also normal with mean $\mu_{i,t}$ and precision $\gamma_{i,t}$ where,

$$\mu_{i,t} = \rho y_{t-1} + a_{i,t-1}\frac{\gamma(\epsilon_t + \eta_{i,t})}{u_{t-1}^{-1} + \gamma}\,, \quad \gamma_{i,t} = u_{t-1}^{-1} + a_{i,t-1}\gamma. \tag{1}$$

Agents are myopic and risk-neutral. Namely, their discount factor is zero, and their utility from a payoff π is $u(\pi) = \pi$. The myopia assumption rules out experimentation, as agents are oblivious to the fact that their payoff today entails information about future payoffs. Without this assumption, agents' strategies may not be monotone in their mean belief, which adds unnecessary complexity to the model.[9]

Aggregate Activity

Note that agent i's expected payoff from investment in period t equals:

$$E(\pi_{i,t+1}|\mu_{i,t}, \gamma_{i,t}) = E(y_{t+1}|\mu_{i,t}, \gamma_{i,t}) = \rho\mu_{i,t}\,.$$

[8]The assumption that agents do not observe others' signals is in line with Bordalo et al. (2020), who find empirical evidence that forecasters do not react to the signals observed by others.

[9]Zohar (2019) studies the role of experimentation in a model with a dynamic state and reversible actions.

Thus, agent i chooses to invest if $\rho\mu_{i,t}$ is greater than zero – the payoff from his outside option. I can now examine the aggregate level of activity:

Definition 1 (Aggregate Activity)**.** $A_t \equiv \int_0^1 a_{i,t} di$ is the aggregate level of activity in period t.

Considering agents' threshold strategy and their belief formation (1), it follows that

$$A_{t+1} = \int_0^1 \mathbb{I}_{\{\mu_{i,t+1} \geq 0\}} di = (1 - A_t)\mathbb{I}_{\{y_t \geq 0\}} + A_t \Pr\left(\mu_{i,t+1} \geq 0 \middle| a_{i,t} = 1\right) =$$
$$(1 - A_t)\mathbb{I}_{\{y_t \geq 0\}} + A_t \Phi\left(\frac{1 + \gamma u_t}{\sqrt{\gamma} u_t} \rho y_t + \sqrt{\gamma}\epsilon_{t+1}\right) =$$
$$\begin{cases} A_t \Phi\left(\frac{1+\gamma u_t}{\sqrt{\gamma} u_t} \rho y_t + \sqrt{\gamma}\epsilon_{t+1}\right) & \text{if } y_t < 0 \\ 1 - A_t \Phi\left(-\frac{1+\gamma u_t}{\sqrt{\gamma} u_t} \rho y_t - \sqrt{\gamma}\epsilon_{t+1}\right) & \text{if } y_t \geq 0 \end{cases}, \quad (2)$$

where $\Phi(\cdot)$ is the standard normal CDF and $\mathbb{I}_{\{X\}}$ is the indicator function.

The following proposition implies that observing y_t is equivalent to observing A_t (all proofs appear in the Appendix). Thus, even though agents do not observe others' actions, they may be thought of as observing the aggregate level of activity.

Proposition 1. *If A_0 and y_0 are common knowledge and $A_0 > 0$, agent i's beliefs almost surely remain the same if he observes A_t instead of y_t at the end of each period t.*

A key role that activity plays in this model is that it produces information as well as noise. Since the level of activity depends on the state of the world, so do learning and the amount of noise. In particular, the following proposition shows that the level of activity is expected to increase with the state of the world, as agents are more optimistic and a larger fraction of them enter the market. As a result, the higher the state is, the more information enters the market, accompanied by more noise. As I show in the following sections, this result has implications for the market's response to fundamental changes such as transitions between uncertainty states.[10]

Proposition 2. *Let $\mathcal{H}_{t-1}^u = \left\{u^L, u^H\right\}^t$ denote the set of possible histories of uncertainty in period $t-1$. For any history $h_{t-1}^u \in \mathcal{H}_{t-1}^u$, $E(A_t | h_{t-1}^u, y_t)$ is increasing in y_t.*

[10] Veldkamp (2005) and Zohar (2019) show that such a mechanism has implications for the market's response to changes in the fundamental state of the world. In their models, changes to the state trigger a larger response at the aggregate level when the original state is favorable than when it is adverse. In the former case, more agents are active and thus observe information about the transition than in the latter case.

Disagreement

Disagreement is defined as the dispersion of agents' beliefs as expressed via the variance of agents' point estimates:

Definition 2 (Disagreement)**.** $D_{t+1} \equiv \int_0^1 \left(\mu_{i,t+1} - \int_0^1 \mu_{j,t+1} dj \right)^2 di$ is the disagreement among agents in period $t+1$.

Consider a partition of agents into two groups – active and inactive. Aggregate disagreement consists of disagreement within the groups and between the groups. This decomposition is formulated using the law of total variance:

Corollary 3. *In period $t+1$, disagreement equals*

$$D_{t+1} = \underbrace{A_t \frac{\gamma}{(u_t^{-1} + \gamma)^2}}_{\text{average intra-group disagreement}} + \underbrace{A_t(1 - A_t) \left[\frac{\gamma \epsilon_{t+1}}{u_t^{-1} + \gamma} \right]^2}_{\text{inter-group disagreement}} . \tag{3}$$

The first argument in Equation (3) captures the average disagreement within each group. Since inactive agents rely on the same information, they are in full agreement. However, active agents disagree due to the idiosyncratic noise entering their beliefs. As in standard noisy information models, disagreement among these agents amounts to $\frac{\gamma}{(u_t^{-1}+\gamma)^2}$. The second term captures inter-group disagreement, namely, the spread between inactive and active agents' average beliefs. This spread stems from the fact that inactive agents rely on stale information, while active agents hold timely information. Thus, this term depends on the realization of the shock ϵ_{t+1}. When it equals zero, stale information is useful as the average timely information and inter-group disagreement vanishes.

After defining the main variables of interest, in each of the following subsections I focus on one side of the triangular connection between activity, uncertainty, and disagreement, as illustrated in Figure 2. While the effects of uncertainty on activity and disagreement are discussed in the literature (Section 2), the direct link between activity and disagreement is unique to this model.

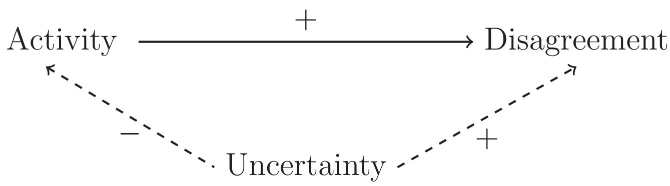
3.1 The Effect of Uncertainty on Disagreement

Following Corollary 3, at the end of period t, expected disagreement in $t+1$ is:

$$E_t(D_{t+1}) = \frac{\gamma}{(u_t^{-1} + \gamma)^2} \left[A_t + \gamma A_t (1 - A_t) u_t \right] . \tag{4}$$

Equation (4) shows that disagreement is positively affected by uncertainty u_t. First, higher levels of uncertainty cause active agents to put less weight on the public signal (lagged realization of the state) and more weight on their noisy private signal. Hence,

Figure 2: The Links between Uncertainty, Activity and Disagreement in the Model



Note: The figure illustrates the links between the three main variables in the model. Dashed arrows depict links discussed in the literature, while the solid arrow depicts a novel link presented in this paper.

disagreement rises. This mechanism also appears in standard noisy information models. Second, uncertainty contributes directly to inter-group disagreement, as it affects the expected magnitude of shocks. This mechanism also appears in sticky information models. When shocks to the state are volatile, agents who receive timely information tend to form different expectations than agents who use stale information. Compared to noisy and sticky information models, the novelty of this model is that disagreement is also affected by lagged activity, as discussed in the following subsection.

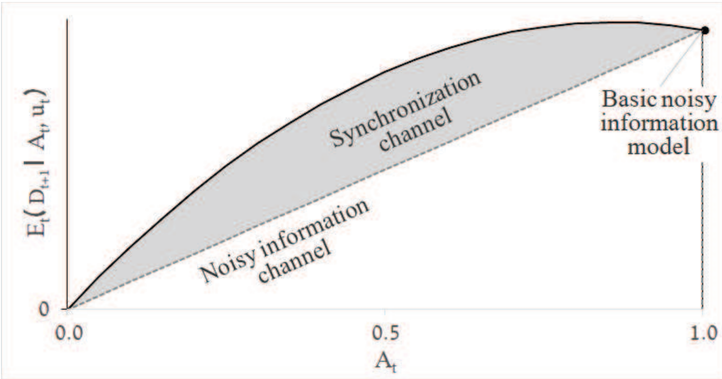
3.2 The Effect of Activity on Disagreement

Figure 3 illustrates the effect of activity on disagreement from Equation (4), holding u_t fixed. At the minimal level of activity ($A_t = 0$), disagreement vanishes since all agents rely solely on publicly-observed information. At the maximal level of activity ($A_t = 1$), the model is equivalent to the basic noisy information model since all agents observe noisy signals. Intermediate levels of activity affect disagreement through two channels:

1. **Noisy information channel:** Activity generates noisy information that draws agents' beliefs apart and increases disagreement. As activity rises, more agents observe noisy signals and disagreement increases. The mechanism operates by increasing intra-group disagreement, and it is captured by the first term in the squared brackets in (4).

2. **Signal synchronization channel:** This channel operates through the intra-group disagreement, and it is captured by the second term in the squared brackets in (4). It generates an inverted U-shaped effect of activity on disagreement that follows from the synchronization of information sources. At extreme levels of activity, agents form expectations based on similar signals, decreasing disagreement. Specifically, if $A_t = 0$, all agents observe only the public signal, and their beliefs perfectly align. At the other extreme, $A_t = 1$, all agents take into account information regarding ϵ_{t+1} and inter-group disagreement vanishes. At intermediate levels of activity, some agents take into account information about ϵ_{t+1}, and some do not, which increases

Figure 3: Effect of Activity on Expected Disagreement



Notes: The figure illustrates the expected link between activity and disagreement, as described in Equation (4), fixing the level of uncertainty.

disagreement. This channel's effect is maximal at $A_t = 0.5$ when exactly half the agents take into account new information, and half do not.

Considering these two channels, we expect that disagreement will rise with activity up to a certain level of activity. As activity increases, the positive effect diminishes and may become negative at high activity levels if the synchronization channel dominates the noisy information channel.

3.3 The Effect of Uncertainty on Activity and its Implications for the Activity-Disagreement Link

The standard channels for explaining a negative impact of uncertainty on activity in the literature are turned off in this model. The agents are risk neutral, so they are oblivious to the variability of the potential profit $\pi_{i,t}$. They are also myopic, so they place no value on the option of future investment (in contrast to Bernanke (1983), Fajgelbaum et al. (2017), and Zohar (2019)).

However, uncertainty does play a role in determining the level of activity in this model through an alternative mechanism - noisy endogenous learning. The belief formation equation (1) implies that uncertainty u_t affects the weight active agents place on the public signal y_t versus their private signal $\pi_{i,t+1}$. As uncertainty rises, agents place less weight on the former and more weight on the latter. The reweighing has a state-dependent effect on activity. When $y_t > 0$, placing less weight on it makes active agents more pessimistic (on average) and pushes them out of the market.[11] In contrast, when $y_t < 0$, the average

[11] Active agents' beliefs are normally distributed with mean $\frac{\rho y_t + \gamma u_t y_{t+1}}{1+\gamma u_t}$ and variance $\frac{\gamma u_t^2}{(1+\gamma u_t)^2}$. So a rise in u_t shifts the entire distribution of beliefs but affects only the behavior of agents with beliefs close to zero (very pessimistic or very optimistic agents will not alter their behavior). Consider the case of $y_t > 0$ and an incremental rise in u_t. If $y_{t+1} = \rho y_t - \epsilon$, where $\epsilon > 0$, then the rise in u_t shifts the distribution downwards and pushes a mass of $\phi\left(\frac{\rho y_t + \gamma u_t y_{t+1}}{\sqrt{\gamma} u_t}\right)$ agents out of the market (those with a

effect of a rise in uncertainty increases activity.

Altogether, uncertainty has an ambiguous effect on activity, depending on the fundamental state. But what happens on average? The effect is no longer ambiguous due to endogenous learning. A rise in uncertainty only affects the beliefs of active agents, who weigh the public signal against their private signal. Since there are more active agents when the fundamental state is high than when it is low (Proposition 2), the adverse effect of uncertainty on activity dominates. Thus, the combination of noisy information with endogenous learning generates a novel mechanism through which uncertainty depresses activity, at least on average. Proposition 4 formulates the connection between A_{t+1} and u_t formally, holding the history of uncertainty $u_0, ..., u_{t-1}$ fixed.

Proposition 4. *For any history of uncertainty* $h^u_{t-1} = \{u_0, ..., u_{t-1}\} \in \mathcal{H}^u_{t-1}$, *expected activity* $E\left(A_{t+1} \middle| h^u_{t-1}, u_t\right)$ *is decreasing in* u_t.

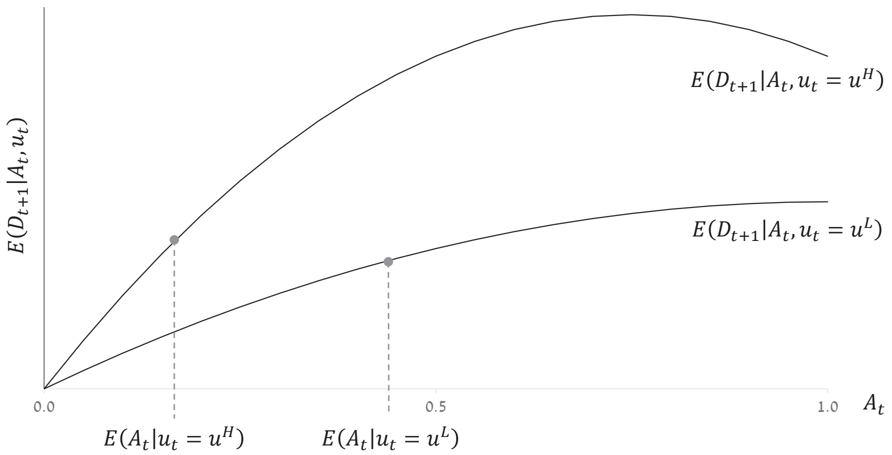
The reason for holding the history of uncertainty fixed is that it affects the distribution of the historical states $y_0, ..., y_{t-1}$, which affects the link between uncertainty and activity before period t. By holding $u_0, ..., u_{t-1}$ fixed, I essentially hold the historical link between uncertainty and activity fixed and consider the effect of a current change in uncertainty. This exercise is equivalent to an impulse response, which is history-dependent since the model is non-linear (as in Gallant et al. (1993) and Koop et al. (1996)).

By setting some limitations on the probability transition matrix of uncertainty, Proposition 4 can be extended to a contemporaneous and history-independent connection between u_t and A_t. If uncertainty is not too dependent on its history ($p^H \approx 1 - p^L$), then the monotonicity result of Proposition 4 extends to a history-independent result. Namely, $E(A_{t+1}|u_t)$ is decreasing in u_t. If, in addition, uncertainty is (even slightly) persistent ($p^H > 1 - p^L$), then the monotonicity is extended to the contemporaneous connection between u_t and $E(A_t|u_t)$. Proposition 5 formalizes this logic and shows that under limited persistence of uncertainty, $E(A_t|u_t)$ is lower when $u_t = u^H$ than when $u_t = u^L$.

The difference in expected activity has implications for the link between activity and disagreement (4). Specifically, Proposition 5 shows that at the high uncertainty state, the connection between activity and disagreement is expected to be stronger than at the low uncertainty state. High uncertainty incentivizes agents to put less weight on the public signal and more on their private signals. Thus, when uncertainty is high, disagreement responds more to changes in activity than is the case when uncertainty is low. The larger response is both through the noisy information channel and through the synchronization channel. However, since activity is expected to be low at the high uncertainty state, both channels operate to increase disagreement around the conditional

zero belief). In the symmetric case of $y_{t+1} = \rho y_t + \epsilon$, beliefs are shifted upwards in the same size, but the mass of affected agents is smaller. Thus, on average, a rise in u_t reduces activity when $y_t > 0$. A similar argument shows that when $y_t < 0$, a rise in uncertainty increases activity.

Figure 4: Effect of Activity on Expected Disagreement at Two Uncertainty States



mean $E(A_t|u_t = u^H)$. Thus, the total response is larger when uncertainty is high than when it is low, as illustrated in Figure 4.

Proposition 5. *Denote by* $\bar{A}_t(u) \equiv E(A_t|u_t = u)$. *There exists some* $\nu > 0$, *such that if* $p^H + p^L - 1 \in (0, \nu)$, *then*

1. $\bar{A}_t(u^H) \leq \bar{A}_t(u^L) \leq \frac{1}{2}$.
2. $\frac{\partial}{\partial A_t} E\left(D_{t+1} \middle| \bar{A}_t(u^H), u^H\right) \geq \frac{\partial}{\partial A_t} E\left(D_{t+1} \middle| \bar{A}_t(u^L), u^L\right) \geq 0$.

4 Empirical Evidence of the Drivers of Disagreement

This section provides empirical evidence for the model's main predictions regarding the drivers of disagreement, focusing on Equation (4). There are several challenges to consider when taking this equation to the data. It contains non-linear interactions between A_t and u_t that cannot be decomposed using standard methods such as log-linearization. The sample size also makes it difficult to estimate the model non-linearly. Furthermore, considering that the model is highly stylized, it is hard to justify estimating Equation (4) as is. Specifically, it is a bit of a stretch to take these non-linear interactions rigorously. Thus, this section focuses on evidence that supports the key economic mechanisms in the model.

To this end, I consider several empirical approaches that differ in the extent to which they adhere to Equation (4). First, I consider the simple correlations implied by Equation (4), namely, that after controlling for uncertainty, disagreement is positively correlated with lagged activity. Second, I tackle the non-linear interactions between A_t and u_t by considering two uncertainty states, as in the theoretical model's Markov chain specification. Finally, abstracting from the static connection implied by Equation (4), I test the dynamic effects of uncertainty and activity on disagreement.

4.1 Data

The analysis focuses on disagreement among professional forecasters in the US, which are surveyed by the Federal Reserve Bank of Philadelphia. The reason for using this measure is twofold. First, it is a common proxy for uncertainty in the literature, and so it is essential to understand other forces that drive it. Second, Coibion and Gorodnichenko (2012) find that a noisy information model best characterizes professional forecasters' expectations in the US. Since noisy information stands at the base of my model, this set of expectations is suitable for testing its predictions.[12] I use two alternative measures of disagreement, each relating to forecasts of a different variable: real GDP growth and inflation. For each variable $y \in \{gdp, infl\}$, let IQR_t^y denote the interquartile range of one-year ahead forecasts of y. I use the quarterly growth of real gross domestic product, GDP_t, to capture aggregate activity. As for uncertainty, since this variable is unobservable, I use several common measures in the literature (Table 1). All measures of disagreement and uncertainty are standardized to facilitate the comparison of results. The analysis is performed on the common sample of all these variables, which is 1986Q1-2019Q4.

4.2 Linear Model

This section focuses on the main quantitative prediction of the model. Recall that controlling for uncertainty, below some upper bound on activity, its effect on disagreement is positive, both through the noisy information channel and through the synchronization channel (Figure 4). Furthermore, around the mean level of activity, the effect is positive (Proposition 5). If activity does not deviate substantially from its mean and crosses the upper bound too often in the data, the prevailing correlation between activity and disagreement will be positive.

Prediction 1. *Controlling for uncertainty, disagreement is positively correlated with lagged activity.*

The prediction is tested using the linear model:

$$IQR_{t+1}^y = \beta_0^y + \beta_1^y GDP_t + \Gamma^y \vec{U}_t + \varepsilon_{t+1}^y \,, \tag{5}$$

[12] Admittedly, when applying the empirical analysis presented in this section to European surveys of professional forecasters, the main results do not hold. This could be due to the shorter sample for which these expectations are available or because sticky information better characterizes these forecasters' disagreement (Andrade and Le Bihan, 2013).

Table 1: Measures of Uncertainty

Variable	Description	Sample	Source
EMV	Newspaper-based equity market volatility tracker	1985Q1-2019Q4 (quarterly average of monthly data)	Baker et al. (2019) at www.policyuncertainty.com
EPUnews	News-based policy uncertainty index for the US	1985Q1-2019Q4 (quarterly average of monthly data)	Baker et al. (2016) at www.policyuncertainty.com
EPUtax	Changes in US tax codes: present value of future scheduled tax code expirations	1985Q1-2019Q4 (quarterly average of monthly data)	Baker et al. (2016) at www.policyuncertainty.com
GPR	Geopolitical risk index based on international news-papers	1985Q1-2019Q4 (quarterly average of monthly data)	Caldara and Iacoviello (2018) at www2.bc.edu/matteo-iacoviello/gpr.htm
JLN	Macroeconomic uncertainty: common factor of forecast uncertainty one year ahead, across various economic series	1960Q2-2019Q4 (quarterly average of monthly data)	Ludvigson et al. (2015) at www.sydneyludvigson.com
VXO	CBOE S&P 100 Volatility Index	1986Q1-2019Q4 (quarterly average of daily data)	Federal Reserve Bank of St. Louis at fred.stlouisfed.org
WUI	Recurrence of the word "uncertain" in the Economist Intelligence Unit reports	1960Q1-2019Q4 (quarterly data)	Ahir et al. (2018) at worlduncertaintyindex.com

Table 2: Correlation between Lagged Activity and Disagreement

Dep. Var	IQR^{gdp}_{t+1}			IQR^{infl}_{t+1}		
	(1)	(2)	(3)	(4)	(5)	(6)
GDP_t	0.02	0.24***	0.36***	-0.11	0.19**	0.23**
	(0.11)	(0.08)	(0.08)	(0.16)	(0.08)	(0.10)
GDP_t^2			-0.13***			-0.05
			(0.04)			(0.07)
Const	-0.54***	-0.60***	-0.57***	-0.21	-0.28***	-0.27***
	(0.09)	(0.05)	(0.05)	(0.17)	(0.07)	(0.08)
Uncertainty measures	No	Yes	Yes	No	Yes	Yes
Obs.	136	136	136	136	136	136
Adj. R-sq.	-0.01	0.39	0.42	0.00	0.33	0.33
F-stat.	0.08	11.81	11.94	1.35	9.36	8.32

Notes: The table shows the OLS estimation results of variation of Equation (5). The full results appear in Appendix A. Newey-West standard errors are reported in parentheses. *** $p < 1\%$, ** $p < 5\%$, * $p < 10\%$.

where $y \in \{gdp, infl\}$ and $\vec{U}_t$ is the vector of uncertainty measures. Prediction 1 implies that $\beta_1^y > 0$.

Table 2 shows a summary of the OLS estimation results of Equation (5) for each of the forecasted variables $y \in \{gdp,\ infl\}$ (full results appear in Appendix A). The results indicate that both measures of disagreement are only weakly correlated with activity without controlling for uncertainty (Columns 1 and 4). Namely, the coefficient of lagged GDP growth is not significant, as illustrated in Figure 1. However, when controlling for uncertainty in Columns 2 and 5, IQR_t^y shows a positive correlation with lagged GDP growth. Namely, the coefficient β_1^y is significantly positive, which supports Prediction 1.

Appendix A provides robustness tests for these results. One concern is that I cannot fully control uncertainty, and my results are sensitive to the measures I have available. To address this issue, I preform a bootstrap procedure and estimate Equation (5) using different subsets of uncertainty measures as controls. I find that the correlation between activity and IQR^{gdp} is positive regardless of the set of uncertainty measures used as controls. Admittedly, the results for IQR^{infl} are more sensitive to the choice of controls. However, when I restrict attention to all specifications that control for the JLN measure (Jurado et al., 2015), the correlation of IQR^{infl} and activity is indeed positive. The JLN estimate directly measures the standard deviation of forecast errors based on public information. Since this is the uncertainty concept that underlies my model, this estimate is likely to be an important control variable.

The second set of robustness tests is a type of placebo test. I estimate Equation (5) using one of the uncertainty measures as the dependent variable instead of a control. I show that no other measure of uncertainty generates positive correlations with GDP, even after controlling for other measures of uncertainty. I conclude that the pro-cyclical component of disagreement is a unique feature of this variable, not related to the uncertainty factor embedded in it.

In the third set of tests, I use alternative activity measures: growth in real investment and the inverse of change in the unemployment rate. Both measures of activity exhibit a positive correlation with disagreement, after controlling for uncertainty. Several other robustness tests appear in Appendix A.

While Prediction 1 focuses on the positive linear connection between activity and disagreement, Equation (4) implies that this connection is concave. To test for this hypothesis, Table 2 also reports estimations of Equation (5), including a quadratic term of GDP growth (Columns 3 and 6). Indeed, the linear term of activity is positive, and the quadratic term is negative (significant for IQR_t^{gdp} but not for IQR_t^{infl}). However, the concavity result is less robust to alternative specifications like those mentioned above. The fragility of the concave shape is consistent with Proposition 5, which implies that activity is expected to affect disagreement positively around the mean level of activity. Namely, around the mean level of activity, both the synchronization and the noisy information channels positively affect disagreement, which might hamper the ability to detect the concave effect empirically.

4.3 Two Uncertainty States

While Proposition 5 implies that the concave effect of activity on disagreement might be hard to detect in the data, Equation (4) still entails another type of non-linearity that was not handled in the previous subsection. Specifically, it includes multiplicative interactions between activity and uncertainty. In this section, I deal with this non-linearity by considering the switch between uncertainty states.

In the theoretical model, uncertainty followed a Markov chain process with two states, high and low uncertainty. Proposition 5 showed that at the high uncertainty state, the mean level of activity is lower and the positive effect of activity on disagreement is stronger. These predictions of are summarized as follows:

Prediction 2. *Let $\mathbb{I}_{\{u_t=u^H\}}$ denote the indicator function that equals one if $u_t = u^H$ and zero otherwise, then*

a. $E\left(GDP_t \middle| u_t = u^H\right) < E\left(GDP_t \middle| u_t = u^L\right)$.

b. The link between activity and disagreement is

$$IQR_{t+1}^y = \beta_0^y + \beta_1^y GDP_t + \beta_2^y GDP_t \, \mathbb{I}_{\{u_t=u^H\}} + \Gamma^y \vec{U}_t + \varepsilon_{t+1}^y, \tag{6}$$

where, $\beta_1 \geq 0$ *and* $\beta_2 > 0$.

I test this prediction in two ways: First, by defining high uncertainty periods exogenously, and second, by estimating them endogenously. For the exogenous approach, I identify high uncertainty periods using the various uncertainty measures. I define a dummy variable for each measure that equals one when the measure is above its eighty-fifth percentile. I use this threshold because around fifteen percent of the observations are NBER recession periods, which are generally accompanied by spikes in uncertainty. As an alternative indicator for high uncertainty periods, I also consider a dummy variable for recessions. Panel A in Table 3 shows that high uncertainty states are associated with lower GDP growth so that Prediction 2.a holds. Panels B and C show OLS estimation results of Equation (6) for IQR^{gdp} and IQR^{infl} as explanatory variables, respectively. In both panels, I can reject the hypothesis that GDP growth's coefficient is negative, consistent with prediction 2.b. The coefficient on the interaction between GDP growth and the dummy for high uncertainty is positive and generally significant for IQR^{gdp}, but mostly insignificant for IQR^{infl}. Altogether, the evidence for Prediction 2.b is robust for GDP forecasts but partial for inflation forecasts.

The second approach I consider to test Prediction 2 estimates the probability of being in a high uncertainty state as part of the model. I consider the following Markov Switching regression with two states $s \in \{1, 2\}$:

$$IQR_{t+1} = \beta_0^s + \beta_1^s GDP_t + \Gamma \vec{U}_t + \sigma^s \varepsilon_{t+1}, \tag{7}$$

$$\text{where, } \varepsilon_{t+1} \sim N(0,1) \text{ and } \Pr(s_t = s | s_{t-1} = s) = p^s.$$

The procedure yields estimators of Γ and $\beta_0^s, \beta_1^s, \sigma^s$ and p^s for each state $s \in \{1, 2\}$. I identify the high uncertainty state as the one with the higher intercept. Namely, state 1 is identified as u^H if and only if $\beta_0^1 > \beta_0^2$. Table 4 shows the estimation results of Model (7). As in the exogenous approach, I find strong evidence for Prediction 2.b for GDP forecasts, as the coefficent of GDP growth is non-negative in the low state and higher in the high state. Again, I find weaker evidence for inflation forecasts, as the coefficient of GDP growth is not significant in either state.

To test Prediction2.a, Panel A in Table 5 shows the estimated probability of being in state u^H in each period. The probability spikes during recessions and is generally close to zero in non-recession periods. Furthermore, Panel B in Table 5 reports the results of a regression of GDP_t on the estimated probability, which yields a significantly negative coefficient. Thus, as in the exogenous identification of uncertainty states, the endogenous approach also shows lower average growth as the probability of high uncertainty increases, supporting Prediction 2.a.

So far, I followed the theoretical model by considering two states of uncertainty. However, the uncertainty measures I use are continuous. In Appendix B, I deviate slightly from the theoretical model's specification and examine the interaction between activity

Table 3: Exogenous High and Low Uncertainty States

$I_{H,t}$	EMV	$EPUnews$	$EPUtax$	GPR	JLN	VXO	WUI	Rec
				A. Dependent Variable: GDP_t				
Const	0.71***	0.70***	0.69***	0.68***	0.73***	0.68***	0.65***	0.74***
	(0.06)	(0.07)	(0.09)	(0.10)	(0.06)	(0.06)	(0.11)	(0.05)
$I_{H,t}$	-0.44	-0.40*	-0.30***	-0.23	-0.63**	-0.27	-0.09	-1.15***
	(0.32)	(0.21)	(0.09)	(0.16)	(0.25)	(0.31)	(0.12)	(0.20)
Obs.	136	136	136	136	136	136	136	136
Adj. R^2	0.07	0.06	0.03	0.01	0.15	0.02	-0.00	0.33
F-stat.	11.01	9.02	4.95	2.86	25.28	3.98	0.41	67.50
				B. Dependent Variable: IQR^{gdp}_{t+1}				
GDP_t	0.09	0.12**	0.19***	0.18**	0.11	0.05	0.19***	0.12*
	(0.05)	(0.06)	(0.07)	(0.07)	(0.07)	(0.06)	(0.07)	(0.07)
$GDP_tI_{H,t}$	0.30***	0.27**	0.00	0.04	0.22*	0.38***	-0.07	0.30**
	(0.09)	(0.10)	(0.09)	(0.11)	(0.13)	(0.08)	(0.20)	(0.15)
$I_{H,t}$	-0.02	-0.07	-0.09	-0.17*	-0.12	-0.29***	0.10	-0.05
	(0.10)	(0.13)	(0.11)	(0.09)	(0.14)	(0.08)	(0.15)	(0.08)
Const	0.88***	0.87***	0.84***	0.85***	0.89***	0.94***	0.82***	0.89***
	(0.04)	(0.04)	(0.05)	(0.05)	(0.06)	(0.04)	(0.04)	(0.05)
Uncertainty measures	+	+	+	+	+	+	+	+
Obs.	136	136	136	136	136	136	136	136
Adj. R^2.	0.44	0.42	0.38	0.39	0.40	0.48	0.38	0.41
F-stat.	11.59	10.80	9.36	9.70	10.15	13.38	9.38	10.35
				C. Dependent Variable: IQR^{infl}_{t+1}				
GDP_t	0.04	0.03	0.06**	0.06**	0.07*	0.06**	0.06**	0.06
	(0.04)	(0.03)	(0.03)	(0.03)	(0.04)	(0.02)	(0.03)	(0.04)
$GDP_tI_{H,t}$	0.04	0.11***	-0.04	-0.03	-0.02	-0.00	0.17	0.01
	(0.06)	(0.04)	(0.04)	(0.06)	(0.05)	(0.05)	(0.13)	(0.13)
$I_{H,t}$	-0.02	-0.06	-0.09	0.06	-0.05	-0.08	-0.20*	0.04
	(0.07)	(0.05)	(0.08)	(0.04)	(0.05)	(0.07)	(0.11)	(0.10)
Const	0.70***	0.71***	0.70***	0.67***	0.69***	0.70***	0.69***	0.68***
	(0.03)	(0.02)	(0.02)	(0.02)	(0.03)	(0.02)	(0.02)	(0.02)
Uncertainty measures	+	+	+	+	+	+	+	+
Obs.	136	136	136	136	136	136	136	136
Adj. R^2	0.32	0.34	0.33	0.32	0.33	0.33	0.34	0.32
F-stat.	7.45	7.95	7.58	7.49	7.57	7.69	7.92	7.42

Notes: The table shows the OLS estimation results of Equation (6). $I_{H,t}$ is a dummy variable that equals one when an uncertainty measure is above its 85th percentile, and in each column, it is defined based on a different measure. Newey-West standard errors are reported in parentheses. *** $p < 1\%$, ** $p < 5\%$, * $p < 10\%$.

Table 4: Markov Switching Regressions

Dep Var.	IQR^{gdp}_{t+1}		IQR^{infl}_{t+1}	
State	u^L	u^H	u^L	u^H
GDP_t	0.07	0.48***	0.15	0.04
	(0.05)	(0.14)	(0.1)	(0.16)
Const	-0.58***	-0.38***	-0.56***	0.27*
	(0.06)	(0.14)	(0.08)	(0.15)
$log(\sigma)$	-1.72***	-0.85***	-1.15***	-0.49***
	(0.09)	(0.16)	(0.09)	(0.12)
Uncertainty Measures	Yes	Yes	Yes	Yes

Notes: The Table shows the estimation results of the Markov switching regression (7). Standard errors based on the observed Hessian are reported in parenthesis. *** $p < 1\%$, ** $p < 5\%$, * $p < 10\%$.

and uncertainty over the continuum of uncertainty values. In this specification, I also find that the relationship between activity and disagreement about future GDP growth strengthens as the level of uncertainty increases. However, for inflation expectations, I find that the relationship does not depend significantly on the uncertainty level.

4.4 Dynamic Effects

The theoretical model and the empirical tests presented so far focus on a static connection between uncertainty, activity, and disagreement. In this section, I test for the dynamic connections between these variables using the local projection method (Jordà, 2005). I estimate the following three-variable local projection model:

$$x_{t+h} = \beta_0 + \sum_{i=0}^{P-1} B_i^h x_{t-i} + \varepsilon_{t+h}^h, \quad h = 1, ..., H,$$

where,

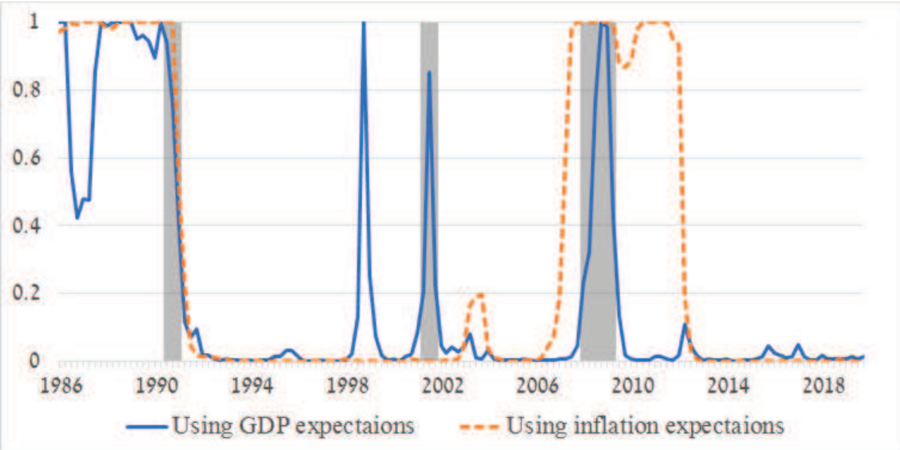
$$x_t = \begin{bmatrix} JLN_t \\ IQR_t^{gdp} \\ GDP_t \end{bmatrix}.$$

The number of lags P is set to eight and structural shocks are identified using the Cholesky ordering specified in the definition of x_t.

Panel A in Figure 5 shows the response of disagreement to a one standard deviation change in the JLN uncertainty measure. As predicted by the model, the effect of the shock is positive. Panel B shows the response of disagreement to a one percentage point increase in GDP growth. Again, I find a positive effect, which is consistent with the model's prediction.

Table 5: Probability of the High Uncertainty State from Markov Switching Regressions

A. Estimated Probability of being in State u^H (Smoothed)



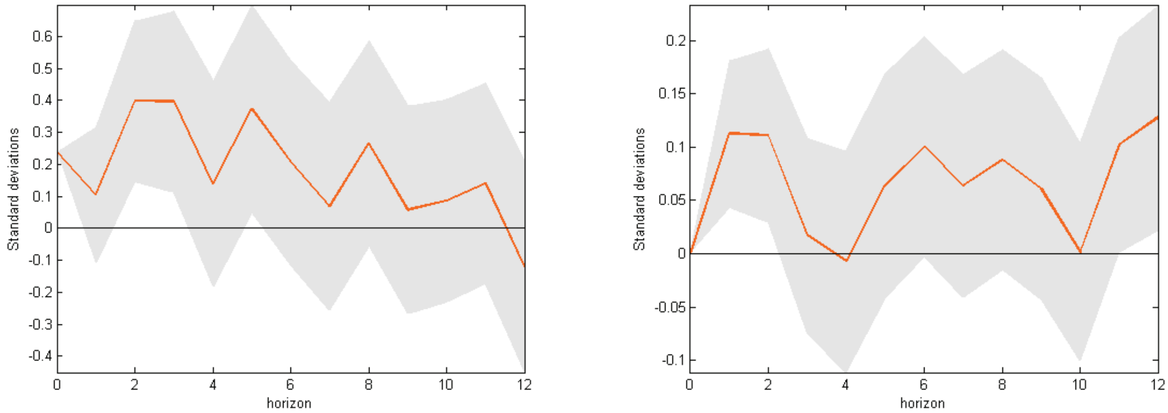
B. GDP Growth on Probability of State u^H (Dep. Var. GDP_t)

	Using GDP expectations	Using inflation expectations
Const	0.70***	0.73***
	(0.05)	(0.06)
PH_t	-0.32**	-0.29***
	(0.14)	(0.11)
Obs.	136	136
Adj. R^2	0.03	0.04
F-stat.	5.30	7.11

Notes: Panel A depicts the smoothed probability of state u^H in each quarter, as estimated in the Markov switching regressions (7). Shaded areas represent NBER recessions. In Panel B, this probability (PH_t) is used as an explanatory variable for GDP growth in an OLS regression. Standard errors are reported in parenthesis. *** $p < 1\%$, ** $p < 5\%$, * $p < 10\%$.

Figure 5: Impulse Responses of Disagreement to Uncertainty and Activity Shocks

A. Response of IQR^{gdp} to JLN B. Response of IQR^{gdp} to GDP



Notes: The figure shows impulse responses using the local projections method. The shock sizes are one unit of the respective variable (one standard deviation increase in the JLN measure in Panel A and one percentage points increase in GDP growth in Panel B). Shaded areas depict 90% confidence intervals based on Newey-West standard errors. The model includes eight lags and three variables with the following Cholesky ordering to identify structural shocks: $[JLN, IQR^{gdp}, GDP]$.

5 Discussion: Why Pay Attention to Disagreement?

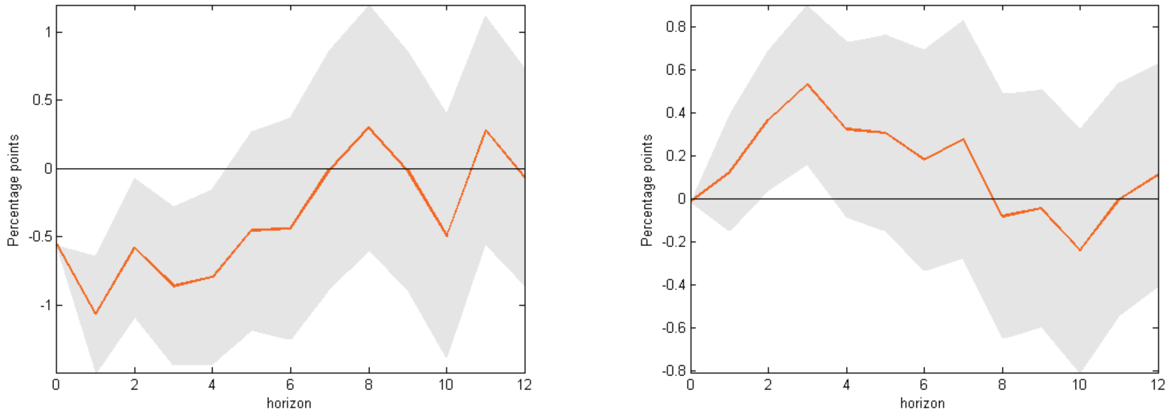
The results shown so far indicate that disagreement may not be a good proxy for uncertainty as it is also affected by changes in activity. This conclusion is in line with previous indications about the validity of this proxy. For example, Lahiri and Sheng (2010) show that when aggregate shocks are volatile, disagreement is not a useful proxy for uncertainty. Thus, when trying to assess uncertainty in real-time, it is preferable to look at the common movement of disagreement and other proxies of uncertainty, as Baker et al. (2016) do in their compound measure of uncertainty.

The question that arises is whether it is at all important to monitor disagreement by itself? I argue that it is, as changes in disagreement, even if not stemming from uncertainty, may affect activity. In the theoretical model, disagreement varied only if uncertainty or activity did. However, one might consider an exogenous change in disagreement, stemming from a change in the parameter γ, which captures the precision of private signals. When the private signals are more precise, active agents put more weight on them and less on the public signal. Thus, an increase in γ affects activity similarly to an increase in u_t, an effect discussed in Section 3.3. The equivalence is formally presented and proved in Appendix D.

A rise in γ also affects disagreement but in two conflicting ways. On the one hand, having agents put more weight on their private signals versus the public signal increases disagreement. On the other hand, if the precision of the private signal improves, then agents receive more similar signals (closer to the true value of the state), and disagreement declines. Thus, if the first mechanism dominates, then a change in the precision of private signals will have opposite effects on activity and disagreement. If the second mechanism

Figure 6: Impulse Responses of GDP Growth to Uncertainty and Disagreement Shocks

A. Response of GDP to JLN B. Response of GDP to IQR^{gdp}



Notes: The figure shows impulse responses using the local projections method. The shock sizes are scaled to produce a one standard deviation increase in the JLN uncertainty measure (Panel A) and in disagreement (Panel B). Shaded areas depict 90% confidence intervals based on Newey-West standard errors. The model includes eight lags and three variables with the following Cholesky ordering to identify structural shocks: $[JLN, IQR^{gdp}, GDP]$.

dominates, the effects will have similar signs.

Since the theory has no clear-cut prediction, I turn to the data to test which effect dominates. Panel B in Figure 6 shows the impulse response of GDP growth to a positive shock to disagreement. The effect is positive, which indicates that in the data, the second channel dominates. Thus, the positive shock to disagreement is consistent with a decline in private signals' precision that increases both activity and disagreement.

The impulse response in Panel B implies that an exogenous change in disagreement creates a positive correlation between it and activity. This is in addition to the direct effect of activity on disagreement (Section 3.2). Therefore, while the empirical evidence in Section 4 supports the existence of the direct effect, it may also capture changes in the precision of private signals. Both channels, which are unique to this model, can explain a positive correlation between activity and disagreement, a phenomenon that the literature has so far not addressed. Furthermore, the direct effect of activity on disagreement amplifies the effect of the precision on disagreement. So regardless of the exogenous source of the increase in disagreement and activity, the direct relationship between them amplifies the initial impact and strengthens the positive relationship between the variables.

Panel A of Figure 6 shows the impulse response of GDP growth to a positive shock to the JLN uncertainty measure. In contrast to the disagreement shock, the uncertainty shock has a mitigating effect on activity.[13] This result highlights another difference between uncertainty and disagreement. However, while the effect of uncertainty on aggregate activity received a lot of attention in the literature, the effect of disagreement remained unnoticed and can direct future research.

[13] Jurado et al. (2015) show a similar result.

6 Conclusion

The existing literature focuses on the role of disagreement as a proxy for uncertainty. This paper shows that disagreement has a distinct feature not shared by common uncertainty measures: it has a pro-cyclical component. This finding is explained using a novel model of noisy information with endogenous learning. In the model, agents privately observe noisy signals about the state only when they are active. As activity rises, more agents receive noisy private signals, and their beliefs draw further apart. This effect is mitigated at high activity levels as agents use more similar information sources, and their beliefs converge. The model's main predictions regarding disagreement are empirically supported using data on professional forecasters in the US.

References

Ahir, H., N. Bloom, and D. Furceri (2018). The world uncertainty index.

Andrade, P. and H. Le Bihan (2013). Inattentive professional forecasters. *Journal of Monetary Economics 60*(8), 967–982.

Atmaz, A. and S. Basak (2018). Belief dispersion in the stock market. *The Journal of Finance 73*(3), 1225–1279.

Bachmann, R., S. Elstner, and E. R. Sims (2013). Uncertainty and economic activity: Evidence from business survey data. *American Economic Journal: Macroeconomics 5*(2), 217–49.

Baker, S. R., N. Bloom, and S. J. Davis (2016). Measuring economic policy uncertainty. *The Quarterly Journal of Economics 131*(4), 1593–1636.

Baker, S. R., N. Bloom, S. J. Davis, and K. J. Kost (2019). Policy news and stock market volatility.

Banerjee, S. (2011). Learning from prices and the dispersion in beliefs. *The Review of Financial Studies 24*(9), 3025–3068.

Banerjee, S. and I. Kremer (2010). Disagreement and learning: Dynamic patterns of trade. *The Journal of Finance 65*(4), 1269–1302.

Bernanke, B. S. (1983). Irreversibility, uncertainty, and cyclical investment. *The Quarterly Journal of Economics 98*(1), 85–106.

Bloom, N. (2009). The impact of uncertainty shocks. *Econometrica 77*(3), 623–685.

Bloom, N. (2014). Fluctuations in uncertainty. *Journal of Economic Perspectives 28*(2), 153–76.

Bloom, N., M. Floetotto, N. Jaimovich, I. Saporta-Eksten, and S. J. Terry (2018). Really uncertain business cycles. *Econometrica 86*(3), 1031–1065.

Bordalo, P., N. Gennaioli, Y. Ma, and A. Shleifer (2020). Overreaction in macroeconomic expectations. *American Economic Review 110*(9), 2748–82.

Borovička, J., L. P. Hansen, and J. A. Scheinkman (2014). Shock elasticities and impulse responses. *Mathematics and Financial Economics 8*(4), 333–354.

Breach, T., S. DAmico, and A. Orphanides (2020). The term structure and inflation uncertainty.

Journal of Financial Economics 138(2), 388–414.

Caldara, D. and M. Iacoviello (2018). Measuring geopolitical risk. *FRB International Finance Discussion Paper* (1222).

Coibion, O. and Y. Gorodnichenko (2012). What can survey forecasts tell us about information rigidities? *Journal of Political Economy 120*(1), 116–159.

Coibion, O. and Y. Gorodnichenko (2015). Information rigidity and the expectations formation process: A simple framework and new facts. *American Economic Review 105*(8), 2644–78.

Fajgelbaum, P. D., E. Schaal, and M. Taschereau-Dumouchel (2017). Uncertainty traps. *The Quarterly Journal of Economics 132*(4), 1641–1692.

Gallant, A. R., P. E. Rossi, and G. Tauchen (1993). Nonlinear dynamic structures. *Econometrica 61*(4), 871–907.

Hong, H. and J. C. Stein (2007). Disagreement and the stock market. *Journal of Economic Perspectives 21*(2), 109–128.

Jordà, Ò. (2005). Estimation and inference of impulse responses by local projections. *American Economic Review 95*(1), 161–182.

Jurado, K., S. C. Ludvigson, and S. Ng (2015). Measuring uncertainty. *American Economic Review 105*(3), 1177–1216.

Koop, G., M. H. Pesaran, and S. M. Potter (1996). Impulse response analysis in nonlinear multivariate models. *Journal of Econometrics 74*(1), 119–147.

Lahiri, K. and X. Sheng (2010). Measuring forecast uncertainty by disagreement: The missing link. *Journal of Applied Econometrics 25*(4), 514–538.

Lucas, R. E. (1973). Some international evidence on output-inflation tradeoffs. *The American Economic Review 63*(3), 326–334.

Ludvigson, S. C., S. Ma, and S. Ng (2015). Uncertainty and business cycles: exogenous impulse or endogenous response? *NBER Working Paper* (21803).

Mackowiak, B. and M. Wiederholt (2009). Optimal sticky prices under rational inattention. *American Economic Review 99*(3), 769–803.

Mankiw, N. G. and R. Reis (2002). Sticky information versus sticky prices: a proposal to replace the New Keynesian Phillips curve. *The Quarterly Journal of Economics 117*(4), 1295–1328.

Mankiw, N. G., R. Reis, and J. Wolfers (2003). Disagreement about inflation expectations. *NBER Macroeconomics Annual 18*, 209–248.

Segal, G., I. Shaliastovich, and A. Yaron (2015). Good and bad uncertainty: Macroeconomic and financial market implications. *Journal of Financial Economics 117*(2), 369–397.

Sims, C. A. (2003). Implications of rational inattention. *Journal of Monetary Economics 50*(3), 665–690.

Veldkamp, L. L. (2005). Slow boom, sudden crash. *Journal of Economic Theory 124*(2), 230–257.

Zohar, O. (2019). Boom-bust cycles of learning, investment and disagreement. *Bank of Israel Discussion Paper 2019*(06).

Appendix A Robustness of the Empirical Linear Model (Equation (5))

Table 6 shows the complete estimation results of Equation (5), which are summarized in Table 2 in the main text. The following sections provide robustness tests for these results, focusing on the positive effect of activity on disagreement that emerges after controlling for uncertainty. First, I test the sensitivity of this result to the inclusion of different uncertainty measures. Second, I perform a placebo test to examine whether there are uncertainty measures, besides disagreement, that show a positive correlation with activity after controlling for other measures of uncertainty. Third, I consider alternative measures of activity instead of GDP growth.

A.1 Alternative Subsets of Uncertainty Measures

In this section, I test the sensitivity of the linear model's results (Equation (5)) to the inclusion of different uncertainty measures. In the form of bootstrap estimation, the test goes over all the subsets of uncertainty measures in Table 1 (total of $2^7 - 1 = 127$ subsets). The measures in each subset are used as controls in Equation (5). Figure 7 presents a histogram of the coefficients of GDP_t in these specifications.

The left panel of Figure 7 shows that the correlation between GDP_t and IQR^{gdp}_{t+1} is positive regardless of the set of uncertainty measures used as controls. Admittedly, the results for IQR^{infl}_{t+1} (right panel) are more sensitive to the choice of controls. However, when I restrict attention to all specifications that control for the JLN measure (Jurado et al., 2015), the correlation between IQR^{infl}_{t+1} and activity is indeed positive. The JLN estimate directly measures the standard deviation of forecast errors based on public information. Since this is the uncertainty concept that underlies my model, this estimate is likely to be an important control variable.

A.2 Placebo Test

Table 7 shows estimations of Equation (5) with measures of uncertainty as dependent variables. In each column of the table, one uncertainty measure is taken as a dependent variable instead of a control. None of these measures exhibits a positive correlation with lagged GDP growth, while measures of disagreement do have a positive correlation. This placebo test indicates that the pro-cyclical component of disagreement is a unique feature of this variable.

Table 6: Effect of Activity on Disagreement - Full Results

Dep. Var	IQR^{gdp}_{t+1}			IQR^{infl}_{t+1}		
GDP_t	0.02	0.24***	0.36***	-0.11	0.19**	0.23**
	(0.11)	(0.08)	(0.08)	(0.16)	(0.08)	(0.10)
GDP_t^2			-0.13***			-0.05
			(0.04)			(0.07)
EMV_t		-0.24***	-0.24***		-0.29***	-0.29***
		(0.06)	(0.07)		(0.11)	(0.11)
$EPUnews_t$		0.22***	0.23***		0.21**	0.21**
		(0.04)	(0.06)		(0.09)	(0.09)
$EPUtax_t$		-0.05*	-0.06**		0.10	0.10
		(0.03)	(0.03)		(0.07)	(0.07)
GPR_t		-0.06**	-0.08***		-0.12***	-0.13***
		(0.03)	(0.03)		(0.04)	(0.04)
JLN_t		0.21***	0.24***		0.37***	0.38***
		(0.05)	(0.06)		(0.10)	(0.11)
VXO_t		0.26**	0.30***		0.23**	0.25**
		(0.10)	(0.08)		(0.11)	(0.10)
WUI_t		-0.11***	-0.11**		-0.16**	-0.16**
		(0.04)	(0.04)		(0.07)	(0.07)
Const	-0.54***	-0.60***	-0.57***	-0.21	-0.28***	-0.27***
	(0.09)	(0.05)	(0.05)	(0.17)	(0.07)	(0.08)
Obs.	136	136	136	136	136	136
Adj. R-sq.	-0.01	0.39	0.42	0.00	0.33	0.33
F-stat.	0.08	11.81	11.94	1.35	9.36	8.32

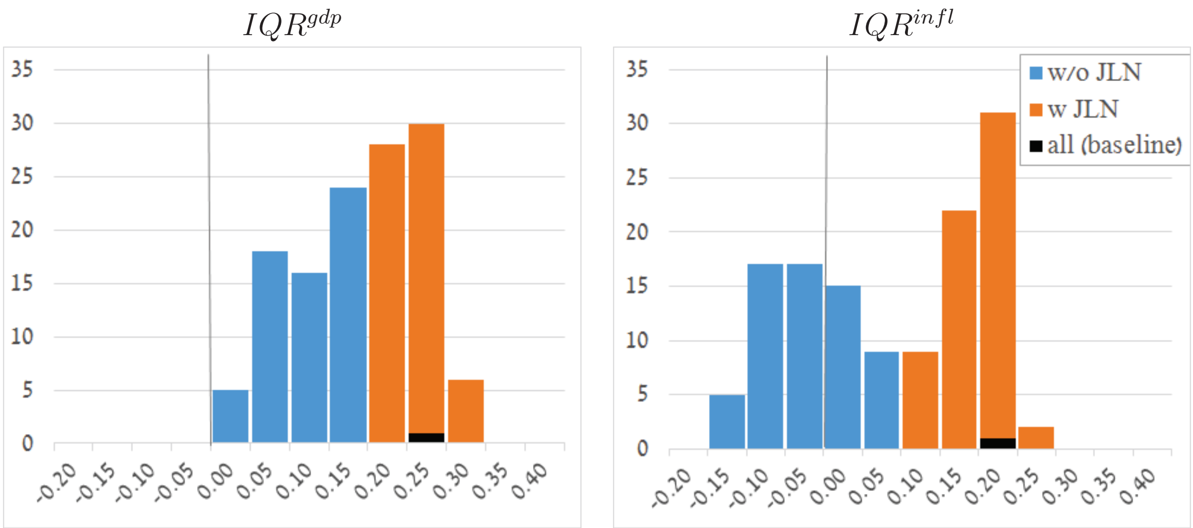
Notes: Newey-West standard errors are reported in parentheses. *** $p < 1\%$, ** $p < 5\%$, * $p < 10\%$..

Table 7: Correlation between Aggregate Activity and Different Uncertainty Measures

Dep. var.	EMV_{t+1}	$EPUnews_{t+1}$	$EPUtax_{t+1}$	GPR_{t+1}	JLN_{t+1}	VXO_{t+1}	WUI_{t+1}
GDP_t	-0.13	-0.35***	-0.03	-0.23	-0.47**	-0.03	-0.08
	(0.15)	(0.12)	(0.29)	(0.21)	(0.19)	(0.11)	(0.16)
EMV_t		-0.03	-0.21	-0.21*	0.05	0.49***	0.36**
		(0.12)	(0.24)	(0.11)	(0.14)	(0.09)	(0.16)
$EPUnews_t$	-0.36***		0.41	0.24	-0.26**	-0.16	0.55
	(0.13)		(0.39)	(0.18)	(0.11)	(0.16)	(0.34)
$EPUtax_t$	-0.01	0.12		-0.34***	0.02	-0.01	0.22
	(0.10)	(0.10)		(0.09)	(0.08)	(0.07)	(0.15)
GPR_t	0.01	0.08	-0.35*		0.02	0.06	0.39***
	(0.07)	(0.08)	(0.21)		(0.06)	(0.08)	(0.08)
JLN_t	0.05	-0.14	0.23	-0.25		0.58***	-0.10
	(0.32)	(0.19)	(0.45)	(0.35)		(0.15)	(0.15)
VXO_t	0.48***	0.18	0.12	0.27	0.30***		-0.43*
	(0.16)	(0.20)	(0.54)	(0.37)	(0.09)		(0.24)
WUI_t	0.24*	0.39*	0.25	0.45***	0.02	-0.03	
	(0.13)	(0.21)	(0.26)	(0.14)	(0.04)	(0.10)	
Const.	0.05	0.06	0.04	0.02	0.12	0.10	0.37**
	(0.11)	(0.11)	(0.37)	(0.18)	(0.19)	(0.11)	(0.16)
Obs.	136	136	136	136	135	137	136
Adj. R-sq.	0.18	0.43	0.19	0.33	0.42	0.45	0.52
F-stat.	5.12	15.53	5.49	10.65	14.79	16.70	22.12

Notes: Newey-West standard errors are reported in parentheses. *** $p < 1\%$, ** $p < 5\%$, * $p < 10\%$.

Figure 7: Correlation between Activity and Disagreement Controlling for Different Subsets of Uncertainty Measures



Notes: The figure shows histograms of $\hat{\beta}_1$ estimated from equation: $IQR^y_{t+1} = \beta_0 + \beta_1 GDP_t + \vec{\gamma} \times \vec{U}_t + \epsilon_t$ for $y = gdp, infl$, going over all the subsets of uncertainty measures $\vec{U}_t$ ($2^7 - 1 = 127$ estimations). Orange bars represent specifications that include the uncertainty measure of Jurado et al. (2015), blue bars – specifications that do not. Full results of the baseline specification with all controls are reported in Table 6.

A.3 Alternative Measures of Activity

Table 8 shows estimation of Equation (5) with alternative measures of activity: growth in real investment ($Invest_t$) and the inverse of change in the unemployment rate ($-\Delta(UE_t)$). Both measures of activity exhibit a positive correlation with disagreement, after controlling for uncertainty. Admittedly, the correlations are less significant than in the baseline estimation with GDP_t.

A.4 Additional Robustness Tests

Table 9 reports several other robustness tests. First, a specification that includes lagged disagreement IQR^y_t. Another specification in Table 9 includes realized quarterly inflation ($Infl_t$) and a measure of inflation uncertainty ($InflVar_t$). Following Breach et al. (2020), the latter variable is estimated as the conditional one-quarter-ahead variance from a GARCH(1,1) model of quarterly inflation on its four lags.[14]

[14] Breach et al. (2020) measure "individual inflation uncertainty" as the conditional variance of forecasts, derived from density forecasts of professional forecasters. They find that average individual uncertainty is highly correlated with conditional variance estimated with a GARCH(1,1) model.

Table 8: Effects of Different Measures of Activity on Disagreement

Dependent var.	IQR^{gdp}_{t+1}			IQR^{infl}_{t+1}		
GDP_t	0.24***			0.19**		
	(0.08)			(0.08)		
$-\Delta(UE_t)$		0.25			0.13	
		(0.15)			(0.22)	
$Invest_t$			0.03***			0.02
			(0.01)			(0.02)
Const.	-0.24***	-0.23***	-0.25***	-0.29***	-0.28***	-0.29***
	(0.06)	(0.08)	(0.08)	(0.11)	(0.11)	(0.11)
Uncertainty measures	Yes	Yes	Yes	Yes	Yes	Yes
Obs.	136	136	136	136	136	136
Adj. R-sq.	0.39	0.33	0.36	0.33	0.31	0.32
F-stat.	11.81	9.33	10.54	9.36	8.72	8.87

Notes: Newey-West standard errors are reported in parentheses. *** $p < 1\%$, ** $p < 5\%$, * $p < 10\%$.

Table 9: Effect of Activity on Disagreement, Controlling for Lagged Disagreement, Inflation, and Inflation Uncertainty

Dep. var.	IQR^{gdp}_{t+1}	IQR^{infl}_{t+1}	IQR^{gdp}_{t+1}	IQR^{infl}_{t+1}
GDP_t	0.17**	0.20**	0.22***	0.15*
	(0.07)	(0.08)	(0.08)	(0.09)
IQR^{gdp}_t	0.38***			
	(0.10)			
IQR^{infl}_t		0.26***		
		(0.09)		
$Infl_t$			0.11***	0.26**
			(0.04)	(0.11)
$InflVar_t$			-0.12*	-0.13
			(0.06)	(0.12)
$InflVar_{t-1}$			0.03	0.15**
			(0.03)	(0.06)
Const.	-0.39***	-0.24***	-0.63***	-0.42***
	(0.05)	(0.06)	(0.08)	(0.11)
Uncertainty measures	Yes	Yes	Yes	Yes
Obs.	136	136	136	136
Adj. R-sq.	0.49	0.37	0.41	0.38
F-stat.	15.30	9.81	6.21	6.55

Notes: Newey-West standard errors are reported in parentheses. *** $p < 1\%$, ** $p < 5\%$, * $p < 10\%$.

Appendix B Continuum of Uncertainty States

In this section I test Prediction 2 using the continuum values of the uncertainty measures, instead of a binary division into two uncertainty states. Panel A in Table 10 shows that all uncertainty measures are negatively correlated with GDP growth. This is in accordance with Prediction 2.a, but also a well known property of the link between activity and uncertainty. Panels B and C of the table show OLS estimation results of Equation 6 but with the uncertainty measures themselves instead of a dummy variable for high levels of the measures. Similarly to the baseline estimation, I find that when it comes to expectations about future GDP growth, disagreement shows a stronger connection to activity the higher the level of uncertainty is. However, in the context of inflation expectations, the link is not significantly affected by the level of uncertainty. All together, I find strong support for Prediction 2.b for GDP expectations, but not for inflation expectations.

Appendix C The Variability of Disagreement

Section 3 showed that disagreement is combined of inter and intra group disagreement:

$$D_{t+1} = \underbrace{A_t \frac{\gamma}{(u_t^{-1}+\gamma)^2}}_{\text{intra-group disagreement}} + \underbrace{A_t(1-A_t)\left[\frac{\gamma\epsilon_{t+1}}{u_t^{-1}+\gamma}\right]^2}_{\text{inter-group disagreement}}. \tag{8}$$

Intra-group disagreement is perfectly predictable in period t given the level of activity A_t and uncertainty u_t. However, inter-group disagreement generates variability in D_{t+1} as it depends on the innovation ϵ_{t+1}. Larger shocks (in absolute terms) increase inter-group disagreement, while smaller shocks decrease it. Equation (8) implies that $STD_t(D_{t+1}) = \frac{\sqrt{2}\gamma^2 u_t}{(u_t^{-1}+\gamma)^2}A_t(1-A_t)$. Specifically, the variability of disagreement is a concave function of activity, as specified in the following prediction:

Prediction 3. *Controlling for uncertainty, the variability of disagreement is a concave function of activity.*

To test this prediction I estimate the dispersion of disagreement as a function of activity as follows. First, I estimate the first and fourth quartiles of the distribution of disagreement conditional on lagged GDP growth and squared lagged growth. Let $Q_{t+1}^{y,\tau}$ denote the τ-quantile of disagreement about $y \in \{gdp, infl\}$ for $\tau = 0.25, 0.75$. I estimate the quantile regressions:

$$Q_{t+1}^{y,\tau} = \sum_{i=0}^{2} \beta_i^{y,\tau} GDP_t^i + \varepsilon_{t+1}^{y,\tau}. \tag{9}$$

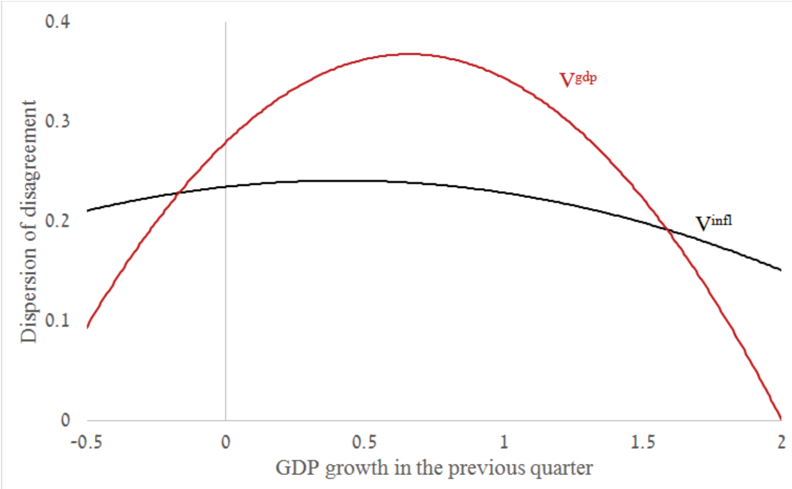
To keep the level of uncertainty relatively constant, I estimate these regressions on non-recessions periods. Estimation results are detailed in Table 11. Second, using the

Table 10: Exogenous High and Low Uncertainty States

$U_t =$	EMV	$EPUnews$	$EPUtax$	GPR	JLN	VXO	WUI
			A. Dependent Variable: GDP_t				
Const	0.64***	0.63***	0.64***	0.64***	0.57***	0.64***	0.67***
	(0.09)	(0.08)	(0.09)	(0.09)	(0.07)	(0.09)	(0.08)
U_t	-0.12	-0.27***	-0.11***	-0.09*	-0.46***	-0.17	-0.09**
	(0.15)	(0.09)	(0.04)	(0.05)	(0.10)	(0.15)	(0.04)
Obs.	136	136	136	136	136	136	136
Adj. R-sq.	0.03	0.15	0.03	0.02	0.26	0.08	0.02
F-stat.	5.77	24.43	5.52	3.78	47.38	13.16	4.15
			B. Dependent Variable: IQR^{gdp}_{t+1}				
GDP_t	0.00	-0.07	0.19***	0.19**	0.01	-0.03	0.19**
	(0.06)	(0.08)	(0.07)	(0.10)	(0.06)	(0.05)	(0.09)
$GDP_t U_t$	0.09***	0.16***	0.00	-0.00	0.13***	0.09***	0.00
	(0.02)	(0.04)	(0.05)	(0.07)	(0.02)	(0.01)	(0.06)
Const	0.89***	0.88***	0.83***	0.83***	0.89***	0.91***	0.83***
	(0.04)	(0.03)	(0.04)	(0.04)	(0.03)	(0.03)	(0.04)
Uncertainty Measures	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Obs.	136	136	136	136	136	136	136
Adj. R-sq.	0.45	0.44	0.39	0.39	0.45	0.47	0.39
F-stat.	13.21	12.78	10.42	10.42	13.23	14.45	10.42
			C. Dependent Variable: IQR^{infl}_{t+1}				
GDP_t	0.04	-0.01	0.06*	0.05	0.04	0.03	-0.00
	(0.04)	(0.06)	(0.04)	(0.04)	(0.04)	(0.04)	(0.05)
$GDP_t U_t$	0.01	0.04	-0.01	0.01	0.02	0.01	0.05*
	(0.01)	(0.03)	(0.02)	(0.03)	(0.02)	(0.01)	(0.03)
Const	0.69***	0.70***	0.68***	0.69***	0.69***	0.70***	0.69***
	(0.03)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.02)
Uncertainty Measures	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Obs.	136	136	136	136	136	136	136
Adj. R-sq.	0.33	0.34	0.33	0.33	0.33	0.33	0.33
F-stat.	8.32	8.66	8.27	8.27	8.34	8.42	8.54

Notes: In each column the variable U_t equals a different uncertainty measure. Panel A tests the correlation between GDP growth and U_t. Panels B and C show OLS estimation results of Equation (6) but with U_t replacing the indicator for high uncertianty. Newey-West standard errors are reported in parentheses. *** $p < 1\%$, ** $p < 5\%$, * $p < 10\%$.

Figure 8: Estimated Variability of Disagreement Conditional on Lagged GDP Growth



Notes: The figure shows the gap between the fourth and first quartiles of disagreement as a function of lagged GDP growth. The functions are estimated using quantile regressions of disagreement on lagged GDP growth and squared lagged growth, in non-recession periods between 1986-2019. The red line refers to disagreement about future GDP growth, and the black line refers to disagreement about future inflation.

estimated coefficients $\hat{\beta}_i^{x,\tau}$, I obtain the interquartile range of disagreement as a quadratic function of GDP growth:

$$V^y(GDP_t) \equiv \sum_{i=0}^{2} \left(\hat{\beta}_i^{y,0.75} - \hat{\beta}_i^{y,0.25} \right) GDP_t^i.$$

Figure 8 shows the estimated V^y functions, extrapolated to the entire range of non-recession GDP growth. In accordance with Prediction 3, I find that both functions are concave.

Table 11: Quantile Regressions

Dep. Var	IQR^{gdp}_{t+1}		IQR^{infl}_{t+1}	
Quantile	0.25	0.75	0.25	0.75
GDP_t	-0.05	-0.04	-0.22***	-0.12
	(0.05)	(0.05)	(0.08)	(0.08)
GDP_t^2	0.07**	0.01	0.20***	0.09
	(0.03)	(0.04)	(0.05)	(0.08)
Const	-0.82***	-0.37***	-0.71***	0.04
	(0.05)	(0.08)	(0.08)	(0.15)
Obs.	136	136	136	136
Adj. R^2	0.01	-0.01	0.03	0.01

Notes: The table shows the estimation results of the quantile regressions (9). Huber-Sandwich standard errors are reported in parentheses. *** $p < 1\%$, ** $p < 5\%$, * $p < 10\%$.

Appendix D The Effect of Noise on Activity

In the baseline model, private signals included noise that was constant over time. Namely, the precision of the private signal γ was constant. In this section I explore the role of noise in this model by studying the effect of a change in γ on activity.

To this end, I take the approach of Gallant et al. (1993), which was also employed by Borovička et al. (2014), to study the impulse responses in a non-linear model. Consider a shift λ in the precision of the distribution of t-period idiosyncratic shocks $\eta_{i,t}^{\lambda} \sim N\big(0, [\lambda\gamma]^{-1}\big)$. The response of A_t to this shift is defined as the expected value of A_t under the perturbed shocks $\eta_{i,t}^{\lambda}$, minus its expected value under the original distribution of shocks. Formally,

$$IR(\lambda) \equiv E_{\eta_{i,t}^{\lambda}}(A_t) - E(A_t)$$

The following proposition shows that $IR(\lambda)$ is decreasing (the proof appears in Appendix E). Since $IR(1) = 0$, it implies that $IR(\lambda) > 0$ for $\lambda > 1$. Namely, an increase in the precision γ decreases activity in the same period. The intuition is that an increase in the precision of the private signals makes active agents put more weight on it and less on the public signal. Thus, a rise in γ has a similar effect to a rise in uncertainty, as discussed in Section 3.3.

Proposition 6. *For any $\underline{\lambda} > 0$, the impulse response function $IR(\lambda)$ is decreasing on $(\underline{\lambda}, \infty)$.*

Appendix E Proofs

E.1 Proof of Proposition 1

The proposition follows by induction from the following Lemma:

Lemma 7. *Given y_t, u_t and $A_t > 0$,*

1. *$A_{t+1} > 0$ almost surely, i.e., $Pr(A_{t+1} = 0|A_t, y_t, u_t) = 0$*

2. *A_{t+1} reveals y_{t+1}, i.e.,* $\Pr(y_{t+1}|y_t, u_t A_t, A_{t+1}) = \mathbb{I}_{\big\{A_{t+1}=A(A_t,y_t,y_{t+1},u_t)\big\}}$.

Proof.

1. $\Pr(A_{t+1} = 0|A_t, y_t, u_t) = \Pr\left((1 - A_t)\mathbb{I}_{\big\{y_t \geq 0\big\}} + A_{t-1}\Phi\left(\frac{1+\gamma u_t}{\sqrt{\gamma u_t}}\rho y_t + \sqrt{\gamma}\epsilon_{t+1}\right) = 0 \Big| A_t, y_t, u_t\right) = 0$ since $\Phi > 0$.

2. $\Phi\left(\frac{1+\gamma u_t}{\sqrt{\gamma u_t}}\rho y_t + \sqrt{\gamma}\epsilon_{t+1}\right)$ is strictly monotone in ϵ_{t+1}. Thus, given $A_t > 0$, y_t and u_t, the function A_{t+1} is also monotone in ϵ_{t+1} and so reveals y_{t+1}.

□

E.2 Proof of Proposition 2

I will prove by induction that for any $h^u_{t-1} \in \mathcal{H}^u_{t-1}$, $G_t(y_t; h^u_{t-1}) \equiv E(A_t|h^u_{t-1}, y_t)$ increases in y_t.

Let $h^u_0 = u_0 \in \{u^L, u^H\}$,

$$G_1(y_1; h^u_0) = (1 - A_0)\mathbb{I}_{\{y_0 \geq 0\}} + A_0 \Phi\left(\frac{\rho y_0 + \gamma u_0 y_1}{\sqrt{\gamma} u_0}\right).$$

Given A_0 and y_0, the function $G_1(y_1; h^u_0)$ is increasing in y_1.

As for $t > 1$, Let $h^u_{t-1} = \{u_0, ..., u_{t-1}\} \in \mathcal{H}^u_{t-1}$ be an arbitrary history and denote the implied history for period $t-2$ by $h^u_{t-2} \equiv \{u_0, ..., u_{t-1}\}$. Furthermore, denote $\sigma^2 = Var(y_{t-1}|h^u_{t-2}) = \sum_{k=0}^{t-2} \rho^{t-2-k} u_k$. First note that following Bayes rule, the probability density function of y_{t-1} given y_t and h^u_{t-1} is

$$f(y_{t-1}|y_t, h^u_{t-1}) = \frac{f(y_t|y_{t-1}, h^u_{t-1}) f(y_{t-1}|h^u_{t-2})}{f(y_t|h^u_{t-1})} = \frac{\varphi\left(\frac{y_t - \rho y_{t-1}}{\sqrt{u_{t-1}}}\right)\varphi\left(\frac{y_{t-1}}{\sigma}\right)}{\varphi\left(\frac{y_t}{\sqrt{\rho^2\sigma^2 + u_{t-1}}}\right)} = \varphi\left(\frac{(\rho^2\sigma^2 y_{t-1} - \rho\sigma^2 y_t)}{\sqrt{(\rho^2\sigma^2 + u)\sigma^2 u}}\right),$$

where φ is the probability density function of the standard normal distribution. Denote $\hat{\Phi}(y_{t-1}, y_t) \equiv \Phi\left(\frac{\rho y_{t-1} + \gamma u_{t-1} y_t}{\sqrt{\gamma} u_{t-1}}\right)$. Now,

$$G_t(y_t) = E_{y_{t-1}|y_t} E\left[(1 - A_{t-1})\mathbb{I}_{\{y_{t-1} \geq 0\}} + A_{t-1}\hat{\Phi}(y_{t-1}, y_t)\middle| y_{t-1}, y_t\right] = \left[1 - E(A_{t-1}|y_{t-1} \geq 0)\right]\Phi\left(\frac{\rho\sigma^2 y_t}{\sqrt{(\rho^2\sigma^2 + u)\sigma^2 u}}\right) + \int_{-\infty}^{\infty} G_{t-1}(y_{t-1}, h^u_{t-2})\hat{\Phi}(y_{t-1}, y_t) f(y_{t-1}|y_t, h^u_{t-1}) dy_{t-1}$$

The first argument increases with y_t, and $\left|\frac{\partial}{\partial y_t} G_{t-1}(y_{t-1}, h^u_{t-2})\hat{\Phi}(y_{t-1}, y_t)\right| \leq \sqrt{\gamma}\phi(0)$. Thus, the dominated convergence theorem implies that

$$\frac{\partial}{\partial y_t} G_t(y_t; h^u_{t-1}) \geq \frac{\partial}{\partial y_t}\int_{-\infty}^{\infty} G_{t-1}(y_{t-1})\hat{\Phi}(y_{t-1}, y_t) f(y_{t-1}|y_t, h^u_{t-1}) dy_{t-1} = \int_{-\infty}^{\infty} G_{t-1}(y_{t-1}; h^u_{t-2})\varphi\left(\frac{1}{\sqrt{\gamma}}\rho y_{t-1} + \sqrt{\gamma} y_t)\right) f(y_{t-1}|y_t, h^u_{t-1}) dy_{t-1} + \frac{\rho\sigma^2}{\sqrt{(\rho^2\sigma^2 + u)\sigma^2 u}}\int_{-\infty}^{\infty} G_{t-1}(y_{t-1}; h^u_{t-2})\hat{\Phi}(y_{t-1}, y_t)\varphi'\left(\frac{\rho\sigma^2 y_t - \rho^2\sigma^2 y_{t-1}}{\sqrt{(\rho^2\sigma^2 + u)\sigma^2 u}}\right) dy_{t-1}$$

Since the first argument is positive, it remains to be shown that second one is also positive. Using integration by substitution with $z = \alpha y_{t-1} - \beta y_t$, where $\alpha = \frac{\rho^2\sigma^2}{\sqrt{(\rho^2\sigma^2+u)\sigma^2 u}}$ and $\beta = \frac{\rho\sigma^2}{\sqrt{(\rho^2\sigma^2+u)\sigma^2 u}}$, the second argument is proportional to

$$\int_{-\infty}^{\infty} G_{t-1}\left(\frac{z+\beta y_t}{\alpha}; h_{t-1}^u\right)\hat{\Phi}\left(\frac{z+\beta y_t}{\alpha}, y_t\right) z\varphi(z)dz = \int_0^{\infty}\left[G_{t-1}\left(\frac{z+\beta y_t}{\alpha}; h_{t-1}^u\right)\hat{\Phi}\left(\frac{z+\beta y_t}{\alpha}, y_t; h_{t-1}^u\right) - G_{t-1}\left(\frac{-z+\beta y_t}{\alpha}; h_{t-1}^u\right)\hat{\Phi}\left(\frac{-z+\beta y_t}{\alpha}, y_t; h_{t-1}^u\right)\right] z\varphi(z)dz \geq 0,$$

where the inequality follows from the fact that G_{t-1} and $\hat{\Phi}$ are increasing in the first argument, so the expression in squared brackets is non-negative for any $z > 0$.

□

E.3 Proof of Corollary 3

$$D_{t+1} = (1-A_t)\left[\frac{\gamma}{u_t^{-1}+\gamma}A_t\epsilon_{t+1}\right]^2 + A_t\left[\frac{\gamma}{u_t^{-1}+\gamma}\right]^2\int_0^1\left((1-A_t)\epsilon_{t+1}+\eta_{i,t+1}\right)^2 di = \left[\frac{\gamma}{u_t^{-1}+\gamma}\right]^2\left[(1-A_t)A_t^2\epsilon_{t+1}^2 + (1-A_t)^2 A_t\epsilon_{t+1}^2 + \gamma^{-1}A_t\right] = \frac{\gamma}{(u_t^{-1}+\gamma)^2}\left[A_t + A_t(1-A_t)\gamma\epsilon_{t+1}^2\right]$$

□

E.4 Proof of Proposition 4

Let $h_{t-1}^u = \{u_0, ..., u_{t-1}\}$ be an arbitrary history.

1. $E\left(D_{t+1}\middle|h_{t-1}^u, u_t\right) = \frac{\gamma}{(u_t^{-1}+\gamma)^2}\left[E\left(A_t\middle|h_{t-1}^u\right) + \gamma E\left(A_t(1-A_t)\middle|h_{t-1}^u\right)u_t\right]$ is increasing in u_t.

2. In this part only, in order to facilitate the analysis, I assume that u_t is a continuous variable and I will prove that $\frac{\partial}{\partial u_t}E\left(A_{t+1}\middle|h_{t-1}^u, u_t\right) < 0$. This will of course imply that $E\left(A_{t+1}\middle|h_{t-1}^u, u_t\right)$ is lower for $u_t = u^H$ than for $u_t = u^L$.

   Note that $y_t|h_{t-1}^u$ is normally distributed with a mean of zero, and denote its probability density function by $f_{y_t|h_{t-1}^u}$.

$$\frac{\partial}{\partial u_t}E\left(A_{t+1}\middle|h_{t-1}^u, u_t\right) = \frac{\partial}{\partial u_t}\int_{-\infty}^{\infty}E\left(A_t\middle|h_{t-1}^u, y_t\right)\Phi\left(\sqrt{\frac{1+\gamma u_t}{\gamma u_t^2}}\rho y_t\right)f_{y_t|h_{t-1}^u}(y_t)dy_t.$$

   Note that $\left|\frac{\partial}{\partial u_t}E\left(A_t\middle|h_{t-1}^u, y_t\right)\Phi\left(\sqrt{\frac{1+\gamma u_t}{\gamma u_t^2}}\rho y_t\right)\right| \leq \left|\frac{\partial}{\partial u_t}\Phi\left(\sqrt{\frac{1+\gamma u_t}{\gamma u_t^2}}\rho y_t\right)\right| \leq$

$\phi(0)\left(\frac{\partial}{\partial u_t}\sqrt{\frac{1+\gamma u_t}{\gamma u_t^2}}\right)\rho|y_t|$. Since $E\left(|y_t|\middle|h_{t-1}^u,u_t\right)<\infty$, the dominated convergence theorem implies that:

$$\frac{\partial}{\partial u_t}E\left(A_{t+1}\middle|h_{t-1}^u,u_t\right)=\int_{-\infty}^{\infty}E\left(A_t\middle|h_{t-1}^u,y_t\right)\frac{\partial}{\partial u_t}\Phi\left(\sqrt{\frac{1+\gamma u_t}{\gamma u_t^2}}\rho y_t\right)f_{y_t|h_{t-1}^u}(y_t)dy_t=$$

$$\int_0^{\infty}\left[E\left(A_t\middle|h_{t-1}^u,y_t\right)-E\left(A_t\middle|h_{t-1}^u,-y_t\right)\right]\phi\left(\sqrt{\frac{1+\gamma u_t}{\gamma u_t^2}}\rho y_t\right)\frac{\partial}{\partial u_t}\left(\sqrt{\frac{1+\gamma u_t}{\gamma u_t^2}}\right)\rho y_t f_{y_t|h_{t-1}^u}(y_t)dy_t$$

$\frac{\partial}{\partial u_t}\left(\sqrt{\frac{1+\gamma u_t}{\gamma u_t^2}}\right)<0$, and following Proposition 2 , the argument in squared brackets is positive. Thus, $\frac{\partial}{\partial u_t}E\left(A_{t+1}\middle|h_{t-1}^u,u_t\right)<0$.

□

E.5 Proof of Proposition 5

1. Denote by $\mathcal{H}_{t-1}^H$ the set of histories that end with $u_{t-1}=u^H$, namely, $\mathcal{H}_{t-1}^H=\{(u_0,...,u_{t-1})\in\mathcal{H}_{t-1}^u|u_{t-1}=u^H\}$. Similarly, denote by $\mathcal{H}_{t-1}^L$ the set of histories that end with $u_{t-1}=u^L$. Thus, $\mathcal{H}_{t-1}^u=\mathcal{H}_{t-1}^H\uplus\mathcal{H}_{t-1}^L$.

$$\begin{aligned}E(A_{t+1}|u_t=u^H)=&\sum_{h_{t-1}^u\in\mathcal{H}_{t-1}^u}\Pr(h_{t-1}^u|u_t=u^H)E(A_{t+1}|h_{t-1}^u,u^H)=\\&(2-p^H-p^L)\left[\frac{p^H}{1-p^L}\sum_{h_{t-1}^H\in\mathcal{H}_{t-1}^H}\Pr(h_{t-1}^H)E(A_{t+1}|h_{t-1}^H,u^H)+\right.\\&\left.\sum_{h_{t-1}^L\in\mathcal{H}_{t-1}^L}\Pr(h_{t-1}^L)E(A_{t+1}|h_{t-1}^L,u^H)\right]=\\&(2-p^H-p^L)\left[\sum_{h_{t-1}^u\in\mathcal{H}_{t-1}^u}\Pr(h_{t-1}^u)E(A_{t+1}|h_{t-1}^u,u^H)+\right.\\&\left.\frac{p^H+p^L-1}{1-p^L}\sum_{h_{t-1}^H\in\mathcal{H}_{t-1}^H}\Pr(h_{t-1}^H)E(A_{t+1}|h_{t-1}^H,u^H)\right].\end{aligned}\quad(10)$$

Similarly,

$$E(A_{t+1}|u_t = u^L) = (2 - p^H - p^L)\Bigg[\sum_{h^u_{t-1}\in\mathcal{H}^u_{t-1}} \Pr(h^u_{t-1})E(A_{t+1}|h^u_{t-1}, u^L)+$$

$$\frac{p^H + p^L - 1}{1 - p^H} \sum_{h^L_{t-1}\in\mathcal{H}^L_{t-1}} \Pr(h^L_{t-1})E(A_{t+1}|h^L_{t-1}, u^L)\Bigg]. \quad (11)$$

$\underline{\Delta} \equiv \min_{h^u_{t-1}\in\mathcal{H}^u_{t-1}} \Big[E(A_{t+1}|h^u_{t-1}, u^L) - E(A_{t+1}|h^u_{t-1}, u^H)\Big]$. Proposition 4 implies that $\underline{\Delta} > 0$. Now, subtracting (10) from (11) yields

$$E(A_{t+1}|u_t = u^L) - E(A_{t+1}|u_t = u^H) \propto$$

$$\sum_{h^u_{t-1}\in\mathcal{H}^u_{t-1}} \Pr(h^u_{t-1})\Big[E(A_{t+1}|h^u_{t-1}, u^L) - E(A_{t+1}|h^u_{t-1}, u^H)\Big]+$$

$$(p^H + p^L - 1)\Bigg[\frac{1}{1 - p^H} \sum_{h^L_{t-1}\in\mathcal{H}^L_{t-1}} \Pr(h^L_{t-1})E(A_{t+1}|h^L_{t-1}, u^H)-$$

$$\frac{1}{1 - p^L} \sum_{h^H_{t-1}\in\mathcal{H}^H_{t-1}} \Pr(h^H_{t-1})E(A_{t+1}|h^H_{t-1}, u^L)\Bigg] \geq$$

$$\underline{\Delta} - \frac{(p^H + p^L - 1)\Pr(u_{t-1} = u^H)}{1 - p^L} = \underline{\Delta} - \frac{p^H + p^L - 1}{2 - p^L - p^H}.$$

As can bee seen in the proof of Proposition 4, $E(A_{t+1}|h^u_{t-1}, u_t)$ is independent of p^H and p^L, which implies that $\underline{\Delta}$ is also independent of them. Thus, for $p^H + p^L$ close enough to one, the final expression is positive, so $E(A_{t+1}|u_t = u^L) > E(A_{t+1}|u_t = u^H)$.

Now,

$$E\left(A_{t+1}|u_{t+1} = u^L\right) - E\left(A_{t+1}|u_{t+1} = u^H\right) =$$

$$p^L E(A_{t+1}|u_t = u^L) + (1 - p^L)E(A_{t+1}|u_t = u^H)-$$

$$(1 - p^H)E(A_{t+1}|u_t = u^L) - p^H E(A_{t+1}|u_t = u^H) =$$

$$(p^L + p^H - 1)\Big[E(A_{t+1}|u_t = u^L) - E(A_{t+1}|u_t = u^H)\Big],$$

where the final expression is positive if $p^L + p^H > 1$.

The next step is to show that $E\left(A_{t+1}|u_{t+1}\right) \leq 0.5$. It suffices to show that $E(A_{t+1}|h^u_t) \leq 0.5$ for any $h^u_t \in \mathcal{H}^u_t$, as $E\left(A_{t+1}|u_{t+1}\right)$ is weighted average of $\left\{E(A_{t+1}|h^u_t)\right\}_{h^u_t\in\mathcal{H}^u_t}$. Let $h^u_t \in \mathcal{H}^u_t$ be an arbitrary history.

$$E(A_{t+1}|h_t^u) = E\left[(1-A_t)\mathbb{I}_{\{y_t\geq 0\}} + A_t\Phi\left(\frac{1+\gamma u_t}{\sqrt{\gamma}u_t}\rho y_t + \sqrt{\gamma}\epsilon_{t+1}\right)\Bigg|h_t^u\right] =$$

$$\Big[1 - E(A_t|h_t^u, y_t \geq 0)\Big]\frac{1}{2} + E\left[A_t\Phi\left(\sqrt{\frac{1+\gamma u_t}{\gamma u_t^2}}\rho y_t\right)\Bigg|h_t^u\right] =$$

$$\frac{1}{2}\Bigg(1 - E(A_t|h_t^u, y_t \geq 0) + E\left[A_t\Phi\left(\sqrt{\frac{1+\gamma u_t}{\gamma u_t^2}}\rho y_t\right)\Bigg|h_t^u, y_t \geq 0\right] +$$

$$E\left[A_t\Phi\left(\sqrt{\frac{1+\gamma u_t}{\gamma u_t^2}}\rho y_t\right)\Bigg|h_t^u, y_t < 0\right]\Bigg) =$$

$$\frac{1}{2}\Bigg(1 - E\left[A_t\Phi\left(-\sqrt{\frac{1+\gamma u_t}{\gamma u_t^2}}\rho y_t\right)\Bigg|h_t^u, y_t \geq 0\right] + E\left[A_t\Phi\left(\sqrt{\frac{1+\gamma u_t}{\gamma u_t^2}}\rho y_t\right)\Bigg|h_t^u, y_t < 0\right]\Bigg) =$$

$$\frac{1}{2} - \frac{1}{2}\int_0^\infty \Big[E(A_t|h_{t-1}^u, y_t) - E(A_t|h_{t-1}^u, -y_t)\Big]\Phi\left(-\sqrt{\frac{1+\gamma u_t}{\gamma u_t^2}}\rho y_t\right) f(y_t|h_t^u)dy_t < \frac{1}{2}$$

where the final inequality is due to $E(A_t|h_{t-1}^u, y_t) > E(A_t|h_{t-1}^u, -y_t)$ for $y_t > 0$ (Proposition 2).

2.

$$\frac{\partial}{\partial A_t}E\left(D_{t+1}|\bar{A}_t(u^H), u^H\right) = \left(\frac{\gamma u^H}{1+\gamma u^H}\right)^2\Big[\gamma^{-1} + u^H\big(1 - 2\bar{A}(u^H)\big)\Big] \overset{\bar{A}(u^H)\leq\bar{A}(u^L)}{\geq}$$

$$\left(\frac{\gamma u^H}{1+\gamma u^H}\right)^2\Big[\gamma^{-1} + u^H\big(1 - 2\bar{A}(u^L)\big)\Big] \overset{\bar{A}(u^L)\leq 0.5}{\geq} \left(\frac{\gamma u^L}{1+\gamma u^L}\right)^2\Big[\gamma^{-1} + u^L\big(1 - 2\bar{A}(u^L)\big)\Big] =$$

$$\frac{\partial}{\partial A_t}E\left(D_{t+1}|\bar{A}_t(u^L), u^L\right) \overset{\bar{A}(u^L)\leq 0.5}{\geq} 0$$

□

E.6 Proof of Proposition 6

Using iterated expectations:

$$IR(\lambda) =$$

$$E\Big(E_{\eta_{i,t}^\lambda}(A_t|h_{t-1}^u, y_{t-1}) - E(A_t|h_{t-1}^u, y_{t-1})\Big) =$$

$$E\left(E(A_{t-1}|h_{t-1}^u, y_{t-1})\left[\Phi\left(\sqrt{\frac{1+\lambda\gamma u_{t-1}}{\lambda\gamma u_{t-1}^2}}\rho y_{t-1}\right) - \Phi\left(\sqrt{\frac{1+\gamma u_{t-1}}{\gamma u_{t-1}^2}}\rho y_{t-1}\right)\right]\right).$$

Denote by $g(\lambda; u_{t-1}) = \rho\sqrt{\frac{1+\lambda\gamma u_{t-1}}{\lambda\gamma u_{t-1}^2}}$. Since $\left|\frac{\partial}{\partial\lambda}E(A_{t-1}|h_{t-1}^u, y_{t-1})\Phi\big(g(\lambda; u_{t-1})\big)\right| \leq$

$\phi(0)\left|\frac{\partial}{\partial\lambda}g\left(\underline{\lambda};u^L\right)\right|$, the dominated convergence theorem implies that

$$IR'(\lambda) = E_{h^u_{t-1},y_{t-1}}\left(E(A_{t-1}|h^u_{t-1},y_{t-1})\phi\left(g(\lambda;u_{t-1})y_{t-1}\right)\frac{\partial}{\partial\lambda}g(\lambda;u_{t-1})y_{t-1}\right) =$$

$$E_{h^u_{t-1}}\Bigg(\int_0^\infty \Big[E(A_{t-1}|h^u_{t-1},y_{t-1}) - E(A_{t-1}|h^u_{t-1},-y_{t-1})\Big]$$

$$\phi\left(g(\lambda;u_{t-1})y_{t-1}\right)\frac{\partial}{\partial\lambda}g(\lambda;u_{t-1})y_{t-1}f_{y_{t-1}|h^u_{t-1}}(y_{t-1})dy_{t-1}\Bigg).$$

Following Proposition 2, the term in squared brackets is positive. Furthermore, $\frac{\partial}{\partial\lambda}g(\lambda;u_{t-1}) < 0$ for any u_{t-1}. Thus, $IR'(\lambda) < 0$.

□