ADVERSE SELECTION AND

THE MARKET FOR ANNUITIES

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Abstract

Adverse selection is often blamed for the thinness of the annuities market. We study alternative types of annuity contracts that differ in the survival information structure, and explore their welfare implications. We show that, in principle, it is preferable to contract before the survival information is revealed, i.e., the deferred annuities equilibrium is better than the adverse selection equilibrium of immediate annuities. Quantitatively, however, the two arrangements are very close in terms of expected welfare. Our simulations show a welfare loss of around one percent for annuitants using the immediate annuities adverse selection market, relative to the first best allocation. We conclude that adverse selection is not the cause for the thinness of the annuities market.

Key words: adverse selection, annuities, insurance, information, Social Security reform, Defined Benefits, Defined Contribution.

JEL Classification: H55, G22, G28

ADVERSE SELECTION AND THE MARKET FOR ANNUITIES

1. Introduction

Several studies document that annuities markets are susceptible to adverse selection.[1] Annuity contracts promise to pay their owners predetermined monthly installments as long as they live.[2] The later individuals purchase annuity contracts, the more likely are they to be informed about their longevity prospects. Several studies attribute the small size of the annuity market to adverse selection.[3]

The recent interest in the functioning of private annuities markets partly stems from its policy implications. A frequently mentioned advantage of the universality of Social Security programs is the avoidance of the adverse selection problem.[4] The magnitude of the adverse selection problem is thus of special interest in the context of the public debate regarding the partial privatization of the Social Security system.[5] Assessing the magnitude of the adverse selection problem is also important for the welfare implications of the recent move from Defined Benefit (DB) pension plans to

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[1] See, for example, Poterba (2001) and Finkelstein and Poterba (2002). The lower mortality rates of annuitants, compared with those for the general population, are also evident in, among others, Mitchell, Poterba, Warshawsky and Brown (1999).

[2] Other variations on the contract include joint survivorship, guaranteed ten-year payments, etc.

[3] Akerloff (1970) introduced the process by which a market may disappear due to asymmetric information. Eichenbaum and Peled (1987), among others, imply that asymmetric information may be a major reason for the small size of the annuity market. Warshawsky (1988) and Friedman and Warshawsky (1990) indicate that a bequest motive and the low yields on the assets in which insurance companies are allowed to invest may contribute to the small size of the annuity market.

[4] Stiglitz (1988, p. 332) states: "Adverse selection may provide part of the explanation for high premiums charged for annuities. The government, however, can force all individuals to purchase the insurance, and thus avoid the problem of adverse selection."

[5] Kotlikoff-Smetters-Walliser (1998) discuss the impact of adverse selection on social security privatization. Their discussion relates opting out of the system to income and age, and is not based on information..

Defined Contribution (DC) pension plans. DB pension plans are usually contracted at a relatively young age, when insureds have little private information regarding their survival probabilities. Thus, these contracts should be more immune to the adverse selection problem than DC plans that allow each participant to choose between lump sum and annuity distributions at the time of retirement.[6]

Assessing the importance of adverse selection is the main objective of this paper. We assess the qualitative and quantitative effects of adverse selection under four alternative types of annuity contracts. These contracts differ in the availability, to the insured and the insurer, of the information that may help predict the insured's longevity. We rank the welfare associated with these contracts and discuss the policy implications of this ranking.

While the qualitative analysis reviews several alternatives to limit the harmful effects of asymmetric information, the quantitative analysis obtains measures of its importance by simulating standard consumer behavior under adverse selection in annuity markets. Our simulations indicate that adverse selection increases the price of annuity by between one percent and seven percent, as compared to the no-adverse selection case. The induced welfare loss is even smaller.

Mitchell, Poterba, Warshawsky and Brown (1999) show that the cost of an annuity in the US exceeds its fair actuarial value by 6 to 10 percent if the annuitants life tables are used. They note that these margins include marketing costs, corporate overhead and profits, in addition to the impact of adverse selection. James and Vittas (1999) find similar figures in an international comparison. Our simulated results are within these bounds. Our study also explains why, in contrast to other insurance

[6] Poterba (2001) shows that a requirement to annuitize at least a fraction of the accumulation in DC plans may also mitigate the impact of adverse selection.

markets, asymmetric information should not be detrimental to the existence of the annuities market.

The rest of the paper is organized as follows. In section 2 we present the four types of annuity contracts. In section 3 we compare the insured's expected utility under the four types of annuity contracts. In Section 4 we present simulations that evaluate the effect of adverse selection under our four contract types. In Section 5 we present the multiperiod model and simulate its asymmetric information equilibrium. Section 6 deals with the impacts of the existence of a bequest motive and of a Social Security system. Section 7 concludes the paper. The Appendix includes the more technical aspects of the analysis.

2. Contract in Annuity Markets

We assume that at a young age insureds believe that their expected longevity equals the population's average. However, their information at the time of retirement is more precise. We consider four alternative types of annuity contracts. The "precommitment" annuity contract is initiated at a young age, before the information on longevity is revealed. It corresponds to a deferred annuity contract, a DB plan or the existing Social Security system. In contrast, the second and third annuity contracts are initiated at the time of retirement (similar to a DC plan or the proposed privatized element of the Social Security system). The "public information" annuity contract is assumed to be initiated when information regarding the insured's survival probabilities is known to both the insured and to the insurer (i.e., information is also symmetric). The "asymmetric information"annuity contract is assumed to be initiated when this information is known only to the insured. In the "partial redemption"contracts insureds initiate the contract at a young age when they do not know their survival probability (as in a DB plan). However, at the time of retirement, when they know these probability, they may redeem part of their annuity subject to some penalty.[7]

Our insureds live for two periods, and consume at the end of each period, at dates t=1,2. In the first period they lives with certainty, but their survival probability through period 2 is $q < 1$. We assume two types of insureds: high survival type and low survival type (denoted as H-insureds and L-insureds, respectively) with $q^H > q^L$ and $(q^H+q^L)/2=$ ½.[8] At date t=0 insureds only know that they can be of either type with a probability of ½, but between t=0 and t=1 they finds out their type. C_1 and C_2 denote consumption levels at the end of periods 1 and 2, respectively.

The insureds maximize a time separable expected utility $u(c)$, $u' > 0$, $u'' < 0$, $u'(0)=\infty$, with a time preference factor $\beta < 1$. It is assumed that they derive utility from consumption at date t=2 only if they survive, indicating the absence of a bequest motive.[9] The interest rate is denoted by r. The exposition and interpretation of our results are simplified by assuming that $\beta(1+r)=1$.[10] Thus, the insured's expected utility is: $u(C_1)+q\beta u(C_2)$, where q is the probability known to the insured at the time the allocation is made. At t=0 insureds use the probability q=0.5, while at t=1 they use the probabilities q_H or q_L according to their type. Under the public information contract insurers know the type information of each insured at t=1, and are allowed to

[7] The redemption plan also adds a liquidity option which is valuable for meeting the contingency of unexpected expenditures. This issue is not explored explicitly in the current paper.

[8] The equal type probabilities maximize the variance of the type distribution. Thus, it is a conservative assumption for demonstrating that the asymmetric information problem does not have a major effect.

[9] The absence of a bequest motive is consistent with the findings in Altonji *et al.* (1997). It also simplifies the initial presentation. In Section 6 we incorporate a bequest motive and demonstrate that our main results are robust to this change.

use it in determining premiums. In contrast, under the asymmetric information and partial redemption contracts insurers either do not have the information or are not allowed to use it. The premium set by the insurance company depends on its information structure and on the behavior of the insureds. We assume that competition in the insurance market guarantees that insurers balance actuarially.

Insureds can use two assets as saving vehicles: a regular, non-annuitized, financial asset *D* and an annuity *A*, with the respective prices P_D and P_A. The annuity pays out one consumption unit in the second period contingent upon survival, while the non-annuitized financial asset pays out one consumption unit unconditionally. Assuming competition between insurers and no overhead costs, $P_D=1/(1+r)$ and $P_A=q/(1+r)$, where q is the insurer's estimate of the survival probability. The price of the annuity is lower than that of the non-annuitized financial asset, because $q<1$[11]; hence, the only rationale for holding a non-annuitized asset in a two-period model is the desire to leave a bequest in the event of death before t=2.[12] Thus, in a two-period model in which a bequest motive is absent, insureds annuitize all their wealth. We thus simplify the presentation in this paper by ignoring the non-annuitized asset in all two-period models in which individuals have no bequest motive. Next we present, for each of the four contract types the information structure, the annuity contract and the resulting consumption levels.

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[10] This assumption implies that an individual who lives with certainty would choose identical consumption levels at dates t=1 and t=2.

[11] Formally, $q \leq 1$, hence $P_A \leq P_D$, so an annuity is always purchased.

[12] In a multi period model individuals may purchase a non-annuitized asset to generate a decreasing consumption pattern.

Contract 1: Full precommitment.

Each insured purchases at date t=0 the consumption for dates t=1 and t=2, denoted as C_1 and C_2. Consumption at t=0 takes place with certainty, but at date t= 2 it is contingent upon the survival of the insured. The timing of information arrival and individual actions are summarized in the following time line:

t=0		t=1		t=2
Purchases an annuity	Type is revealed	Consumes C_1	Longevity is revealed	Consumes C_2 if survives

Because the type information is not known at the time the contract is made, at date t=0, both insureds and insurers use ½ (the average survival probability) as the relevant probability in calculating the expected utility, the budget constraint, and the annuity price. All insureds buy the same annuity amount, and thus the annuity price is: $P_A=0.5/(1+r)$. Recall that all of the second period consumption is bought as an annuity.

The insured's problem is thus:

$$\max u(C_1)+0.5\beta u(C_2)$$

$$\text{s.t. } C_1+P_A C_2=W,$$

where W is the wealth at the beginning of period 1. Later, we normalize the units and set W=1. The conditions for the insured's optimal consumption imply that the standard result of equal consumption in both periods exists. Hence we obtain:

(1) $$C_1 = C_2 =1/(1+0.5/(1+r)) = C_{PR}.$$

The expected utility of this contract, denoted by $EUpr$, is:

(2) $EUpr = (1+0.5\beta)u(C_{PR})$.

Contract 2: Public information

The timing of information arrival and individual actions under this contract is:

t=0		t=1		t=2
Type is revealed to insured and insurer	Purchases an annuity	Consumes C_1	Longevity is revealed	Consumes C_2 if survives

Annuities are purchased when *both* insureds and insurers know the survival probabilities. Insurers thus charge each insured an actuarially fair premium. Insurers use q_H for the H-insureds and q_L for the L-insureds. Thus, an *i*- (i =H, L*)* insured solves the following maximization problem:

(3) $\max u\,(C_{1i})+q_i\beta u(C_{2i})$

s.t. $C_{1i}+ C_{2i}q_i/(1+r) = 1$

$i= H, L$

The solution for each type is a fixed lifetime consumption level. However, the fixed consumption level of L-insureds exceeds the corresponding level for H-insureds:

(4) $C_{1i} = C_{2i} =1/(1+q_i/(1+r))$, $i= H, L$.

Insurers have *two* budget constraints, one for each type of insureds, which are identical to the respective budget constraints (and they balance actuarially for each type).

Contract 3: Adverse selection - asymmetric information

The equilibrium under this contract is assumed to be a pooling equilibrium, where insurers cannot distinguish between the two types of insureds. In this equilibrium insurers cannot observe the total quantities of annuities bought by each individual (from various insurers), and thus a separating equilibrium (in which annuity prices are positively related to the total quantities purchased) is not possible. Although not modeled explicitly, income variation may also hinder the insurers' ability to infer the type of market behavior.[13]

The timing of information arrival and individual actions under the asymmetric information contract is:

t=0 t=1 t=2

Type is revealed to insured, but insurer cannot use it	Purchases an annuity	Consumes C_1	Longevity is revealed	Consumes C_2 if survives

Insureds decide on their purchases *after* their types are revealed. However, insurers cannot condition the premium on the insured's type due to asymmetric information or legal constraints. Thus, insurers charge all insureds an equal annuity price, q_{AD}, which reflects a weighted average of the survival probabilities. The weights for q_H and q_L are C_{2H} and C_{2L}, respectively. Note that in such pooling equilibrium, the L-insureds subsidize the H-insureds because the annuity price is higher than the fair price for L-insureds, while it is lower than the fair price for H-insureds.

[13] Abel (1986), likewise, uses a pooling equilibrium. Eichenbaum and Peled (1987) use a Rothschild-Stiglitz quantity-constrained separating equilibrium.

Consequently, L-insureds purchase less annuity than H-insureds, raising the weighted average q_{AD} and causing it to exceed ½. In that case, the insured's problem is:

$$\max u(C_{1i})+q_i\beta u(C_{2i})$$

$$\text{s.t. } C_{1i}+ C_{2i}q_{AD}/(1+r) = 1$$
$$i= H, L .$$

The Langrangean for this case is:

$$L_i= u(C_{1i})+q_i\beta u(C_{2i})-\lambda_i [C_{1i}+ C_{2i}q_{AD}/(1+r) -1], i= H, L .$$

The first order conditions for obtaining the maximum are:

$$\begin{aligned} &u'(C_{1i})= \lambda_i \\ (5) \qquad &u'(C_{2i})= \lambda_i q_{AD}/q_i \\ &C_{1i}+ C_{2i}q_{AD}/(1+r) = 1 \\ &i= H, L . \end{aligned}$$

Because $q_L \leq q_{AD} \leq q_H$, $C_{2L}/C_{1L} \leq C_{2H} / C_{1H}$. Furthermore, because all insureds face the same budget constraint, $C_{2L} \leq C_{2H}$. Note that the consumption levels are always strictly positive because $u'(0)$ is assumed to be unbounded.

In addition to these first order conditions, competition implies that the equilibrium solution should satisfy the zero profit condition for the insurer:

$$\pi=(q_{AD}/(1+r))(0.5C_{2L}+0.5C_{2H})- (q_L 0.5C_{2L}+q_H 0.5C_{2H})/(1+r)=0.$$

The annuities C_{2L} and C_{2H} are purchased at the same price $P_A= q_{AD}/(1+r)$. The first term in the above expression represents the revenue of the insurer, while the second term represents his expected capitalized expenses.

Equivalently:

$$(6) \quad q_{AD} = \frac{q_L C_{2L} + q_H C_{2H}}{C_{2L} + C_{2H}}$$

A positive C_{2L} implies that $q_{AD}<1$ even if $q_H=1$. As explained above, as long as $q_{AD}<1$, the annuity contract strongly dominates the non-annuitized financial investment. The fundamental reason for the preference of annuities is the absence of the bequest motive. That is, under the non-annuitized investment individuals who die prior to the end of their planning horizon leave unintended bequests.

This special feature of the annuity market is also the basis for the existence of a non-trivial equilibrium, where all agents participate in the annuities market.

Definition: A participating adverse selection equilibrium is an annuity price P_A and annuity purchases $C_{2L} >0$, $C_{2H}>0$, such that conditions (5) and (6) are met.

Proposition 1: There exists a participating adverse selection equilibrium.

Proof: Insureds prefer to annuitize all their savings. (When $q_{AD}=1$, they are indifferent). It is well known that the demand for consumption functions C_{1L}, C_{1H}, C_{2L} and C_{2H} are continuous in q_{AD}, hence the insurer's profit function π is continuous in q_{AD}. For $q_{AD} = q_L$, the insurer breaks even on the L-insureds and loses on the H-insureds, hence $\pi <0$. Similarly, for $q_{AD}= q_H$, the insurer breaks even on the H-insureds and profits on the L-insureds (because $C_{2L}>0$), hence $\pi >0$. The proposition then follows by the continuity of the profit function in q_{AD}.[14] The method used in the proof indicates that the proposition holds for any distribution of survival probabilities.

Proposition 1 contrasts with the well-known result of Akerloff (1970) for the non-existence of equilibrium in the market for lemons, the seminal contribution to the asymmetric information literature. In Akerloff's case the equilibrium fails because

[14] In the pathological case $q_H=1$ (and thus $q_L=0$), there also exists a trivial equilibrium where $q_{AD}=1$, and $C_{2L}=0$.

the insurer loses regardless of the premium he charges. When the insurer tries to raise the premiums in order to break even, low risk insureds leave the market at a sufficiently fast rate so as to frustrate the insurer's attempt for achieving an actuarial balance. The process of attempting to achieve actuarial balance ends when the price of insurance is prohibitively high, driving the demand for insurance and the profit to zero. In contrast, in the absence of a bequest motive, the demand for annuities by all insureds is positive and bounded away from zero for any $q_{AD} < 1$, even if it is very close to 1. This positive demand guarantees that insurers can achieve an actuarial surplus for some q_{AD}s and keeps the equilibrium q_{AD} away from 1 (and strictly below q_H in our model).

These different behavior patterns reveal a fundamental difference between annuity insurance and traditional insurance. In the former, every dollar invested in annuity yields more than a dollar invested in a non-annuitized asset because only the insureds who survive share the total returns.[15] As compared with the non-annuitized asset, the annuity contract is a first order stochastic improvement. Annuity insurance eliminates wasting resources on unintended bequests and thus moves out the insured's budget constraint. In contrast, in the traditional insurance market all insureds share the cost. Thus, the insurance contract replaces a random variable with its expected value, a second order stochastic improvement. This reasoning may not hold with the introduction of a bequest motive, as we show in Section 6.

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[15] This interpretation of annuities is similar to the *tontine,* an arrangement for sharing bequests among survivors that was popular in France before the Revolution. We thank Olivia Mitchell for pointing this out to us.

Contract 4: Partial redemption.

The timing of information arrival and individual actions under the partial redemption contract is:

t=0			t=1		t=2
Purchases an annuity	Type is revealed to insured, but insurer cannot use it	May partially redeem the the annuity	Consumes C_1	Longevity is revealed	Consumes C_2 if survives

At date t=0, insureds purchase the contract (C_1, C_2). At date t=1, after they find out their type, they may redeem some or all of the second period consumption and use the proceeds to increase the first period consumption. The annuity price at t=0 reflects an adjusted survival probability $q_{R0,}$ while the refund t=1 is priced using another adjusted survival probability, q_{R1}. . In order to prevent arbitrage, $q_{R0} \geq q_{R1}$. Equivalently, insureds pay a redemption fine of $(q_{R0}-q_{R1})/q_{R0}$ percent. Insureds plan ahead to redeem part of their C_2 if the low probability type is realized, but to retain all of C_2 if the high probability type is realized. Therefore (C_1,C_2) is the planned consumption in case the H type is realized, and (C_{1L} ,C_{2L}) is the planned consumption in the event that the L type is realized. The insured thus solves:

$$\max \tfrac{1}{2}[u(C_1)+\beta q_H u(C_2)] + \tfrac{1}{2}[u(C_{1L})+\beta q_L u(C_{2L})]$$
$$s.t. \quad C_1 + C_2 q_{R0}/(1+r) = 1$$
$$C_{1L} + C_{2L} q_{R1}/(1+r) = C_1 + C_2 q_{R1}/(1+r)$$
$$C_{2L} \leq C_2$$

The individual maximizes the expected utility from the two states of nature, subject to three constraints. The first budget constraint is related to the purchase at date t=0, and the second to the redemption at date t=1. The third constraint states that agents cannot purchase additional annuity at t=1, they can only redeem it. The rationale for the partial redemption contract is that, as long as $q_{R1} < q_{R0}$, it imposes on the insured the cost of using the information revealed at date 1, and thus limits the effect of adverse selection. Additionally, as long as $q_{R1} > q_L$, the option to redeem some of the second period consumption mitigates the subsidy to H-insureds at the expense of L-insureds that is inherent in a pooling equilibrium. The equilibrium consumption levels should also satisfy the insurer's zero profit condition:

$$1/2(C_1+C_{1L})+1/2\ [q_H C_2+q_L C_{2L}]\ /(1+r) = 1.$$

More technically, the insured maximizes the following Langrangean:

$$L= [u(C_1)+\beta q_H u(C_2)] + [u(C_{1L})+\beta q_L u(C_{2L})] - \lambda\ [C_1+ C_2 q_{R0}/(1+r) - 1]$$

$$- \mu\ [C_{1L}+ C_{2L}q_{R1}/(1+r) - C_1- C_2 q_{R1}/(1+r)],\ C_{2L} \leq C_2\ .$$ [16]

The first order Kuhn-Tucker conditions for a maximum are:

$$u'(C_1)= \lambda-\mu$$

$$u'(C_2)\leq \lambda q_{R0}/q_H-\mu q_{R1}/q_H$$ (inequality implies $C_{2L}=C_2$)

$$u'(C_{1L})=\mu$$

$$u'(C_{2L})\geq \mu q_{R1}/q_L$$ (inequality implies $C_{2L}=C_2$)

$$C_1+ C_2 q_{R0}/(1+r) = 1$$

$$C_{1L}+ C_{2L}q_{R1}/(1+r) = C_1+ C_2 q_{R1}/(1+r)$$

$$C_{2L} \leq C_2\ .$$

It is possible to show that $C_{2L} = C_2$ if and only if $q_{R1} \leq q_L$.

[16] To simplify the notation, we multiplied the objective function by 2, a linear transformation of the utility that does not alter the outcome of the maximization problem.

It should be noted that the partial redemption contract is identical to the asymmetric information contract when $q_{R0} = q_{R1}$. In this case both are equal to q_{AD}. The insured can fully annuitize his wealth, and then redeem as much as he wants at no penalty (because $q_{R0} = q_{R1}$). Thus, because $q_H > q_{AD} = q_{R1} > q_L$, the inequality constraint $C_{2L} \leq C_2$ is not binding in this case. We can also see the equivalence of the partial redemption and asymmetric information contracts in this case by noting that for the first order conditions, $\lambda_H = \lambda - \mu$, and $\lambda_L = \mu$. At the other end of the spectrum, for $q_{R0} = 0.5$ and $q_{R1} \leq q_L$, the insured is at the corner solution $C_{2L} = C_2$ and $C_{1L} = C_1$. Thus, in this case, the partial redemption contract is identical to the precommitment contract.

3. Hierarchy of the Contract Types

In the previous section we presented four types of annuity contracts, where the partial redemption contract includes the precommitment and asymmetric information as two extreme cases. In this section we rank the utility level of the consumer under these contract types. This ranking and the simulations presented in the following sections should help policy makers evaluate the welfare loss due to adverse selection that is generated by alternative annuity contracts. We evaluate the expected utility of the insured, derived from consumption in periods 1 and 2, from the vantage point of date 0, before the information on the type is revealed to insureds.

Our first result is general and strong, stating the superiority of precommitment contract to any other contract:

Proposition 2: The precommitment contract is superior to any other equilibrium contract.

Proof: Let C'_{ti} denote the consumption at time *t* and state *i* of the other equilibrium. The expected utility of the consumption under that equilibrium, denoted by EU_{other}, is:

$$EU_{other} = 0.5(u(C'_{1H}) + u(C'_{1L})) + 0.5\beta(q_H u(C'_{2H}) + q_L u(C'_{2L})) .$$

The consumption path satisfies the budget constraint of the insurer:

$$0.5(C'_{1H} + C'_{1L}) + 0.5\,(q_H C'_{2H} + q_L C'_{2L})\,/(1+r) = 1$$

Let $\underline{C}_1 = 0.5C'_{1H} + 0.5C'_{1L}$ and $\underline{C}_2 = q_H C'_{2H} + q_L C'_{2L}$ be the respective averages of the two-period consumption levels. If we replace the consumption in both states of nature by their average for both periods, they satisfy the budget constraint and are preferred to the original consumption because of risk aversion:

$$EU(\underline{C}_1,\underline{C}_2) = u(\underline{C}_1) + 0.5\beta u(\underline{C}_2) = u(0.5C'_{1H} + 0.5C'_{1L}) + 0.5\beta u(q_H C'_{2H} + q_L C'_{2L}) \geq$$

$$0.5(u(C'_{1H}) + u(C'_{1L})) + 0.5\beta(q_H u(C'_{2H}) + q_L u(C'_{2L})) = EU_{other} .$$

Note that $\underline{C}_1 + 0.5\underline{C}_2/(1+r) = 1$.

Thus, by its optimality, C_{PR} is preferred to $(\underline{C}_1,\underline{C}_2)$. Hence:

$$EUpr \geq EU(\underline{C}_1,\underline{C}_2) \geq EU_{other} .$$

It follows that precommitment contract is preferred to all other contracts, namely, public information, asymmetric information and partial redemption. The reason for this result is that precommitment provides insurance both against the uncertainty of the insured's type and her longevity. This result is reminiscent of Hirshleifer's (1971) model, where the revelation of information reduces welfare because it destroys the insurance markets.[17] Eckwert and Zilcha (2000) stress the same point.

[17] Sheshinski (1999) also concludes that early contracting is preferred to annuitizing at retirement. However, he focuses on the optimal retirement age and the unintended bequest of individuals who die prior to their retirement date. In our model, retirement age is given exogenously, and no one dies prior to that date.

Asymmetric information vs. public information

The disadvantage of the asymmetric information contract is that all insureds pay the high premium that the insurer has to charge because of the relatively high consumption of the H-insureds. However, some subsidization of the H-insureds at the expense of the L-insureds is also an advantage of the asymmetric information contract because it provides some insurance against the uncertainty of the type. No such insurance is provided by the public information contract. The simulations we present in the next section indicate that there is no clear hierarchy between these two contractual arrangements. In the standard example of the CRRA (Constant Relative Risk Aversion) utility function $u(C) = [1/(1-\gamma)]C^{1-\gamma}$, where $\gamma > 0$ is the measure of relative risk aversion, the ordering depends on γ. For low levels of γ, the public information contract is preferred, while for high levels of γ the asymmetric information contract is preferred. The break-even point is obtained at a level of γ close to two from below, such as 1.95.[18] The impact of changes in γ on the expected utility of the asymmetric information contract, relative to that of the public information contract, has the following intuition. The allocation under the public information contract is independent of γ. In contrast, the difference between C_{2H} and C_{2L}, and thus the cost of an annuity under the asymmetric information contract, are negatively related to γ. As γ increases, the difference shrinks and the asymmetric information contract dominates the public information contract.

Because γ is empirically found to be more than 2 in many empirical studies (see Kocherlakota (1996)), the asymmetric information contract should be preferred by the insureds to the public information contract.

Partial redemption vs. asymmetric information, precommitment and public information

As explained above, the consumption allocation under the partial redemption contract depends on the fine for redemption. When the fine for redemption is prohibitive, the equilibrium is characterized by $q_{R0} = 0.5$ and $q_{R1} \leq q_L$, and is identical to that under the precommitment contract. When the fine is zero, the equilibrium becomes the asymmetric information equilibrium, with $q_{R0} = q_{R1} = q_{AD} > 0.5$. We know from Proposition 2 that from the insured's viewpoint the best option is the prohibitive fine (equivalent to the precommitment contract). The simulations support the intuition that the expected utility is increasing with the fine, or equivalently, that the expected utility is decreasing with q_{R0}.

Empirical regularity: *EUrd* is a decreasing function in q_{R0}.

We can summarize the hierarchy of schemes in the following:

Proposition 3: For $\gamma \geq 2$, $EUpr \geq EUrd \geq EUad > EUpi$.

This ordering of the four contractual arrangements have practical implications concerning the value of information. The best contract is initiated before the information is known, without the ability to renege once the information is received. The second best contract is initiated before the information is known, but also allows the redemption of some of the annuity. The third best contract is initiated when the

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[18] In some other applications the critical value is 1. In these cases, the underlying mechanism is the equality of the substitution and income effects under the log-utility case.

information may be used by the insured, but not by the insurer. The worst contract is when both the insured and the insurer use the information on the insured's type.

One application of Proposition 3 concerns medical and genetic information that may contain relevant information on survival probability. Another example is the use of unisex tables for annuity insurance. Annuity providers in the US are not allowed to charge gender-based premiums for annuities that are issued within a "qualified pension plan", although life tables vary significantly across genders.[19] Our analysis justifies this practice. Even if the information is known to the insurer, for $\gamma \geq 2$, welfare is higher when the insurer cannot use gender information. Notice that the criterion used is the expected utility of the fetus before its gender is known. Otherwise, this policy involves income redistribution between genders, and the welfare implications are less evident.

4. Simulation of the Two Period Model

In this section we present the behavior of insureds as well as market equilibria under the alternative contracts. We also present measures for welfare loss due to deviation from the ideal contract - the precommitment at date 0. The simulations of the two-period model are useful for investigating the hierarchy of alternative plans that cannot be ordered based on theoretical considerations. They also illustrate the quantitative impact of adverse selection, although this will be further investigated in a more realistic multi-period model in the next section.

Our basic example is the CRRA utility function $u(C)= [1/(1-\gamma)]C^{1-\gamma}$, where $\gamma>0$ is the measure of relative risk aversion. We simplify the presentation by assuming

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[19] For other policies, the ability to issue gender-based premiums is regulated by state law and varies across states.

that $1/(1+r) = \beta = ½$. The latter corresponds to a rate of 6% compounded annually during a 12-year period. Thus, our two-period model covers about twenty-four years of retirement[20].

The Appendix contains the more technical aspects of the solutions. Here we present the results and their implications. As indicated by Equations (1) and (2), the utility of the precommitment scheme is:

$$EUpr = 1.25*[1/(1-\gamma)]0.8^{1-\gamma} = [1/(1-\gamma)]0.8^{-\gamma}.$$

Equations (3) and (4) imply that the utility under the public information scheme is:

$$EUpi = [0.5*(1+q_H/(1+r))]*u(C_H) + [0.5*(1+q_L/(1+r))]*u(C_L) =$$

$$(0.5/(1-\gamma))[(1+q_H/(1+r))^{\gamma} + (1+q_L/(1+r))^{\gamma}] .$$

By Proposition 2, expected utility under the public information contract is lower than that under the pre-commitment contract. To obtain a quantitative measure of the utility loss, we calculate the pre-commitment wealth that provides the same expected utility as that provided by a $1 wealth under a public information contract for alternative values of q_H, q_L and γ. We refer to this measure as the Equivalent Variation (EV). Note that under the CRRA utility function this EV represents the ratio between the required wealth levels under the two contracts for any fixed utility level. An increase in q_H represents a mean preserving spread of the survival probabilities, since $q_L = 1 - q_H$. Thus, the higher q_H and γ are, the smaller should be the equivalent wealth levels. These values are presented in Table 1:

[20] Under this interpretation the consumption at t=1 is the lump sum equivalent of the 12 years that immediately follow retirement, and the consumption at t=2 is the lump sum equivalent of the following 12 years.

Table 1: Equivalent Variation of the Public Information Contract

Numbers represent the wealth under the precommitment contract that yields the same welfare as one unit of wealth under the Public Information contract.

q_H \ γ	0.5	1.5	2	2.5	3	4	5	6
0.55	0.9999	0.9997	0.9996	0.9995	0.9994	0.9992	0.9990	0.9988
0.60	0.9996	0.9988	0.9984	0.9980	0.9976	0.9968	0.9960	0.9953
0.65	0.9991	0.9973	0.9964	0.9955	0.9946	0.9929	0.9912	0.9895
0.70	0.9984	0.9952	0.9936	0.9921	0.9905	0.9875	0.9846	0.9817
0.75	0.9975	0.9925	0.9901	0.9877	0.9853	0.9807	0.9763	0.9722
0.80	0.9964	0.9893	0.9858	0.9824	0.9791	0.9727	0.9667	0.9612
0.85	0.9951	0.9854	0.9808	0.9762	0.9718	0.9635	0.9559	0.9489
0.90	0.9936	0.9810	0.9750	0.9692	0.9637	0.9533	0.9440	0.9357
0.95	0.9918	0.9761	0.9686	0.9615	0.9547	0.9422	0.9313	0.9219
1.00	0.9899	0.9706	0.9615	0.9530	0.9449	0.9304	0.9180	0.9076

Proposition 1 ensures the existence of equilibrium for the asymmetric information contract and the Appendix provides the equations for obtaining a solution for the equilibrium q_{AD}, given q_H and γ. Table 2 presents the corresponding EV measures for the asymmetric information contract.

Table 2: Equivalent Variation of the Asymmetric Information Contract Numbers represent the wealth under the precommitment contract that yields the same welfare as one unit of wealth under the Asymmetric Information contract.								
γ q_H	**0.5**	**1.5**	**2**	**2.5**	**3**	**4**	**5**	**6**
0.55	**0.9984**	**0.9995**	**0.9996**	**0.9997**	**0.9997**	**0.9998**	**0.9998**	**0.9999**
0.60	**0.9941**	**0.9979**	**0.9984**	**0.9987**	**0.9989**	**0.9992**	**0.9994**	**0.9995**
0.65	**0.9882**	**0.9953**	**0.9964**	**0.9971**	**0.9976**	**0.9982**	**0.9985**	**0.9988**
0.70	**0.9820**	**0.9919**	**0.9937**	**0.9949**	**0.9957**	**0.9968**	**0.9974**	**0.9978**
0.75	**0.9771**	**0.9876**	**0.9903**	**0.9921**	**0.9933**	**0.9949**	**0.9959**	**0.9966**
0.80	**0.9743**	**0.9828**	**0.9863**	**0.9887**	**0.9904**	**0.9926**	**0.9940**	**0.9950**
0.85	**0.9742**	**0.9775**	**0.9816**	**0.9846**	**0.9868**	**0.9898**	**0.9917**	**0.9930**
0.90	**0.9768**	**0.9723**	**0.9765**	**0.9799**	**0.9825**	**0.9862**	**0.9887**	**0.9904**
0.95	**0.9821**	**0.9677**	**0.9707**	**0.9740**	**0.9770**	**0.9815**	**0.9846**	**0.9868**

A comparison of Tables 1 and 2 does not reveal a clear hierarchy between the public information and the asymmetric information contracts. It turns out that for low levels of γ the public information contract is preferred to the asymmetric information contract, while for high levels of γ the asymmetric information contract is preferred.

Table 3 presents the Equivalent Variations for the asymmetric and public information contracts, when these measures are close to one another. As indicated in Table 3, for $\gamma = 1.9$ a public information contract is preferred to an asymmetric information contract, while the reverse holds for γ=2. For γ=1.95, a public information contract yields higher expected utility levels for low levels of q_H, while the reverse holds for high levels of q_H. As indicated by Kocherlakota (1996), the literature concludes that γ is likely to exceed 2. Thus, we conclude that the asymmetric information contract should be preferred to the public information contract.

Table 3: Hierarchy of the Public Information and Asymmetric Information Contracts									
	γ=1.9			γ=1.95			γ=2		
q_H	q_{AD}	*EVad*	*EVpi*	q_{AD}	*EVad*	*EVpi*	q_{AD}	*EVad*	*EVpi*
0.55	0.5021	0.9996	0.9996	0.5021	0.9996	0.9996	0.5020	0.9996	0.9996
0.60	0.5085	0.9983	0.9985	0.5083	0.9984	0.9984	0.5081	0.9984	0.9984
0.65	0.5195	0.9963	0.9966	0.5190	0.9964	0.9965	0.5185	0.9964	0.9964
0.70	0.5354	0.9934	0.9940	0.5345	0.9936	0.9938	0.5336	0.9937	0.9936
0.75	0.5570	0.9899	0.9906	0.5556	0.9901	0.9903	0.5542	0.9903	0.9901
0.80	0.5856	0.9857	0.9865	0.5835	0.9860	0.9861	0.5815	0.9863	0.9858
0.85	0.6236	0.9809	0.9817	0.6206	0.9813	0.9812	0.6177	0.9816	0.9808
0.90	0.6755	0.9757	0.9762	0.6715	0.9761	0.9756	0.6675	0.9765	0.9750
0.95	0.7544	0.9700	0.9701	0.7490	0.9703	0.9694	0.7438	0.9707	0.9686

Note that as γ increases, the equivalent wealth for the public information contract decreases. This takes place because the consumer becomes more risk averse and hence is willing to pay more to eliminate the uncertainty of the type (which is not eliminated by the public information contract). In contrast, as γ increases, the equivalent wealth for the asymmetric information contract increases because the extent to which insureds take advantage of their private information is negatively related to γ.

The literature has used both the Money's worth Ratio (MWR) and the equivalent wealth measure, to assess the impact on the insureds of adverse selection (in conjunction with expenses and profits that are assumed to equal zero in the current paper). The MWR is the expected capitalized value of the income stream ensuing from a one-dollar annuity purchased by an individual with average survival probabilities (where "average" may refer to the general population or to a subset). In our case MWR= $0.5/q_{AD}$. Note that under the precommitment contract, MWR=1.

In Tables 4 and 5 we present the MWR measures, along with the consumption levels of the H- and L-insureds, for a variety of parameter combinations. In Table 4 we demonstrate the sensitivity of the MWR and consumption levels to the degree of risk aversion of agents. Thus, we present these variables for our base case of q_H=0.75 and alternative values of the risk aversion measure. The MWR values for reasonable values of the degree of risk aversion (between 2 and 4) are consistent with the 6 to 10 percent excess of the cost of an annuity over its fair actuarial value that are documented in the literature (see, for example, Mitchell et al. (1999) when annuitants life tables are used).

Table 4: Money's Worth Ratios and Consumption Levels Under the Asymmetric Information Contract for Various Levels of Risk Aversion When q_H=0.75

γ	q_{AD}	MWR	C_{1H}	C_{1L}	C_{2H}	C_{2L}
0.5	**0.6848**	**0.7301**	**0.7089**	**0.9564**	**0.8502**	**0.1275**
1.5	**0.5717**	**0.8746**	**0.7448**	**0.8586**	**0.8926**	**0.4947**
1.9	**0.5570**	**0.8977**	**0.7543**	**0.8455**	**0.8822**	**0.5547**
1.95	**0.5556**	**0.9000**	**0.7553**	**0.8443**	**0.8809**	**0.5606**
2.0	**0.5542**	**0.9022**	**0.7562**	**0.8431**	**0.8797**	**0.5663**
2.5	**0.5435**	**0.9199**	**0.7639**	**0.8339**	**0.8689**	**0.6112**
3.0	**0.5363**	**0.9322**	**0.7693**	**0.8279**	**0.8603**	**0.6419**
4.0	**0.5273**	**0.9482**	**0.7764**	**0.8205**	**0.8479**	**0.6808**
5.0	**0.5219**	**0.9581**	**0.7809**	**0.8162**	**0.8396**	**0.7045**
6.0	**0.5183**	**0.9648**	**0.7840**	**0.8134**	**0.8338**	**0.7203**

In Table 5 we demonstrate the sensitivity of the MWR and consumption levels to the difference between the expected longevity of H- and L- insureds. Thus, we present these variables for our base case of γ=3 for alternative values of q_H.

Table 5: Money's Worth Ratios and Consumption Levels under the Asymmetric Information Contract for Various Levels of q_H When γ=3.						
q_H	q_{AD}	MWR	C_{1H}	C_{1L}	C_{2H}	C_{2L}
0.55	**0.5013**	**0.9973**	**0.7946**	**0.8053**	**0.8195**	**0.7768**
0.60	**0.5054**	**0.9893**	**0.7889**	**0.8105**	**0.8353**	**0.7497**
0.65	**0.5123**	**0.9759**	**0.7829**	**0.8159**	**0.8475**	**0.7186**
0.70	**0.5225**	**0.9570**	**0.7764**	**0.8216**	**0.8559**	**0.6829**
0.75	**0.5363**	**0.9322**	**0.7693**	**0.8279**	**0.8603**	**0.6419**
0.80	**0.5548**	**0.9012**	**0.7614**	**0.8351**	**0.8602**	**0.5943**
0.85	**0.5796**	**0.8626**	**0.7523**	**0.8441**	**0.8547**	**0.5379**
0.90	**0.6144**	**0.8139**	**0.7414**	**0.8564**	**0.8420**	**0.4676**
0.95	**0.6698**	**0.7465**	**0.7266**	**0.8764**	**0.8164**	**0.3690**

In the partial redemption contract the zero profits assumption implies a monotonic correspondence between the two prices for insurance, q_{R0} and q_{R1}. The first is the price before the type is known, and the second is the redemption price after the insured finds out that he is L-insured. Obviously, to prevent arbitrage, the redemption price should be lower than the purchase price.

In Table 6 we introduce simulations with γ=3, q_H=0.75, and alternative values of q_{R0}. The table presents the redemption price, the consumption levels that H- and L-insureds obtain, and the wealth under a precommitment contract that yields the same expected utility as the redemption contract (EV_{rd}). When q_{R0}=0.5, the redemption price may be between 0 and 0.25, the insured does not redeem any of his annuity, and the contract is equivalent to full precommitment. At the other end in this table, when q_{R0}=0.536, q_{R0}=q_{R1}, the contract is equivalent to an asymmetric information contract. The rest of the table includes the intermediate values for q_{R0} and q_{R1}. Clearly, the desirability of the contract, as measured by the equivalent variation, decreases as both

q_{R0} and q_{R1} increase, with the highest value at the precommitment end and the lowest value at the asymmetric information end.

Table 6: Annuity Prices, Consumption Levels and Equivalent Variations under the Partial Redemption Contract - q_H =0.75 and γ=3

q_{R0}	q_{R1}	C_1	C_2	C_{1L}	C_{2L}	*EVrd*	
0.50	0-0.25	0.8	0.8	0.8	0.8	1	precommitment
0.51	0.3797	0.7871	0.8347	0.8116	0.7060	0.9981	
0.515	0.4147	0.7832	0.8418	0.8150	0.6885	0.9972	
0.52	0.4461	0.7797	0.8475	0.8182	0.6746	0.9963	
0.525	0.4752	0.7763	0.8522	0.8213	0.6630	0.9954	
0.53	0.5028	0.7731	0.8561	0.8242	0.6529	0.9945	
0.535	0.5294	0.7701	0.8595	0.8271	0.6441	0.9936	
0.536	0.5363	0.7693	0.8603	0.8279	0.6419	0.9933	asymmetric information

5. Multi-period Simulations under the Asymmetric Equilibrium Contract

In the previous section we demonstrated in a simple model that adverse selection in the annuity market has a minor impact on welfare. The welfare loss due to adverse selection for the case where γ=3 and q_H=0.75 is two thirds of one percent (see Table 2 and the last line in Table 6). The impact is of the same order of magnitude for other likely parameter combinations. In the literature, the MWR is usually related to the excess of the annuity price over the fair price based on the life table of either the annuitants or the general population. This excess is represented in our simulations by the difference between 1 and the MWR. In the simulations presented in the previous section, this difference (which corresponds to both definitions of the excess cost because all agents purchase annuities) equals about 6.8 percent (see the line for

q_H=0.75 in Table 5). In this section and in the next section we examine whether our quantitative results are sensitive to the introduction of a multi-period model, a bequest motive (where not all insureds purchase annuities) and a Social Security System.

The multi-period problem is different, because insureds have significantly less flexibility. The annuity contract usually limits its owner to either a fixed nominal, or an approximately fixed real, annual distribution.[21] As will be explained below, this structure is optimal for the fair annuity buyer, but H-insureds would like an *increasing* real annuity, while the L-insureds would like a *decreasing* real annuity. Thus, the institutional set-up of annuities is an incomplete substitute for precommitment. Although H-insureds buy the annuity after their type is revealed, they are limited to a contract with little flexibility so they cannot fully exploit their type information.[22]

More formally, we denote by q_t the death probability between age t and age $t+1$ of an individual who is alive at age t. Consider an individual who contemplates the purchase of an immediate annuity contract at the retirement age of 65. Denote the survival probabilities as of age 65 by $P_1, P_2, \ldots, P_T$ where P_1=1 is the probability to survive through age 65; P_2 is the probability to survive through age 66; etc. We denote by T the last period that an individual may be alive. Given the series $\{q_t\}$, $P_2=1-q_{65}$, $P_i=(1-q_{65})*(1-q_{66})\ldots*(1-q_{65+i-2})$, we assume that the death probabilities of the L-insureds are, at all ages, higher than the corresponding probabilities for the H-insureds. Denoting these probabilities by q_{Lt} and q_{Ht} respectively, $q_{Lt} > q_{Ht}$ for all t.

The survival probabilities as of age 65 are similarly denoted by P_{Lt} and P_{Ht}, with $P_{Lt} < P_{Ht}$ for $t \geq 2$, and $P_{L1}=P_{H1}=1$. Both P_{Lt} and P_{Ht} are decreasing series, where the

[21] Annuities that increase in a fixed percentage are available in the U.K., and consist about four percent of the annuities studied by Finkelstein and Poterba (2000).

[22] However, in our model L-insureds procure a decreasing consumption stream by combining a fixed annuity with a decreasing stream of consumption that is generated by non-annuitized financial asset.

elements of the first are lower than the corresponding elements of the second, and are decreasing at a faster rate. Given that at t=0 the L-type and the H-type have equal weights in the market, the population life table survival probabilities denoted as P_t satisfy $P_t = 1/2P_{Lt} + 1/2P_{Ht}$, for all t.

The insured evaluates his consumption series C_t with the monotonic concave separable utility function stated below and the discount factor β. We maintain the assumption that $\beta=1/(1+r)$, where r is the annual interest rate.

The *fixed* annuity contract is an obligation of the insurer to pay 1 unit of consumption at every age as long as the consumer survives, where the first payment is at age 65. Based exclusively on the life table for the general population, the fair price of an annuity is $P_{Afair}= \Sigma^{T}_{t=1} P_t\, (1/(1+r))^{t-1}$. However, the annuity price P_A that is consistent with the zero profit condition is higher than P_{Afair} if H-insureds buy more annuities than L-insureds. The Money's Worth Ratio that is mentioned above is now: $MWR=P_{Afair}/P_A$. We also define the fair annuity price for each type of insureds,

$$P_{iAfair}= \Sigma^{T}_{t=1} P_{it}\, (1/(1+r))^{t-1},\ i=L,H,$$

and the corresponding Money's worth Ratios,

$$MWR_i= P_{iAfair}/\ P_A.$$

Notice that this definition of MWR_i is according to the life table of each type. Because the insurer is assumed to break even, the insurer profits from the contracts with the L-insureds and loses on the contract with the H-insureds. Thus, $MWR_H > 1$. We also conclude that $P_A<\Sigma^{T}_{t=1}(1/1+r))^{t-1}$, because otherwise purchasing the non-annuitized asset dominates purchasing the annuity contract.

The insured may choose not to annuitize all his wealth, so in addition to the a units of annuity that he purchases, he also buys a stream of non-annuitized income b_t.

Thus, $C_t=a+b_t$ for all t. In that case, the insured solves the following maximization problem:

$$\max\sum_{t=1}^{T}\beta^{t-1}P_t u(C_t)$$
$$S.T.\;\; C_t = a + b_t$$
$$W = P_A a + \sum_{t=1}^{T} b_t\left(\frac{1}{1+r}\right)^{t-1}$$
$$a, b_t \geq 0$$

We assume $u'(0)=+\infty$. It is well known that when the insurance is fair (i.e. MWR_i=1), the insured chooses to annuitize all his wealth (i.e., b_t =0 for all t). Proposition 4 generalizes the annuitization choice for all other possibilities:

Proposition 4: For insureds with $MWR_i \geq 1$, b_t =0 for all t. For insureds with MWR_i<1, $b_t \geq 0$, $b_1 > 0$, and b_t is a decreasing series, b_T=0. It is always true that a>0. The proof is presented in the Appendix.

The simulations

We obtain the death probability, q_t, from the standard unisex life tables[23]. These tables indicate that, for the general population, half of the agents that are alive at age 65 reach the age of 81. Thus, age 81 corresponds to t=1 in our two period model. We set $q_{Ht}=0.45*q_t$, and $q_{Lt}=2.07*q_t$, making the H-type less likely to die and thus live longer, and the reverse for the L-type. These constants are chosen to replicate the two-period model, with the resulting survival probabilities at age 81 equal to 0.25 for the L-type and 0.75 for the H-type. The life expectancies as of age 65 for the two types also diverge significantly in a manner similar to the assumption in the two

[23]Bowers et al. (1986).

period model: 23.9 years for the H-type and 11.1 for the L-type. We use the standard interest rate used in the actuarial literature of 6% compounded annually.

The fair prices for a \$1 annuity for the two types are P_{HAfair}=\$12.43 and P_{LAfair}=\$7.97. Thus, although each death probability of the L-insureds is 4.6 times larger than the corresponding death probability of the H-insureds, the fair annuity price of the L-insureds is only 36% lower than the corresponding price for the H-insureds. To understand this discrepancy, recall that fair annuity prices are the present values of future cash flows, contingent upon survival. The discounting of future cash flows implies that annuities obtain their values mostly from the cash flows during the initial retirement years. For example, our simulations span a 45-year retirement horizon. However, the cash flows during the first 17 retirement years contribute 82% of the annuity fair value for the H-insureds and 96% for the L-insureds. During these years, the ratio of the survival probabilities for the H- and L-insureds monotonically increases from one to three. As explained above, we assume that all insureds receive the first cash flow at age 65 (i.e., $P_{L1}=P_{H1}=1$), and that the survival probabilities to age 81 are $P_{L17}=0.25$ *and* $P_{H17}=0.75$. The relatively small difference in fair annuity values between the H- and L-insureds is consistent with the assertion that asymmetric information in the annuity market should not have a major welfare effect.

Proposition 4 states that the lack of a bequest motive generates a positive demand for annuities by all insureds. Proposition 1, claiming that equilibrium in the annuities market always exists, may thus be extended to cover this case. The equilibrium value of P_A is obtained by successive approximations, calculating the demand for annuities of both types, and then plugging them into the insurer's budget constraint, which is similar to the two period model.

Tables 7 and 8 present the results for three alternative values of γ. Table 7 presents the consumption levels and the impact of the asymmetric information on welfare for alternative levels of the risk aversion parameter γ.

Table 7: Multiperiod Simulation: Annuity Prices, the Demand for Annuities and Measures of Adverse Selection

P_A is the equilibrium price of an annuity under the multiperiod asymmetric information contract. P_{Afair} is the annuity price under a precommitment contract (i.e., based exclusively on the life table for the general population). a_i is the annuity purchased by type i (i=H,L) individual. EV_{MU} represents the wealth under a corresponding precommitment contract that yields the same welfare as one unit of wealth under the asymmetric information contract in the multiperiod model.

γ	P_A	P_{Afair}	*MWR*	$a_H=w/P_A$	a_L	EV_{MU}
3.0	**10.2730**	**10.201**	**0.9930**	**0.0973*w**	**0.0912*w**	**0.9949**
1.5	**10.3607**	**10.201**	**0.9846**	**0.0965*w**	**0.0837*w**	**0.9889**
0.5	**11.1645**	**10.201**	**0.9137**	**0.0896*w**	**0.0355*w**	**0.9393**

The H-insureds annuitize all their wealth while the L-insureds annuitize only a fraction of their wealth, in line with Proposition 4. This difference between the purchases of the H- and L-insureds accounts for the adverse selection. For γ=3 or even γ=1.5, this difference is not sizable and thus the MWR ratio is close to 1. However, for γ=0.5 the L-insureds annuitize only 40 percent of their wealth, and the MWR drops to 91 percent. Notice also that the EV_{MU} is, again, closer to 1 than the MWR, because the L-insureds substitute away from annuities when their price increases.

Table 8 takes a closer look at the L-insured's consumption profile. As the table shows, the higher is γ, the less desirable is the substitution, and hence more consumption is annuitized.

Table 8: Multiperiod Simulation: The Ratio between Consumption Levels and Annuity Levels									
γ	C_{L1}/a_L	C_{L2}/a_L	C_{L3}/a_L	C_{L4}/a_L	C_{L5}/a_L	C_{L6}/a_L	C_{L7}/a_L	C_{L8}/a_L	C_{L9}/a_L
3.0	**1.165**	**1.148**	**1.130**	**1.111**	**1.090**	**1.069**	**1.046**	**1.021**	**1**
1.5	**1.379**	**1.340**	**1.299**	**1.256**	**1.210**	**1.163**	**1.113**	**1.061**	**1.008**
0.5	**4.610**	**4.224**	**3.845**	**3.473**	**3.111**	**2.759**	**2.421**	**2.100**	**1.798**

The utility loss from adverse selection should be compared to the case when there is no access to the annuity market. The utility losses from having no access to annuity markets compared with utility under the precommitment contract, as measured by the equivalent variation, are given in Table 9. For γ=3 this loss is equivalent to 31% of wealth, as opposed to two thirds of one percent in the adverse selection case. This comparison further demonstrates that adverse selection and asymmetric information in the annuity market should not reduce welfare significantly.

Table 9: Equivalent Variation for the No Insurance Case

Numbers represent the wealth under the precommitment contract that yields the same welfare as one unit of wealth in the absence of an annuity market for alternative parameter values. $EV_{no\text{-}mu}$ and $EV_{no\text{-}two}$ are the equivalent variations for the multiperiod and two period models, respectively.

	$EV_{no\text{-}mu}$		$EV_{no\text{-}two}$
γ	***r=6%***	***r=1%***	***r=100% ; q_H=.75***
0.5	**0.7924**	**0.7088**	**0.9223**
1.5	**0.7226**	**0.5895**	**0.8732**
3.0	**0.6873**	**0.5293**	**0.8545**

6. Robustness to a Bequest Motive and to a Social Security System

In Proposition 1 we argue that in the absence of a bequest motive all agents participate in the annuities market. The annuity contract provides more consumption than a non-annuitized asset by eliminating unintended bequests. In this section we study a two-period model of the annuity market with a bequest motive. The model is identical to the asymmetric information model described above with two exceptions. First, a bequest motive, denoted by δ, appears in the utility of agents who die before date t=2.[24] Second, we assume more than two possible realizations of the survival probability. We demonstrate that, in contrast to the no bequest case (where all agents purchase annuities), agents with sufficiently low expected longevity and a strong bequest motive do not purchase annuities. We find a threshold value for the parameter representing the strength of the bequest motive, denoted by δ', that depends on the equilibrium price of annuities and the survival probability of the agent. If the bequest motive parameter is below δ', the agent purchases annuities and thus participates in the annuities market. Conversely, if the bequest motive parameter is above δ', the agent does not purchase annuities and does not participate in the annuities market. We show that this behavior may lead to non-existence of equilibrium in the continuous distribution case.[25]

We simulate the equilibrium by using the CRRA utility function, and two discrete approximations of the uniform distribution for the survival probability and the bequest motive parameter. In this equilibrium, much like in reality, a large fraction of the agents do not participate in the annuities market. Thus, we calculate

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[24] For a discussion of the modeling of a bequest motive, see: Abel and Warshawsky (1988).

[25] More formally, only the group of agents with the highest survival probability purchases annuities.

two MWR measures: one relative to the life table of the general population and the other relative to the life table of annuitants. Although these MWRs are considerably lower than the corresponding values obtained in the previous sections, the expected welfare loss is still very small.

A formal model of an asymmetric information contract with a bequest motive

The model is an adaptation of the asymmetric information model from Section 2. Recall that the insured can invest in two assets: a regular, non-annuitized, financial asset D and an annuity A, with the respective prices $P_D=1/(1+r)$ and $P_A=q_{AD}/(1+r)$. The annuity price is lower because $q_{AD}<1$; hence, the only rationale for holding the other asset is the desire to leave a bequest in the event of death before t=2.

We assume that the agent evaluates the utility of her heirs by the same utility function as her own, except that she applies a discount factor $\delta_j, 0 \le \delta_j \le 1$. We assume m possible values for δ_j and n possible values for the probability survival q_i. Thus, each agent is characterized as belonging to one of $n*m$ equally likely types. Recall that the wealth W is assumed to be 1, and that the subjective discount factor β is assumed to equal $1/(1+r)$. The agent's maximization problem is:

$$\max u(C_{1ij})+ \beta [q_i u(C_{2ij})+(1-q_i)\delta_j u(B_{ij})]$$

$$\text{s.t. } C_{1ij}+A_{ij}P_A+D_{ij}P_D=1$$

$$C_{2ij}=A_{ij}+D_{ij}$$

$$B_{ij}=D_{ij}$$

$$i=1..n, j=1..m.$$

The budget constraint may also be written as:

$$C_{1ij}+C_{2ij}q_{AD}/(1+r)+B_{ij}(1-q_{AD})/(1+r)=1.$$

Since $u'(0)$ is unbounded, the bequest B_{ij} vanishes if and only if $\delta_j = 0$.

To derive the first order conditions and solve for the optimum, we distinguish between two cases: $A>0$ and $A=0$.

Case 1 : $A > 0$.

$u'(C_{2ij}) = u'(C_{1ij}) q_{AD}/q_i$.

$\delta_j u'(B_{ij}) = u'(C_{1ij})(1-q_{AD})/(1-q_i)$.

Case 2: $A=0$.

$B_{ij} = C_{2ij}$.

$u'(B_{ij}) = u'(C_{1ij})/(q_i + \delta_j(1-q_i))$.

These conditions in conjunction with the budget constraint yield solutions for the consumption and bequest levels as functions of q_{AD}. When δ is sufficiently large, A vanishes. The threshold value of δ, denoted as δ', is found when the solution of the two cases obtain the same value. Hence:

$$\delta' = ((1/q_{AD})-1)/((1/q_i)-1).$$

It follows that:

for $\delta_j < \delta'$, $A_i > 0$, and for $\delta_j \geq \delta'$, $A_i = 0$.

The insurance industry equilibrium condition is:

$$\Sigma_{i=1..n\ j=1..m} A_{ij}\, q_i = q_{AD} \Sigma_{i=1..n\ j=1..m} A_{ij}.$$

Simulation of the model

We use the CRRA utility function as in the previous sections. The details of the calculations are reported in the Appendix.

The participation of the ij-agent in the annuity market depends on the strength of her bequest motive. For low levels of δ, below the critical value δ' (which depends on q_i and q_{AD}), the agent will participate. Conversely, for high levels of δ, above δ', the agent will not participate. In the simulations δ' and q_{AD} are determined simultaneously.

We assume that q_i and δ_j are distributed evenly on the interval [0,1]. For calculation purposes we approximate the distribution by ten intervals. The values q=0 and q=1 are trivial in our framework, hence we consider only nine possible values for q_i : $q_i=0.1,0.2,...,0.9$. We consider eleven possible values for δ_j : $\delta_j=0,0.1,...,1.0$ (i.e., $n=9$ and $m=11$).

In Tables 10 and 11 we report the results of the simulation. For our base case of γ=3, we present in Table 10 the threshold value of δ' for each q_i, and the demand for annuity for each agent type (i.e., a combination of q_i and δ_j). Only 54 out of the 99 agent types (54.5 percent) participate in the market, and all others demand zero annuity. This pattern of demand for annuities results in a more substantial adverse selection: higher values of q_{AD} and lower values of MWR than their counterparts at the no-bequest models studied in the previous sections. However, if we construct the life-table of participating agents only, the figures change significantly.

We define an indicator function: $a_j = \{1 \text{ whenever } A_{ij}>0 \text{ and } 0 \text{ whenever } A_{ij}=0\}$.

We calculate $q_{particp}$ as the average q_i within the group of the participating agents: $q_{particp} = \left\lfloor \sum_{i=1..n \;\; j=1..m} a_{ij} q_i \right\rfloor / \left\lfloor \sum_{i=1..n \;\; j=1..m} a_{ij} \right\rfloor$. We denote by MWR' the value of MWR relative to $q_{particp}$. Table 11 reports the results for alternative values of the risk parameter γ. The annuity cost declines and the MWR' increases as γ increases. For the sufficiently high value of $\gamma=5$, the MWR' even exceeds 1.[26]

Table 10: Asymmetric information with Bequest: Annuity purchases A_{ij} and Equivalent Variations EV_b.

$\gamma=3$ qAD = .684 MWR=.731

q δ	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1.0**	**δ**
0.1	**0.45**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0.05**
0.2	**0.54**	**0.02**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0.12**
0.3	**0.60**	**0.11**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0.20**
0.4	**0.65**	**0.19**	**0.08**	**0.01**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0.31**
0.5	**0.69**	**0.26**	**0.16**	**0.08**	**0.03**	**0**	**0**	**0**	**0**	**0**	**0**	**0.46**
0.6	**0.72**	**0.32**	**0.23**	**0.16**	**0.11**	**0.07**	**0.03**	**0**	**0**	**0**	**0**	**0.69**
0.7	**0.75**	**0.39**	**0.30**	**0.24**	**0.19**	**0.16**	**0.12**	**0.09**	**0.06**	**0.04**	**0.02**	**1.08**
0.8	**0.77**	**0.46**	**0.38**	**0.33**	**0.29**	**0.25**	**0.22**	**0.20**	**0.17**	**0.15**	**0.13**	**1.85**
0.9	**0.80**	**0.55**	**0.49**	**0.44**	**0.41**	**0.38**	**0.36**	**0.33**	**0.31**	**0.30**	**0.28**	**4.16**
												average
EV_b	**0.94**	**0.98**	**0.99**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**0.99**

Table 11: Measures of Adverse Selection for Alternative Values of the risk parameter γ

γ	q_{AD}	**MWR**	$q_{particp}$	**MWR'**
0.5	**0.789**	**0.633**	**0.693**	**0.878**
2.0	**0.709**	**0.705**	**0.667**	**0.941**
3.0	**0.684**	**0.731**	**0.663**	**0.970**
5.0	**0.651**	**0.768**	**0.652**	**1.001**

[26] This is not a calculation error. Note that, among annuitants, the average annuity purchase of agents with q_i= .1 (only those with δ=0 are annuitants) is 0.45. The corresponding average for agents with q_i= .9 is 0.42. Thus, considering annuitants only, the average annuity purchased is not necessarily increasing in q_i . Consequently, the weighted average of the q_i's of annuitants with the A_{ij}'s serving as weights (=q_{AD}) may be lower than the simple average of the q_i's of annuitants (=$q_{particp}$). Thus, MWR'=$q_{particp}/q_{AD}$ >1. This is another indication that MWRs do not always reflect welfare loss.

The average equivalent variation for agents with a given δ_j is presented in the last row of Table 10. At date 0 there are eleven types of agents, with values of δ_j between zero and one. For each type we calculate the Equivalent Variation (i.e., the level of wealth that under a precommitment contract yields the same level of expected utility as generated by a $1 wealth under the asymmetric information contract).

We find that the welfare loss is the largest for the agent with no bequest motive at all (about a 6 percent loss). For all other agents it is equivalent to about one percent of wealth. A closer look reveals that the demand for annuities of agents with positive bequest parameter is very sensitive to the survival probability. When the survival probability is low, agents annuitize only a small portion of their savings and leave most of it in bequeathable form. The bequest is like a private annuity contract that is agreed upon with the heirs. Under asymmetric information this contract is a substitute for the public annuities market. If the agent survives to period 2, she will consume this wealth, if not the heirs will inherit it.[27]

Agents with a bequest parameter larger than six-tenths purchase annuities only if the realization of their q_i is larger than 0.7. Because q_{AD} is less than 0.7, they can only gain from the annuities market. Thus, the adverse selection in the annuities market creates transfer of welfare from agents with a low bequest motive agents to those with a high bequest motive.

An example of the non-existence of equilibrium

In another simulation we approximate a uniform distribution for q on the interval [0,1] by assuming 5001 equally spaced possible realization, but assume an identical bequest parameter for all agents. Assuming that γ=3, an equilibrium in which some agents purchase annuities exists for values of δ below 0.275. For values of δ exceeding 0.28, the only equilibrium we find is when is q_{AD}=1, and agents with q=1 are indifferent between purchasing annuities and not purchasing them.

This example shows that Proposition 1 does not apply when there is a positive bequest motive.

Adverse Selection in the presence of mandatory Social Security annuity insurance

The annuity market may be influenced by the existence of a mandatory Social Security annuity insurance. To assess its impact, we modify our basic adverse selection model of Section 2 to allow for the existence of self-financing Social Security system. The insured now receives a retirement pension of S upon survival to the second period. Since the Social Security system is universal and egalitarian, every individual pays a Social Security tax of S/(2(1+r)). These changes are reflected in the insureds' problem below:

$$\max u(C_{1i})+q_i\beta u(C_{2i})$$

$$\text{s.t. } C_{1i}+(C_{2i}-S)q_{AD}/(1+r) = 1-S/(2(1+r))$$

$$i= H, L .$$

The existence of a Social Security system aggravates the adverse selection problem. The insureds now obtain their retirement income from two sources: the fixed publicly provided pension S, and the privately provided annuity $C_{2i}-S$. The ratio of $(C_{2L}-S)/(C_{2H}-S)$ is negatively related to S, and hence the adverse selection problem is exacerbated with the expansion of the Social Security system. The simulation results

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[27] This point is elaborated upon in Kotlikoff-Spivak (1981).

for our base case, modified to include Social Security insurance, are presented in Table 12. The money's worth ratio declines from .93 to .76 as S increases from 0 to .55. Because this range covers up to 70 percent of retirement income under precommitment, this is the most likely range for Social Security benefits. For $S \geq .6$, the L-insureds do not purchase annuities in the private sector at all, while the H-insureds purchase fairly priced annuities. Our calculations show, however, that the total effect of the introduction of Social Security insurance on insured's welfare is negligible: less than 1 percent as measured by the EV. We conclude that our results are robust to the inclusion of moderate levels of Social Security benefits in the model.

Table 12: Asymmetric information with Social Security System							
Social Security Benefit (S)	q_{AD}	MWR	C_{1H}	C_{1L}	C_{2H}	C_{2L}	EV_{ss}
0.0000	0.5364	0.9322	0.7693	0.8279	0.8603	0.6419	0.9933
0.0500	0.5389	0.9278	0.7695	0.8282	0.8591	0.6411	0.9933
0.1000	0.5419	0.9227	0.7697	0.8286	0.8578	0.6403	0.9933
0.1500	0.5454	0.9168	0.7699	0.8291	0.8562	0.6393	0.9933
0.2000	0.5495	0.9099	0.7702	0.8296	0.8544	0.6381	0.9933
0.2500	0.5545	0.9017	0.7706	0.8303	0.8522	0.6367	0.9933
0.3000	0.5606	0.8918	0.7710	0.8311	0.8495	0.6350	0.9932
0.3500	0.5685	0.8796	0.7715	0.8321	0.8461	0.6328	0.9932
0.4000	0.5788	0.8638	0.7721	0.8334	0.8418	0.6300	0.9931
0.4500	0.5933	0.8428	0.7730	0.8352	0.8359	0.6262	0.9929
0.5000	0.6153	0.8126	0.7744	0.8379	0.8272	0.6206	0.9926
0.5500	0.6557	0.7626	0.7766	0.8425	0.8121	0.6109	0.9919
0.6000	0.7500	0.6667	0.7818	0.8500	0.7818	0.6000	0.9906
0.6500	0.7500	0.6667	0.7864	0.8375	0.7864	0.6500	0.9951
0.7000	0.7500	0.6667	0.7909	0.8250	0.7909	0.7000	0.9979
0.7500	0.7500	0.6667	0.7955	0.8125	0.7955	0.7500	0.9995

7. Conclusion

In this paper we investigate the effect of adverse selection on the functioning of the market for annuities and the resulting welfare implications under alternative contracts and information structures. The annuities that are provided by the Social Security insurance and Defined Benefits pensions are contracted when insureds have little private information regarding their survival probabilities. In contrast, owners of

Defined Contribution contracts (and possibly the proposed privatized portion of the Social Security system) determine their annuities when they retire. At that time they usually have more precise estimates of their survival probabilities. Because insurers either do not know these estimates or are prohibited from using them to set premiums, adverse selection of insureds is introduced into the market.

On a theoretical basis, the Social Security insurance and Defined Benefits pensions are superior to Defined Contribution contracts. We find that, similarly to the conclusion in Hirshleifer's (1971) model, welfare is maximized when annuity contracts are set when information regarding annuitants' survival probabilities is not yet known. Thus, in principle, a privatized Social Security system, that allows insureds to accumulate contributions in a personal account until retirement and then annuitize it, is susceptible to adverse selection.

However, a closer examination shows that the impact of adverse selection on insureds' welfare is rather limited. In a general theoretical framework we demonstrate that, unlike the classic lemon market example (Akerloff ,1970), insureds that do not have a strong bequest motive should participate in the annuities market in equilibrium. Furthermore, our simulated estimates of the impact of the information structure on annuity prices and insureds welfare show a very small effect. Using multi-period simulations of the insureds' behavior, we find that adverse selection increases the price of annuity by about one percent as compared to the no-adverse selection case. The induced welfare loss is even smaller than the loss reflected in the price hike because of the partial substitution of annuities with non-annuitized funds.

We examine the robustness of our results to the simplifying assumptions in our model by incorporating a bequest motive and a Social Security system. The existence of bequest reduces the demand for annuities. We demonstrate the existence of an equilibrium in which some, but not all, individuals purchase annuities. As in the no-bequest case, the effect on welfare as measured by the equivalent variation is relatively small, averaging a one percent wealth decline. We also find that our results are robust to the incorporation of a Social Security system. While the magnitude of adverse selection increases, the overall welfare loss as measured by equivalent variation wealth remains at about one percent.

Our findings are in line with the empirical analysis of annuity markets in the U.S. as well as in other countries. Mitchell, Poterba, Warshawsky and Brown (1999) find that the actual annuities price are higher than the no-cost fair insurance by six to ten percent. However, they note that this margin includes "marketing cost, corporate overhead and income taxes, additions to various company contingency reserves, and profits, as well as the cost of adverse selection"(p. 1300). Thus, their results imply that the impact of adverse selection is bounded from above by ten percent. All our simulations fall within these bounds.

Our model and simulations suggest that, and explain why, adverse selection should not significantly affect the welfare of annuitants. This holds even in the presence of a Social Security system similar to that currently in effect in the U.S.. We thus conclude that adverse selection in the annuities market is not a sufficient reason to maintain Social Security in its present form.

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APPENDIX

Our basic example is the CRRA utility function $u(C)= 1/(1-\gamma)\ C^{1-\gamma}$, where $\gamma > 0$ is the measure of relative risk aversion. We use the equality $1/(1+r) = \beta$ extensively below. In what follows r=1 and thus $\beta= 1/2$.

Contract 1: Full precommitment.

The consumption is independent of the functional form:

$$C_1 = C_2 = 1/(1+0.5/(1+r)) = C_{PR}.$$

$$C_{PR} = 1/1.25 = 0.8$$

The expected utility of this contract, denoted by *EUpr* is:

$$EUpr = (1+0.5/(1+r))u(C_{PR}) .$$

$$EUpr = [1.25/(1-\gamma)]\ 0.8^{\ 1-\gamma} = [(1-\gamma)\ 0.8^{\ \gamma}]^{-1}$$

Contract *2: Public information*

The solution consists of fixed life-time consumption, higher for the *L*-insureds:

$$C_i = C_{1i} = C_{2i} = 1/(1+q_i/(1+r)) , i= H, L.$$

The expected utility of this contract, denoted by *EUpi* is:

$$EUpi = [0.5*(1+q_H/(1+r))]*u(C_H) + [0.5*(1+q_L/(1+r))]*u(C_L)$$

$$= [0.5/(1-\gamma)]\{[1+q_H/(1+r)]^{\gamma} + [1+q_L/(1+r)]^{\gamma}\}$$

Contract 3: Adverse selection- asymmetric information

F.O.C.:

$$C_{1i}^{-\gamma} = \lambda_i \;\Rightarrow\; C_{1i} = \lambda_i^{-1/\gamma}$$

$$C_{2i}^{-\gamma} = \lambda_i q_{AD}/q_i \;\Rightarrow\; C_{2i} = \lambda_i^{-1/\gamma} (q_{AD}/q_i)^{-1/\gamma}$$

$$C_{1i} + C_{2i} q_{AD} \beta = 1 \;\Rightarrow\; C_{1i}(1+(q_{AD}/q_i)^{-1/\gamma} q_{AD}\beta) = 1$$

Therefore,

$$C_{1i} = \frac{1}{1+(q_i/q_{AD})^{1/\gamma} q_{AD}\beta}$$

$$C_{2i} = \frac{(q_i/q_{AD})^{1/\gamma}}{1+(q_i/q_{AD})^{1/\gamma} q_{AD}\beta}$$

$i=H,L$.

Obtaining equilibrium value of q_{AD}

We denote the equilibrium value by q_{AD}. It must satisfy the budget constraint of the insurer:

$$\sum_{i=L,H} \frac{(q_i - q_{AD})(q_i/q_{AD})^{1/\gamma}}{1+(q_i/q_{AD})^{1/\gamma} q_{AD}\beta} = 0$$

Contract 4: Partial redemption.

We consider two cases: one in which inequalities of the Kuhn -Tucker conditions hold as strict inequalities, implying that $C_{2L} = C_2$ and the other when all inequailities hold as equalities and $C_{2L} \leq C_2$.

In the first case, the budget constraint at $t=1$ implies that $C_{1L} = C_1$. The budget constraint at $t=0$ and the insurer's budget constraint imply that $q_{R0} = 0.5$. This implies that $C_2 = C_1$. Given that $q_{R0} = 0.5$ and $C_2 = C_1$, the L-insureds choose not to redeem (i.e., to set $C_{2L} = C_2$ as assumed in this case) if and only if $q_{R1} \leq q_L$.

In the second case we solve the following system:

$$C_1^{-\gamma} = \lambda - \mu$$
$$C_2^{-\gamma} = \lambda q_{R0}/q_H - \mu q_{R1}/q_H$$
$$C_{1L}^{-\gamma} = \mu$$
$$C_{2L}^{-\gamma} = \mu q_{R1}/q_L$$
$$C_1 + C_2 q_{R0}/(1+r) = 1$$
$$C_{1L} + C_{2L} q_{R1}/(1+r) = C_1 + C_2 q_{R1}/(1+r)$$
$$1/2(C_1 + C_{1L}) + 1/2\,[q_H C_2 + q_L C_{2L}]\,/(1+r) = 1.$$
$$C_{2L} \leq C_2\,.$$

The formal proof for the first case and detailed solution for the second case are available upon request.

Proof of Proposition 4:

We define the Lagrangean:

$$L(a,b_1,b_2,\ldots,b_T) = \Sigma^T_{t=1} P_t \beta^{t-1} u(a+b_t) - \lambda(P_A a + \Sigma^T_{t=1} b_t (1/1+r))^{t-1} - W).$$

Differentiating with respect to a b_t and λ, the first order conditions obtained are respectively:

(A1) $\Sigma^T_{t=1} P_t \beta^{t-1} u'(a+b_t) \leq \lambda P_A$. For $a>0$ equality must hold.

(A2) $P_t u'(a+b_t) \leq \lambda$, $t=1,\ldots,T$. For $b_t>0$ equality must hold. (The assumption $\beta=1/(1+r)$ was used here).

(A3) $P_A a + \Sigma^T_{t=1} b_t (1/(1+r))^{t-1} \leq W$. When $u(.)$ is strictly increasing, equality holds.

The Proposition is proved via the following three claims.

Claim 1: The series b_t is strictly decreasing for $b_t>0$, i.e., if $b_t>0$ then $b_t> b_{t+1}$.

Let $b_t>0$. Then, by equation (A2) $u'(a+b_t)=\lambda/P_t$. If $b_{t+1}=0$, the claim is proved. If not, $u'(a+b_{t+1})=\lambda/P_{t+1}$, implying $a+b_{t+1}< a+b_t$, because $P_{t+1}< P_t$, and u' is strictly decreasing.

Claim 2: $b_T=0$ and $a>0$.

The proof is by contradiction. We first prove that $a>0$. Suppose that $\alpha=0$, then $b_t>0$ for all t, because $u'(0)$ is infinity. From equation (A2) it then follows that $\Sigma^T_{t=1} P_t\beta^{t-1}u'(a+b_t)=\lambda\Sigma^T_{t=1}(1/(1+r))^{t-1}>\lambda P_A$. (We assume that $P_A<\Sigma^T_{t=1}(1/(1+r))^{t-1}$, because otherwise the insurer has strictly positive profits.) This contradicts equation *(A1).*

To show that $b_T=0$ notice that if $b_T>0$, then the following consumption plan $b'_t=b_t-b_T$, $a'=a+b_T$ provides the same utility at a lower cost.

Claim 3: If $P_A\leq P_{Afair}$, $b_t=0$, $t=1,\ldots,T$; If $P_A>P_{Afair}$, $b_1>0$.

By the strict concavity of u, there exists only one maximum. We now show that under $P_A \leq P_{Afair}$, $a>0$ *and* $b_t=0$ (for all t) satisfy the first order conditions A(1) and (A2), and thus is the only solution.

By Claim 2 $a>0$ and $\Sigma^T_{t=1} P_t\beta^{t-1}u'(a)=\lambda P_A$. Then $u'(a)\ \Sigma^T_{t=1} P_t\beta^{t-1}=\lambda P_A$, and because $P_{Afair}= \Sigma^T_{t=1} P_t\ (1/(1+r))^{t-1}$ and $\beta=1/(1+r)$, it follows that $\lambda\geq u'(a)$.

Equation (A2) is now met for $b_t=0$, because $P_t\leq 1$, and $\lambda \geq u'(a)$.

In the same way we prove that for $P_A\geq P_{Afair}$, $b_1=0$ does not satisfy the conditions (A1) and (A2).

The Bequest Motive

Recall that:

$\delta' = (1/q_{AD}-1)/(1/q_i-1)$. For $\delta_j<\delta'$, $A_i>0$, and for $\delta_j \geq \delta'$, $A_i=0$.

For the CRRA utility, the f.o.c. are:

For $\delta_j<\delta'$: $A_{ij}+D_{ij}=C_{2i}=C_{1ij}(q_i/q_{AD})^{1/\gamma}$.

$D_{ij}=C_{1ij}(\delta_j(1-q_i)/(1-q_{AD}))^{1/\gamma}$.

(Notice that for $\delta_j=0$, $D=0$.)

For $\delta_j \geq \delta'$: $D_{ij}=C_{1ij}[(q_i+\delta_j(1-q_i)]^{1/\gamma}$.

Recall the budget constraint is:

$C_{1ij}+C_{2ij}q_{AD}/(1+r)+B_{ij}(1-q_{AD})/(1+r)=1$, and that the indicator function:

$\alpha(\delta)=\{1$ for $\delta<\delta'$ and 0 for $\delta \geq \delta'\}$.

Using the budget constraint to solve for optimal consumption, we obtain:

$$C_{1ij} = \frac{W}{1+\alpha(\delta_j)\{\frac{q_i^{1/\gamma} q_{AD}^{1-1/\gamma}}{1+r} + \frac{\delta_j^{1/\gamma}(1-q_{AD})^{1-1/\gamma}(1-q_i)^{1/\gamma}}{1+r}\} + [1-\alpha(\delta_j)]\frac{[q_i+\delta_j(1-q_i)]^{1/\gamma}}{1+r}}$$

We can now re-use the first order conditions to obtain C_{2ij} and D_{ij}, and calculate the q_{AD} from the market equilibrium condition:

$(1/mn)\Sigma_{i=1..n\ j=1..m} [C_{1ij}+ C_{2ij}q_i/(1+r)+B_{ij}(1-q_i)/(1+r)]=1$.