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Decomposing the Israeli Term Structure of Interests Rates

Daniel Nathan

Abstract

This paper decomposes the Israeli term structure of interest rates into two parts: the expected path of real interest rates and the risk premia for 01/1985—12/2019. We carry out the estimation using a discrete-time essentially affine term structure model (ATSM). ATSM models are essentially reduced-form models: they assume that latent factors drive the economy, and are extensively used by major central banks to infer risk premia in the term structure. The results show that part of the decline in real yields since 1985 was accompanied by a substantial decrease in the real risk premium; the compensation investors require to hold government indexed-bonds has gone down substantially. The compensation has been as high as 3% for the 10-year real yield and has gone down to around zero in recent years. The inflation risk premium (an inflation compensation which is part of the nominal yield curve), has also shown a significant drop in recent years. The inflation risk premium has become slightly negative in recent years after being as high as 2.5% in early 2000 for the 10-year nominal yield.

פירוק עקום התשואות העיתי בישראל

דניאל נתן

תקציר

מאמר זה מפרק את עקום התשואות הממשלתי בישראל, בתקופה מינואר 1985 עד דצמבר 2019, לשני חלקים: הריבית הריאלית הצפויה ופרמיות הסיכון. אנו מבצעים את האמידה תוך שימוש במודלים מסוג(ATSM) Affine Term Structure Models. מודלים אלה מניחים שגורמים סמויים מניעים את הכלכלה והם משמשים את הבנקים המרכזיים המובילים, בבואם לאמוד את פרמיות הסיכון בעקום התשואות הממשלתי. מתוצאות האמידה עולה שחלק מהירידה בתשואות הריאליות מאז 1985, לווה בירידה משמעותית בפרמיית הסיכון הריאלית והפיצוי שהמשקיעים דורשים עבור אג״ח ממשלתיות צמודות ירד משמעותית. הפיצוי עמד בתחילת 2001 על 1.5% מהתשואה הריאלית ל-10 שנים וירד בשנים האחרונות לרמות אפסיות. גם פרמיית סיכון האינפלציה (הפיצוי לאינפלציה שמהווה חלק מהתשואה הנומינלית), הציגה ירידה משמעותית בשנים האחרונות. היא הפכה להיות שלילית בשנים האחרונות לאחר שהתשואה הנומינלית ל-10 שנים הייתה בגובה של כ-2.5% בתחילת שנות ה-2000.

1 Introduction

This paper follows several recent studies by decomposing the term structure of interest rates into their expected interest-rates and risk-premia parts while utilizing ex-ante real yields derived from inflation-indexed bonds. Specifically, I decompose the Israeli term structure of interest rates to analyze its developments throughout the years. Given that standard asset pricing theory ascribes changes in nominal yields to either change in real yields, expected inflation, or the inflation risk premium, inflation-indexed bonds have the potential to enrich our understanding of the determinants of bond yields. Furthermore, many market participants, ranging from central banks to professional investors, monitor both nominal and inflation-indexed bonds. One particular use of looking at both types of bonds is the difference between the nominal and inflation-indexed bonds with the same maturity. It is also known as the break-even inflation (BEI), and it is often used as a proxy for the expected inflation. However, as market participants, particularly central banks, know, the BEI also contains an inflation premium so that the "true" inflation expectations can deviate from the BEI.

I estimate a discrete-time essentially affine term structure model (ATSM) to explain the sources of variation in Israeli government yields, using monthly inflation-indexed government bond data for 01/1985 12/2019 and nominal government bond data for the period of 05/2001–12/2019. ATSMs have the advantage to be both tractable and flexible enough to match the observed time-varying behavior of risk premia in the U.S., as shown by Dai and Singleton (2002).

Three characteristics make the Israeli bond market particularly well-suited to the current research:

1. **Issuance amount.** The amount of inflation-indexed bonds the Israeli treasury issues yearly remains high. Inflation-indexed bonds still make up a substantial part of new issues, at around 30%.

2. **Short real yields** - My data consists of real yields with maturities starting at one year. To the best of my knowledge, this is the first paper that estimates the real and nominal yield curves jointly using real rates with maturities of less than four years.

3. **Short indexation lag**. The indexation lag is, at most, a month and a half.[1]

I find that four (latent) factors provide a good fit for both the real and nominal yield curves. In particular, I show that the real yield embedded in Israeli government inflation-indexed bonds reflect real yields that are less affected by illiquidity, which distinguishes the current paper from both Abrahams, Adrian, Crump, Moench, and Yu (2016) and D'Amico, Kim, and Wei (2018) who adjust their model by adding a liquidity factor specific to Treasury Inflation-Protected Securities (TIPS). My main findings are as follows:

- The unconditional term structure of the inflation premium is increasing in maturity.
- Most of the variance in the break-even inflation in the short end is due to changes in the expected inflation. However, in the long end, most of it is due to the inflation term premium.
- Most of the variance in real yields is due to expected real short rates.
- The average expected real short rates are flat throughout the sample period.
- An increasing real term premium gives the unconditional real yield curve a positive slope.

The rest of the paper describes the analysis that led to these conclusions. Section 2 describes the data used for estimating the model in detail and presents summary statistics. Section 3 motivates the choice of four factors. Section 4 describes the essentially affine term structure model I used. Section 5 presents the details of the estimation procedure. Section 6

[1] The reference CPI is the last known CPI before the bond was issued. If an inflation-indexed bond is issued on the 14th of a month, say in May, the last known reference CPI is the 15th of April (i.e., March's inflation), which constitutes an inflation lag of one and a half months.

presents the results. Section 7 reiterates the findings and describes limitations and avenues for further research.

2 Data

In contrast to U.S. and U.K. government bonds, Israeli government bonds are traded in the stock market and not over-the-counter. A striking feature of Israeli government bonds is that inflation-indexed bonds have been trading since the early 1950s, long before long-term nominal bonds were introduced in the market in 1995. This is due to a history of double- and even triple-digit yearly inflation.

The earliest period for which I have complete and reliable real yields data is from 01/1985, and it is of the form of end-of-month yield to maturities up until 01/1995. From 01/1995–12/2019, real yields are smoothed using a cubic spline method. Nominal bonds data spans 05/2001–12/2019 and are smoothed using the cubic spline method as well. Throughout the sample, the yields are adjusted for carry (expectation of the nearest month realized inflation that has yet to be announced). From 2008 onwards, they are adjusted for seasonality (see Stein, 2012, for the methodology).[2]

I use real and nominal yields with maturities of one, three, five, seven, and ten years. For the shorter end of the nominal yield curve (up to one year), I use nominal zero-coupon yields, known as MAKAM for the period of 05/2001–12/2019.[3]I also use the mean of the survey forecasts of Bank of Israel (BOI) interest rate of one month and one year ahead for 05/2013–12/2019.[4] Israeli Central Bureau of Statistics offers seasonally adjusted monthly inflation data for the period of 06/2001–12/2019.[5]

Table 1 presents summary statistics of the nominal and real yield curve. The real term

[2]Even though seasonal adjustment has only been carried out since 2008, this does not imply that before 2008 real yields were biased. Seasonality was likely less of an issue then because many inflation-indexed bonds had been trading in the market and seasonal effects of in neighboring maturities would have canceled each other out.

[3]MAKAM data can be found at http://www.boi.org.il/en/DataAndStatistics/Pages/SeriesSearchBySubject.aspx?Level=3

[4]Twelve forecasters provide these forecasts to the BOI several times during the calendar month.

[5]See http://www.cbs.gov.il/ts/databank/databank_main_func_e

structure is upward sloping with an average slope (ten minus one) of 93 basis points, and the nominal term structure is upward sloping as well (for a shorter period of 05/2001–12/2019) with an average slope of 213 basis points. The term structure of the standard deviation of the changes of real yields is in general downward sloping. The nominal curve shows a similar trend. The term structure has a hump in the three years to maturity and declines afterward.[6] Apart from the one-year real rate, the term structure of standard deviation of the nominal rate changes is higher than the real rate. Roll (2004) documents this phenomenon also in the U.S. data. Roll explains that expected inflation shocks cause nominal yields to increase their standard deviation.

Table 1 also reveals that both real and nominal bonds have a high one-month autocorrelation, with a slower decay in long term real yields compared to their nominal counterparts. The time series in Figures 1, 2, and 3 reveal that all yields in Israel have decreased in the past 16 years. This has been a worldwide phenomenon.[7] These time series also reveal that the BEI has decreased considerably from its high levels in the early 2000s, a period of high inflation, to its low levels of recent years.

3 Preliminary Analysis

3.1 Factor analysis

To choose the appropriate number of factors, I perform a PCA analysis. It is a common finding that three factors explain the majority of the variability of the U.S. nominal term structure. Performing a PCA analysis on the Israeli data for the period of 01/85–12/2019 for real yields and for the period of 05/2001–12/2019 for nominal yields confirms that this is also the case for Israel: three factors explain 96.2% and 98.6% of the variance of monthly changes in the real and nominal yields, respectively. However, as I am interested in modeling both

[6]This latter empirical finding has been documented in Piazzesi (2001) for U.S. data, albeit for a different period. See Singleton (2009, chapter13) for a full discussion of this point.

[7]See, for instance, King and Low (2014).

the real and nominal curves, it is essential to do a PCA analysis combining real and nominal yields. Performing PCA on monthly changes of both the real and nominal yield curves yields that three factors can explain only 92% of the combined variance. In comparison, four and five factors can explain 96.3% and 98.18% of the variance, respectively.[8] Obviously, three factors are not going to be enough to explain the combined term structure. On the other hand, five factors can explain as much variability as three factors can for the real and nominal parts separately. Nevertheless, I chose to model the combined term structure with four factors to favor the parsimony of the model and avoid overfitting.[9]

3.2 Liquidity in the Israeli Government Bond Market

Several studies of the U.S. market, such as Sack and Elsasser (2002), Shen (2006), and D'Amico et al. (2018), conclude that prior to 2004, TIPS yields were too high. This suggests that there might have been a significant liquidity premium that has since declined. A recent paper by Fleckenstein, Longstaff, and Lustig (2014) shows that there are arbitrage opportunities in the TIPS market that were economically significant, particularly during the financial crisis of 2008. The same conclusion is reached by Haubrich, Pennacchi, and Ritchken (2012), albeit in a different way. They estimate an affine term structure model, but use U.S. zero-coupon inflation swaps to infer real yields. They report large deviations between their estimated real yields and those of TIPS. These issues have prompted both D'Amico et al. (2018) and Abrahams et al. (2016) to add an extra liquidity factor, which represents the lack of liquidity of TIPS relative to nominal Treasury securities. This section explores whether there is a lack of liquidity of CPI-linked bonds relative to their nominal counterparts in the Israeli market.

I follow D'Amico et al. (2018) and regress weekly 3-month, 2-year, and 10-year nominal

[8] As my real yields dataset starts earlier, I made sure that they both begin at 05/2001 for the PCA analysis.

[9] In ATSMs this is a real concern as shown in Duffee (2011). He shows that using more than three factors to estimate the U.S. nominal yield curve results in extremely high and unrealistic Sharpe ratios of expected excess returns.

yields on the 10-year BEI for the period of 05/2001–12/2019. If the 10-year real yield embedded in the 10-year inflation-indexed bond accurately captures the real yield, the BEI should contain the 10-year expected inflation and the inflation risk premium: two variables that are part of the 10-year nominal yield. This implies that regressing the 10-year BEI on weekly 3-month, 2-year,10-year nominal yields should result in a high R^2. Unlike D'Amico et al. (2018), I also run the regression on the 1-year, 3-year, 5-year, and 7-year BEI to assess the liquidity of the shorter maturities.

The results are presented in Table 2. They show an R^2 of 86% for the whole period for the 10-year maturity, much higher than the 6% reported in D'Amico et al. (2018). The results also show that the 1-year real-yield might suffer from less liquidity than the longer maturities. However, the general picture is that of a liquid indexed market. This suggests that the variation in the real yields implied from the inflation-indexed bond is due to variation in the "true" real yields. This should come as no surprise as the amount the Israeli treasury issues yearly is still high. Looking at Figure 4, we see that inflation-indexed bonds still make up a substantial part of new issues, at around 30%. For example, the U.K. is frequently cited in the literature as having a high percentage of its issues in indexed bonds, which has been around 20% in recent years.[10]Furthermore, Figure 5 and 6 show a time series of the monthly average of the daily trading volume of nominal and inflation-indexed bonds from 2001 onwards and the monthly average of daily quoted spreads divided by the midpoint price (in percentages) of inflation-indexed and nominal government bonds from 2005 onwards.[11][12]The figures show that even though the nominal market has a higher trading volume and a lower quoted spread, the difference is not as dramatic between them. For example, the ratio (as of June 2018) between the trading volume in the primary dealers market in the U.S. between the nominal and inflation-indexed markets is around 13, which indicates a much larger difference between the two markets.[13]Daily quotes data from Bloomberg for July 2018, reveal that the

[10]Data taken from http://www.dmo.gov.uk/index.aspx?page=Gilts/Portfolio_Statistics

[11]The earliest period I have data for.

[12]The daily quoted spreads are an average of all the quoted spreads in the limit order book.

[13]Data from https://www.newyorkfed.org/markets/gsds/search.html.

ratio between the quoted half-spread of the 5-year on-the-run treasury and the 5-year on-the-run 5 TIPS was 7. For the 10-year maturity, the ratio was 3.5. In contrast, in Israel, it has been two on average for the whole period and all maturities.

4 The Affine Term Structure Model

Both affine models and macroeconomic models derive their equations from equilibrium (and no-arbitrage) assumptions. Whereas in macroeconomic models, the pricing kernel has the form

$$\frac{M_{t+1}}{M_t} = \frac{\beta U^{'}(C_{t+1})}{U^{'}(C_t)}, \tag{1}$$

where β is the subjective discount factor, and $U^{'}(C)$ is the marginal utility of consumption, in affine models, as we shall see, the pricing kernel takes a different form. Many papers that decompose the term structure use ATSMs, such as the one I use here. Duffie and Kan (1996) popularized ATSMs, and Duffee (2002) extended them to include time-varying prices of risk (and named them "essentially affine models"). Dai and Singleton (2002) show that the estimates of a three-factor, essentially affine model, can match the dynamics of the time-varying risk premiums found by Fama and Bliss (1987), and Campbell and Shiller (1991) using U.S. nominal yields. My model is set in discrete-time and builds on the model of D'Amico et al. (2018).[14]

4.1 Real Pricing Kernel

I start with the specification of the real short rate. I assume the real short rate is an affine function of four latent factors

$$r_t^R = \delta_0^R + \delta_1^{R'} X_t,$$

[14]For full details and derivation see Appendix B.

where δ_0^R is a scalar, δ_1^R is a (4×1) vector, and X_t are the latent factors that drive the real yields and have the following form

$$X_{t+1} = \mu + \Phi X_t + \Sigma e_{t+1}, \tag{2}$$

where μ is a (4×1) vector, ϕ is a (4×4) matrix, $\{e_t\}$ is a (4×1) i.i.d $N(0, I)$ process, I is a (4×4) diagonal matrix and Σ is a (4×4) matrix. The model is complete with the specification of the real pricing kernel

$$\frac{M_{t+1}^R}{M_t^R} = \exp\left(-0.5\lambda_t^{R'}\lambda_t^R - r_t^R - \lambda_t^{R'} e_{t+1}\right), \tag{3}$$

where λ_t^R is a (4×1) vector of the real price of risk. In the essentially affine model, the price of risk can vary with the level of the state vector

$$\lambda_t^R = \lambda_0^R + \lambda_1^R X_t, \tag{4}$$

where λ_0^R is a (4×1) vector and λ_1^R is a (4×4) matrix. That is, the real price of risk is also an affine function of the latent factors and can change in magnitude and sign. After the real pricing kernel is specified, we can price real bonds at time t with n periods to maturity. Manipulation of the pricing equation (see Appendix B) shows that

$$y_{t,t+n}^R = A_n^R + B_n^{R'} X_t, \tag{5}$$

where $y_{t,t+n}^R$ is the continuously compounded yield at time t with maturity n, A_n^R is a scalar, and B_n^R is a (4×1) vector.

4.2 Nominal Pricing Kernel

As I a, jointly estimating both real and nominal bonds, I specify the nominal pricing kernel. I assume that the nominal short rate is an affine function of four latent factors

$$r_t^N = \delta_0^N + \delta_1^{N\prime} X_t, \tag{6}$$

where δ_0^N is a scalar, δ_1^N is a (4×1) vector, and X_t, the same latent factors that drive the real yields, also drive the nominal yields. The nominal pricing kernel has the form

$$\frac{M_{t+1}^N}{M_t^N} = \exp\left(-0.5\lambda_t^{N\prime}\lambda_t^N - r_t^N - \lambda_t^{N\prime} e_{t+1}\right), \tag{7}$$

where λ_t^N, the nominal price of risk, is a (4×1) vector. As in the case of the real price of risk, the nominal price of risk also varies with the state of the economy, i.e.,

$$\lambda_t^N = \lambda_0^N + \lambda_1^N X_t, \tag{8}$$

where λ_1^N is a (4×4) matrix.

The specification of the price process, π_t, links the nominal and real pricing kernels

$$\Pi_{t+1} = \Pi_t \exp\left(\delta_0^\pi + \delta_1^{\Pi\prime} X_t + \sigma_q' e_{t+1} + \sigma_v v_{t+1} - 0.5(\sigma_q'\sigma_q + \sigma_v^2)\right), \tag{9}$$

where δ_0^π is a scalar, δ_1^π is a (4×1) vector, σ_q is a (4×1) vector, σ_v is a scalar, and v_t is a white noise process. I assume that $\text{cov}(v_t, e_t) = 0$. Thus, I follow D'Amico et al. (2018) and Risa (2001) and allow for an exogenous shock not spanned by the yield curve to affect the price process. Nominal yields follow the same functional form as real yields, i.e.,

$$y_{t,t+n}^N = A_n^N + B_n^{N\prime} X_t, \tag{10}$$

where $y_{t,t+n}^N$ is the continuously compounded yield at time t with maturity n, A_n^N is a scalar, and B_n^N is a (4×1) vector. To complete the model, the relationship between the real and nominal pricing equations can be derived via the following no-arbitrage argument: the price of an asset that promises 1 unit of consumption in the next period has to be equal under both the real and nominal pricing kernel. That is,

$$E\left(\frac{M_{t+1}^R}{M_t^R} \cdot 1\right) = E\left(\frac{M_{t+1}^N}{M_t^N} \cdot \frac{\Pi_{t+1}}{\Pi_t}\right). \tag{11}$$

Assuming that the market is complete implies that

$$\frac{M_{t+1}^R}{M_t^R} = \frac{M_{t+1}^N}{M_t^N} \cdot \frac{\Pi_{t+1}}{\Pi_t}. \tag{12}$$

Substituting Equations (3), (7) and (9) into (12) and matching coefficients yields the following restrictions on the parameters:

$$\begin{aligned}
\delta_0^N &= \delta_0^R + \delta_0^\pi + 0.5(\sigma_q^{'}\sigma_q + \sigma_v^2) - \lambda_0^{N'}\sigma_q, \\
\delta_1^N &= \delta_1^R + \delta_1^\pi - \lambda_1^{N'}\sigma_q, \\
\lambda_0^N &= \lambda_0^R + \sigma_q, \\
\lambda_1^R &= \lambda_1^N.
\end{aligned} \tag{13}$$

5 Estimation

Following Joyce, Lildholdt, and Sorensen (2010), Kim and Orphanides (2012), and D'Amico et al. (2018), I assume that all yields are estimated with error, and construct the maximum likelihood function using the Kalman filter.[15]

[15]See Appendix A for more details on deriving the maximum likelihood function.

5.1 The State and Measurement Equation

Implementing the Kalman filter requires definition of a state and a measurement equation. ATSMs lend themselves easily to this specification. Equation (2), the law of motion of the latent factors, explicitly defines the state equation. For the measurement equations[16] I will assume that real and nominal yields are observed with a measurement error such that

$$
\begin{aligned}
y^N_{t,t+n} &= A^N_n + B^{N'}_n X_t + \eta_R, \\
y^R_{t,t+n} &= A^R_n + B^{R'}_n X_t + \eta_N,
\end{aligned} \tag{14}
$$

where

$$
\begin{bmatrix} \eta_R \\ \eta_N \end{bmatrix} \overset{N}{\sim} \begin{pmatrix} \sigma_R & 0 \\ 0 & \sigma_N \end{pmatrix}, \tag{15}
$$

where σ_R and σ_N are scalars. Therefore, I assume that nominal and real bonds differ with respect to their measurement error.

5.2 The Price Process

For the price process level, Π_t, I use the seasonally adjusted series. I define the monthly change at time t as the log of the level at time t divided by its level at $t-1$, i.e.,

$$
\pi_{t+1} \equiv \log\left(\frac{\Pi_{t+1}}{\Pi_t}\right) = \delta^\pi_0 + \delta^{\pi'}_1 X_t + \sigma^{'}_q e_{t+1} + \sigma_v v_{t+1} - 0.5(\sigma^{'}_q \sigma_q + \sigma^2_v). \tag{16}
$$

Unlike for yields, I assume that the price process is estimated without error.

[16]Since the data is monthly end-of-month yields, t and n are in monthly units.

5.3 Matching Surveys

Kim and Orphanides (2012) make the point that ATSMs law of motion parameters are estimated imprecisely and biased downwards because estimating persistent factors requires a large sample. This results in a model with faster mean reversion parameters that attribute almost all variation of longer-term forward rates to the term premium. Kim and Orphanides then show that supplementing the estimation of an ATSM with survey forecasts of future T-bill yields reduces the bias in the estimated law of motion parameters considerably. I follow them by using the mean of survey forecast of one month and one year ahead interest rate BOI target rate.

Expected n-month ahead short rates are easily derived and take the form[17]

$$E_t(r^N_{t+n}) = \delta^N_0 + \delta^{N'}_1 \phi^n X_t. \tag{17}$$

I assume that one-month and one-year ahead mean survey forecasts have no bias in their expectations, i.e.,

$$\begin{aligned} I^2_t &= E_t(r^N_{t+1}) + \eta_{i_2}, \\ I^{13}_t &= E_t(r^N_{t+12}) + \eta_{i_{13}}, \end{aligned} \tag{18}$$

and

$$\eta_I \overset{N}{\sim} \begin{pmatrix} \sigma_{i_2} & 0 \\ 0 & \sigma_{i_{13}} \end{pmatrix}. \tag{19}$$

A disadvantage of the Israeli data is that there are no long-term surveys, which may

[17] To show this, note that

$$E_t(r^N_{t+n}) = E_t(\cdots E_{t+n-1}(r^N_{t+n})).$$

This can be solved backwards using the tower property.

bias the estimated law of motion parameters. However, it is not clear how important the long term forecasts are in reducing bias. For instance, Kim and Orphanides (2012) use the expected 3-month T-bill yield between the next 6 and 11 years, but since it is available only twice a year, they downplay its role in their estimation by imposing a large standard error.

5.4 The Augmented State and Measurement Equation

To fix ideas, I conclude this section with a more explicit representation and the identification restriction I impose. The augmented state equation takes the form

$$\begin{bmatrix} x^1_{t+1} \\ x^2_{t+1} \\ x^3_{t+1} \\ x^4_{t+1} \\ \pi_{t+1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \delta^\pi_0 - \frac{1}{2}(\sigma_q^{'}\sigma_q + \sigma_v) \end{bmatrix} \begin{bmatrix} \phi_{1,1} & 0 & 0 & 0 & 0 \\ 0 & \phi_{2,2} & 0 & 0 & 0 \\ 0 & 0 & \phi_{3,3} & 0 & 0 \\ 0 & 0 & 0 & \phi_{4,4} & 0 \\ \delta^\pi_{1,1} & \delta^\pi_{1,2} & \delta^\pi_{1,3} & 0 & 0 \end{bmatrix} \begin{bmatrix} x^1_t \\ x^2_t \\ x^3_t \\ x^4_t \\ q_t \end{bmatrix} + \begin{bmatrix} 0.01 & 0 & 0 & 0 & 0 \\ \sigma_{2,1} & 0.01 & 0 & 0 & 0 \\ \sigma_{3,1} & \sigma_{3,2} & 0.01 & 0 & 0 \\ \sigma_{4,1} & \sigma_{4,2} & \sigma_{4,3} & 0.01 & 0 \\ \sigma^1_q & \sigma^2_q & \sigma^3_q & \sigma^4_q & \sigma_v \end{bmatrix} \begin{bmatrix} \epsilon^1_{t+1} \\ \epsilon^2_{t+1} \\ \epsilon^3_{t+1} \\ \epsilon^4_{t+1} \\ \upsilon_{t+1} \end{bmatrix}. \tag{20}$$

Notice that as in the ATSM, not all the parameters are identified, I set the off-diagonal entries of matrix ϕ to zero, the vector μ to zero and the diagonal of Σ_v to 0.01, while only estimating the cross correlation parameters.[18] I also set the off-diagonal price of risk of λ_1^N to zero.

The augmented measurement equation takes the form

[18] For a discussion on this point see Dai and Singleton (2000).

$$
\begin{pmatrix} y^N_{t,t+6} \\ y^N_{t,t+9} \\ \vdots \\ y^N_{t,t+12} \\ \vdots \\ y^N_{t,t+120} \\ y^R_{t,t+12} \\ \vdots \\ y^R_{t,t+120} \\ \pi_t \\ I^2_t \\ I^{13}_t \end{pmatrix} = \begin{pmatrix} A^N_6 \\ A^N_9 \\ \vdots \\ A^N_{12} \\ \vdots \\ A^N_{120} \\ A^R_{12} \\ \vdots \\ A^R_{120} \\ \delta^\pi_0 - \frac{1}{2}(\sigma^{'}_q\sigma_q + \sigma_v) \\ \delta^N_0 \\ \delta^N_0 \end{pmatrix} + \begin{pmatrix} B^N_{1,6} & \cdots & B^N_{4,6} & 0 \\ B^N_{1,9} & \cdots & B^N_{4,9} & 0 \\ \vdots & \ddots & \vdots & \vdots \\ B^N_{1,12} & \cdots & B^N_{4,12} & 0 \\ \vdots & \ddots & \vdots & \vdots \\ B^R_{1,120} & \cdots & B^R_{4,120} & 0 \\ B^R_{1,12} & \cdots & B^R_{4,12} & 0 \\ \vdots & \ddots & \vdots & \vdots \\ B^R_{1,120} & \cdots & B^R_{4,120} & 0 \\ \delta^\pi_{1,1} & \cdots & \delta^\pi_{1,4} & 0 \\ \delta^N_{1,1}\phi_{1,1} & \cdots & \delta^N_{1,4}\phi_{4,4} & 0 \\ \delta^N_{1,1}\phi^{12}_{1,1} & \cdots & \delta^N_{1,4}\phi^{12}_{4,4} & 0 \end{pmatrix} \begin{bmatrix} X_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \eta_N \\ \eta_R \\ 0 \\ \eta_I \end{bmatrix}. \tag{21}
$$

Using the Kalman filter, I estimate a total of 36 parameters.

$$
\Theta \equiv (\phi, \sigma_v, \sigma_q, \lambda^N_0, \lambda^N_1, \delta^\pi_0, \delta^\pi_1, \sigma_\eta, \sigma_R, \sigma_N, \sigma_I) \tag{22}
$$

6 Results

6.1 General Fit

The mean absolute deviation (MAD) and the root mean square error (RMSE) are reported in Table 3. Except for the one-year real yield, the average RMSE of real yields across maturities is 23 basis points for the period of 01/1985–12/2019. For the period of 05/2001–12/2019, the RMSE of real yields across maturities is lower (15 basis points). The better fit in the later period might be because the yield to maturities I used in the previous period induced more measurement error. For nominal yields, the RMSE is 7.5 basis points for the period of 05/2001–12/2019. In Figures 8 and 9, I compare the time series of the model's fitted real and nominal yields with their actual yields. The figures confirm the model's good fit of the nominal and real yield curves.

The lack of a good fit for the 1-year real rate probably stems from its poor liquidity, as we already saw in the preliminary evidence in subsection 3.2. Another possibility is that the 1-year real yield is still partially affected by seasonality, as its effect on the shortest maturities is highest. This could result in a fitted 1-year real yield closer to the "true" seasonally-adjusted 1-year rate. To test these hypotheses, I follow Pflueger and Viceira (2011) and run the following regression,

$$BEI_t = \alpha + \beta_1 E_t(\pi_t) + \beta_2 \mathrm{BaS}_t + \beta_3 \mathrm{Seas}_t + \epsilon_t, \tag{23}$$

for the period of 01/2005–12/2019, where BEI_t is the end-of-month 1-year BEI, $E_t(\pi_t)$ is the end-of-month average of the survey of professional forecasters for the 1-year inflation, BaS_t is the average monthly quoted spreads divided by the midpoint price (in percentages) that serve as a proxy for the 1-year real yield liquidity, and Seas_t is a series of the seasonal factors.[19]The results of the regression are presented in Table 4. They reveal that the level of liquidity, as proxied by the bid-ask spreads, results in a lower BEI. Therefore, the lack of fit (at least from 2005 onwards) is partially the result of the poor 1-year liquidity. The results also indicate that the BEI is also affected by seasonality. In untabulated results, I find that both have roughly the same effect on the BEI.

6.2 Decomposing the Yield Curve

This subsection presents the main results of the paper, which is the decomposition of the nominal yield into its expectation and risk premium parts:

[19]The sample begins in 2005 because this is the earliest I have bid-ask spreads data.

$$y_t^N = \underbrace{E_t(y_t^R)}_{expected\ real\ rate} + \underbrace{RealRP_t^N}_{real\ risk\ premium} + \underbrace{E_t(\pi_t^N)}_{inflation\ expectations} + \underbrace{RP_t^N}_{inflation\ risk\ premium} . \quad (24)$$

6.3 The Real Rate

Table 1 revealed that the real term structure was upward sloping during the 01/1985–12/2019 period. Figure 10 shows the time-series decomposition of the 1-year, 3-year, 5-year, 7-year, and 10-year real yields into fitted real yields, the average expected short real yields, and the real term premium. It shows that variation in real yields is mostly driven by variation in the expected short rate and not in the real term premium. This is further confirmed by Table 5 shows that, on average, the expected real short rate was flat throughout the period.

It is interesting to compare these results to some theoretical model predictions. Nakamura, Steinsson, Barro, and Ursúa (2013) develop a model of consumption disasters that allows disasters to unfold over multiple years and to be systematically followed by recoveries. Their model implies that the real term premium should be negative and that real term structure should be downward sloping, as real bonds are excellent hedges against disaster risk. Piazzesi and Schneider (2006) solve for a model of a representative agent asset-pricing model with recursive utility preferences and exogenous consumption growth and inflation. In their model, inflation shocks are "bad news" for future consumption growth. Therefore, long indexed bonds pay off when future real interest rates (and future consumption growth) are low so that they provide a hedge. Therefore, the real term structure should be downward sloping. The Israeli data do not corroborate both of the channels described by the two models. However, our results are in line with the model of Wachter (2006) who develops a consumption-based model of the term structure of interest rates. She calibrates her model so that the real risk-free rate is negatively correlated with surplus consumption. The negative correlation between surplus consumption and the risk-free rate leads to positive risk premia on real bonds and an upward-sloping yield curve.

6.4 The Break-Even Term Structure

In this subsection, I analyze the results of the BEI decomposition. As nominal yields data is only from 05/2001, my analysis of the BEI starts accordingly from that point. Figure 13 shows the time series decomposition of the 1-year, 3-year, 5-year, 7-year, and 10-year of the BEI into fitted break-even yields, expected inflation, and the inflation term premium. Table 6 shows the first and second moments of the decomposition. The Table reveals that the inflation term premium for the whole sample has been positive, ranging from a low of 0.18 basis points at the short end (1 year) to 0.54 at the 10-year maturity. The figure and the Table also show that the short rate variation is primarily driven by variation in expected inflation. As we move into longer maturities, the variation is mostly caused by variation in the inflation risk premium.

The results also reveal that in the past five years, the inflation premium has been negative in all years to maturity, which coincides with a period of decline in yearly CPI to negative territory both in Israel and worldwide, as shown in Figure 11. The negative inflation risk premia are consistent with positive inflation shocks correlated with states of low marginal utility of wealth and, therefore, act as a hedge against inflation. To test this empirically, I follow Song (2017) and compute the rolling correlation of five years between seasonally adjusted quarterly CPI and consumption growth. The results are presented in Figure 12. The results of the figure support the empirical finding of the ATSM. They show a regime shift at the beginning of 2013, where the correlation between inflation and consumption growth became procyclical.

The negative inflation risk premium also agree with other empirical findings in the literature outside of Israel. In the U.S., Kitsul and Wright (2013) find that the empirical pricing kernel, which they estimate from options based on the consumer price index, is U-shaped with inflation. Therefore, in low consumption times, a negative inflation shock is associated with higher real yields for nominal bondholders and is a hedge.[20] Song (2017) documents a

[20]The high inflation term premia in the early 2000s is also consistent with this finding.

regime shift in the relation between inflation and consumption growth over the past 15 years. He finds that inflation has become procyclical, implying a negative inflation risk premium. D'Amico et al. (2018) document a trending inflation risk premium in the few years in their sample that ends in 2013.

Figure 14 shows the fitted 5-year/5-year BEI forward rates along with the 5-year/5-year expected inflation. It shows that the inflation term premium drives almost all the variation in the forward BEI measure. It further indicates that the expected inflation is relatively constant at around 2%, which is the BOI target rate. These results are similar to those of Abrahams et al. (2016) who report the same finding for the U.S.: most of the variation is due to the inflation term premia while expected inflation remains stable between 2.1% and 2.5%. As mentioned earlier, the results for Israel show that the inflation term premia has become negative concurrently with the persistent low inflation. Thus, the results for Israel are consistent with the findings of Abrahams et al. (2016) (whose sample ends in November 2014) and D'Amico et al. (2018) (whose sample ends in March 2013) for the U.S. They show a trending inflation term premium that was concurrent with the persistent world inflation.

As we noted in the previous subsection, many models imply a downward sloping nominal curve due to the negative inflation risk premium. An exception is Song (2017) who develops a macroeconomic model that can accommodate a negative inflation risk premium and an upward sloping nominal yield curve within a regime switching framework. However, he doesn't concentrate on the real yield curve, which, as we show per our empirical results, is upward sloping.

7 Conclusions

This paper decomposes the Israeli term structure of interest rates to understand its sources of variation throughout the years. I do the decomposition by using a discrete-time four-factor essentially affine term structure model and utilizing inflation-indexed and nominal government bonds. The real rates data is uniquely long and spans the period of 01/1985–12/2019. The nominal yields data spans the period of 05/2001–12/2019. An increasing inflation term premium accounts for the unconditional positive slope of the nominal curve, while inflation expectations are relatively flat. I also find that the one-year real yield variance accounts for most of the variance in the one-year nominal yield. In the longer maturities, most of the variance is driven by the expected inflation and the inflation term premium.

I also show that contrary to the results of Abrahams et al. (2016), D'Amico et al. (2018) for the U.S., liquidity has a less prominent role in determining real yields in Israel.

A limitation of the model is that it does not identify the four factors. It would be interesting to identify the factors, as macro-finance models do. I leave this to future work.

A Estimating ATSMs with the Kalman Filter

In this appendix, I summarize the general strategy of estimating affine term structure models with the Kalman filter. We start with our state and measurement equations:

$$\begin{aligned} X_{t+1} &= \mu + \Phi X_t + \Sigma \epsilon_{t+1}, \\ y_t^n &= A_n + B_n X_t + u_t, \end{aligned} \tag{25}$$

where $\{u_t\}$ is i.i.d $N(0, \Sigma\Sigma')$. This framework assumes that what drives the yields are k latent factors. In this particular setup, we shall also need some results from the Kalman filter algorithm. The Kalman filter algorithm can produce latent factors given both observed data and the parameters of the model. Although Equation (25) is written in state-space form, we lack the parameters of the dynamics. Thus, the role of the Kalman filter in the estimation process is subtle.

A.1 The Kalman Filter Algorithm

In order to understand the role of the algorithm in the estimation of the essentially affine term structure model, we first present the algorithm. Suppose that we are given the following set of equations:

$$y_t = Hx_t + v_t, \tag{26}$$

$$x_t = Gx_{t-1} + \epsilon_t, \tag{27}$$

where

$$\begin{pmatrix} v_t \\ \epsilon_t \end{pmatrix} \sim N\left(0, \begin{bmatrix} R & 0 \\ 0 & Q \end{bmatrix}\right). \tag{28}$$

Assume that we can observe only the multivariate series $\{y_t\}$, when what we really want to know is the dynamics of the latent series $\{x_t\}$. Suppose also that H,G,R, and Q are given. The Kalman filter algorithm yields:

$$E(x_t|\Omega_t),\ Var(x_t|\Omega_t),\ f(y_t|\Omega_{t-1}).$$

We introduce some useful notation:

$$\begin{aligned} x_{t-1|t-1} &\equiv E(x_{t-1}|\Omega_{t-1}), \\ P_{t-1|t-1} &\equiv V(x_{t-1}|\Omega^{t-1}). \end{aligned}$$

The Kalman filter combines $x_{t-1|t-1}$ and $P_{t-1|t-1}$ with y_t to yield

$$\begin{aligned} x_{t|t} &\equiv E(x_t|\Omega_t), \\ P_{t|t} &\equiv V(x_t|\Omega_t). \end{aligned}$$

Note that we know that $\{x_t\}$ and $\{y_t\}$ are jointly normal processes. Moreover,

$$\begin{pmatrix} x_t & |\Omega_{t-1} \\ y_t & |\Omega_{t-1} \end{pmatrix} \sim N\left(\begin{bmatrix} x_{t|t-1} \\ y_{t|t-1} \end{bmatrix}, \begin{bmatrix} P_{t|t-1} & C'_{t|t-1} \\ C_{t|t-1} & h_{t|t-1} \end{bmatrix}\right) \tag{29}$$

where we have that

$$\begin{aligned} x_{t|t-1} &= Gx_{t-1|t-1}, \\ y_{t|t-1} &= Hx_{t|t-1}, \\ P_{t|t-1} &= GP_{t-1|t-1}G' + Q, \\ C_{t|t-1} &= HP_{t-1|t-1}, \\ h_{t|t-1} &= HP_{t|t-1}H' + R. \end{aligned}$$

The main takeaway is to note that the marginal distribution of y_t is

$$f(\cdot|y_{t|t-1}) \sim N(y_{t|t-1}, h_{t|t-1}). \tag{30}$$

This means that in order to build the likelihood function of the $\{y_t\}$ process, we have to go along and work out the Kalman filter, as it generates the covariance matrix of the process. Next, we run the Kalman filter algorithm.

A.2 The Likelihood Function

We want to estimate the likelihood function

$$f(y_1, \dots, y_T|\Theta), \tag{31}$$

where $\Theta = \{\Sigma, \Sigma_u, A_n, B_n, \mu, \Phi, \lambda\}$. Since the process $\{y_t\}$ has the Markov property, we have

$$f(y_1, \dots, y_T|\Theta) = f(y_1|\Theta) \prod_{t=2}^{T} f(y_t|y_{t-1}, \Theta), \tag{32}$$

where $f(y_t|y_{t-1}, \Theta)$ is a multivariate normal variable (see Equation (30)). The likelihood function is thus of the form

$$L(\Theta|\{y_1, \dots, y_T\}) = (2\pi)^{\frac{-k(T)}{2}} \prod_{t=1}^{T} \det(h_{t|t-1})^{-\frac{1}{2}} \exp\left(-\frac{1}{2}(y_t - y_{t|t-1})' h_{t|t-1}^{-1}(y_t - y_{t|t-1})\right). \tag{33}$$

For better numerical stability, we maximize the log-likelihood function:

$$LogL(\Theta|\{y_1, \dots, y_T\}) \propto -\sum_{t=2}^{T} \log\left(\det(h_{t|t-1})\right) - \sum_{t=2}^{T}(y_t - y_{t|t-1})' h_{t|t-1}^{-1}(y_t - y_{t|t-1}). \tag{34}$$

B Deriving the Pricing Equations

Note that

$$P_t^1 = E(M_{t+1}) = \exp\left(-r_t\right) = \exp\left(-\delta_0 - \delta_1 X_t\right).$$

From this, we guess the general form of the price of a bond to be

$$\log(P_t^n) = p_t^n = \left(-\overline{A}_n - \overline{B}_n X_t\right). \tag{35}$$

We know from no-arbtirage pricing that

$$p_t^n = E(p_{t+1}^{n-1} + m_{t+1}) + \frac{1}{2}\mathrm{var}(p_{t+1}^{n-1} + m_{t+1}). \tag{36}$$

Matching the coefficients of Equation (36) with Equation (35), we have that the coefficients $\overline{A}_n$ and $\overline{B}_n$ follow the difference equations (also known as loading factors):

$$\begin{aligned} \overline{A}_{n+1} &= \overline{A}_n + \overline{B}_n{}'(\mu - \Sigma_v \lambda_0) + 0.5\overline{B}_n{}'\Sigma_v \Sigma_v' \overline{B}_n - \delta_0, \\ \overline{B}_{n+1}{}' &= \overline{B}_n{}'(\Phi - \Sigma_v \lambda_1) - \delta_1'. \end{aligned} \tag{37}$$

Since $P_t^n = \exp\left(-ny_t^n\right)$, we have that

$$y_t^n = A_n + B_n X_t, \tag{38}$$

where $A_n = -\overline{A}_n/n$ and $B_n = -\overline{B}_n/n$.

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Table 1: Summary Statistics. This Table shows summary statistics of nominal and real yields with maturities of one, three, five, seven, and ten. The data frequency is monthly (end-of-month), and it spans the period of 01/1985–12/2019 for real yields and the period of 05/2001–12/2019 for nominal yields. From 05/2001 onwards, the yields are of constant maturity and smoothed. Before that, the yield to maturity of inflation-indexed bonds is used. Real yields are adjusted for both the effect of carry and seasonality. Data is in percent and annualized. Yields are continuously compounded.

(a) Real Yields 01/1985–12/2019

Maturity	**1**	**3**	**5**	**7**	**10**
Mean	2.33	2.62	2.80	2.92	3.26
Std. Dev. (changes)	1.20	0.54	0.45	0.40	0.35
1 Month Autocorrel.	0.892	0.975	0.977	0.977	0.98
3 Month Autocorrel.	0.810	0.931	0.931	0.941	0.943
6 Month Autocorrel.	0.741	0.887	0.888	0.891	0.915
1 Year Autocorrel.	0.704	0.834	0.845	0.839	0.864

(b) Nominal Yields (05/2001–12/2019)

Maturity	**1**	**3**	**5**	**7**	**10**
Mean	2.84	3.48	4.06	4.53	4.97
Std. Dev. (changes)	0.33	0.40	0.40	0.38	0.38
1 Month Autocorrel.	0.991	0.989	0.989	0.989	0.988
3 Month Autocorrel.	0.963	0.963	0.964	0.964	0.959
6 Month Autocorrel.	0.911	0.916	0.924	0.928	0.924
1 Year Autocorrel.	0.841	0.843	0.857	0.864	0.855

Table 2: Dependent variable: break-even inflation. This Table shows a regression of the break-even inflation for 1-year, 3-year, 5-year, 7-year, and 10-year on contemporaneous variables. The variables are the MAKAM, which is the 3-month Israeli t-bill, the 2-year zero-coupon nominal yield, and the 10-year zero-coupon nominal yield. The sample ranges from 05/2001–12/2019 and is in monthly frequency. The t-statistics are in brackets below the coefficients and are corrected for heteroscedasticity using the White (1980) correction. *, **, and *** denote significance at the 10%, 5%, and 1% levels respectively.

	1-year	**3-year**	**5-year**	**7-year**	**10-year**
Intercept	0.55***	0.48***	0.49***	0.60***	0.72***
	(5.32)	(6.86)	(8.61)	(12.57)	(15.44)
MAKAM	-0.59***	-0.26***	-0.17***	-0.11***	-0.02
	(-7.92)	(-5.25)	(-4.52)	(-3.62)	(-0.67)
2-year nom.	0.69***	0.21***	0.07	-0.01	-0.11***
	(6.07)	(2.72)	(1.24)	(-0.17)	(-2.75)
10-year nom.	0.07	0.32***	0.41***	0.43***	0.43***
	(1.17)	(7.82)	(13.22)	(17.83)	(19.46)
Adjusted R-squared (%)	38.48	63.29	78.44	85.95	86.80

Table 3: Fit Statistics. This table shows the RMSE and the MAD of the estimated yields, both real and nominal. Data is in basis points.

(a) Real rates (01/1985-12/2019)

	Real				
	1 Year	**3 Year**	**5 Year**	**7 Year**	**10 Year**
Real RMSE	52.13	19.61	21.57	26.30	24.06
Real mean	31.72	12.68	10.64	14.56	17.68

(b) Nominal rates (05/2001-12/2019)

	Nominal						
	3M	**6M**	**1 Year**	**3 Year**	**5 Year**	**7 Year**	**10 Year**
Nominal RMSE	8.20	3.45	16.80	5.48	4.43	2.29	9.79
Nominal mean	5.44	2.19	10.04	3.75	3.13	1.59	6.88

Table 4: Liquidity of the 1-year real rate. This table presents the results of the regression $BEI_t = \alpha + \beta_1 E_t(\pi_t) + \beta_2 \mathrm{BaS}_t + \epsilon_t$ for the period of 01/2005–12/2019, where BEI_t is the end-of-month 1-year BEI, $E_t(\pi_t)$ is the end-of-month average of the survey of professional forecasters for the 1-year inflation rate, BaS_t is the monthly average quoted spread divided by the midpoint price (in percentages) that serves as a proxy for the 1-year real rate. Seas_t is the series of seasonal factors used to adjust CPI-inflation. The t-statistics are in brackets below the coefficients and are corrected for heteroscedasticity using the White (1980) correction. *, **, and *** denote significance at the 10%, 5%, and 1% levels respectively.

	1-year BEI
Intercept	0.18
	(1.25)
$E_t(\pi_t)$	1.17***
	(32.71)
BaS_t	-8.31***
	(-3.91)
Seas_t	0.23***
	(3.88)
Adjusted R-squared (%)	90.83

Table 5: Decomposing the Real Term Structure. This table presents the first and second moments of the decomposition of fitted real yields implied from the estimated four factor model. Data is in percent annualized.

Maturity	Fitted real yields	std. dev.	Expected real yields	std. dev.	Real term premium	std. dev.
1	2.25	2.55	2.00	2.49	0.25	0.43
3	2.63	2.39	2.19	2.14	0.44	0.32
5	2.80	2.12	2.22	1.85	0.58	0.51
7	2.97	1.90	2.21	1.58	0.76	0.62
10	3.24	1.73	2.16	1.25	1.08	0.64

Table 6: Decomposing the BEI. This table presents the first and second moments of the decomposition of fitted BEI implied from the estimated four factor model. Data is in percent annualized.

Maturity	Fitted BEI	std. dev.	Expected inflation	std. dev.	Inflation term premium	std. dev.
1	1.73	1.05	1.55	0.84	0.18	0.29
3	2.09	0.95	1.77	0.59	0.32	0.47
5	2.31	0.93	1.89	0.45	0.42	0.58
7	2.46	0.92	1.97	0.35	0.49	0.65
10	2.58	0.92	2.03	0.26	0.54	0.72

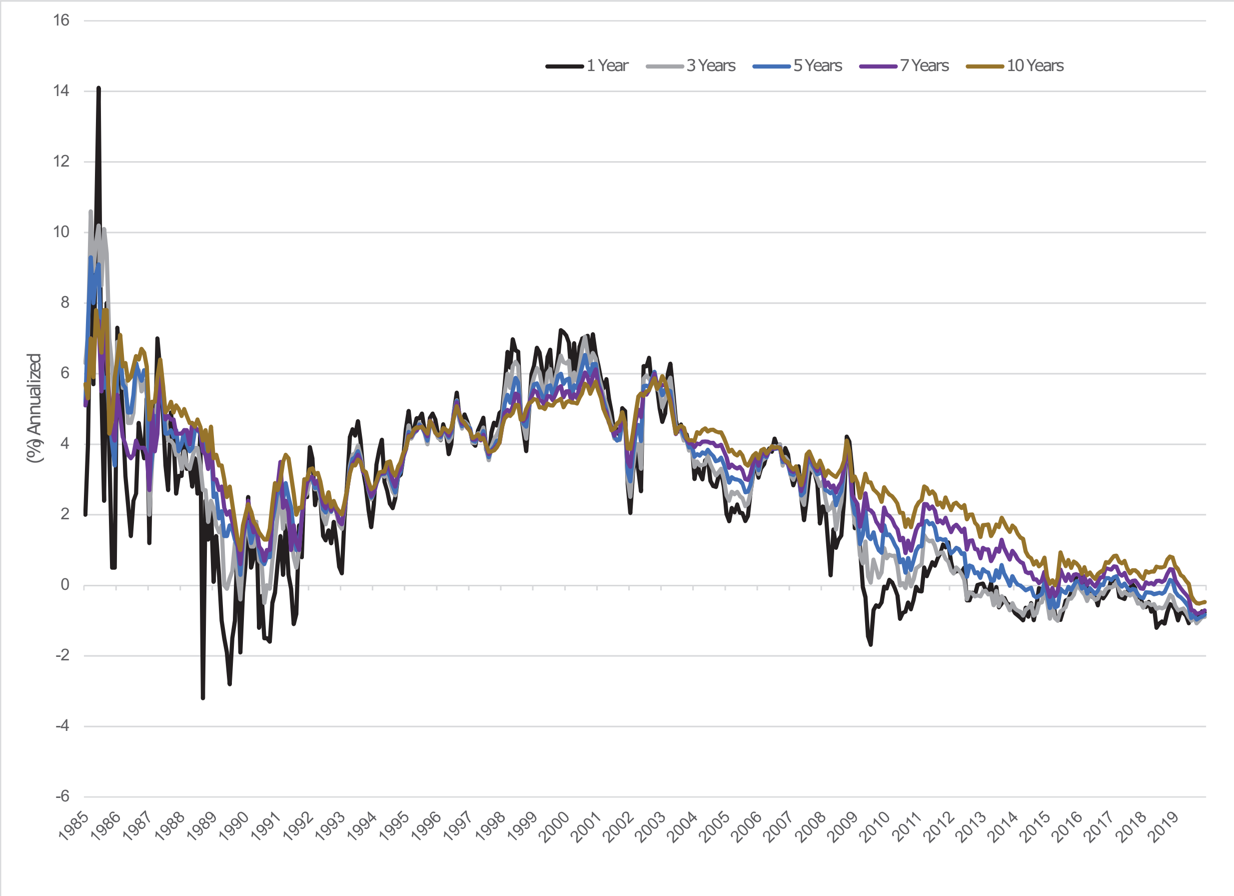

Figure 1: Time Series of Real Yields. This figure presents the time series of real yields (adjusted for the effect of carry and seasonality) with maturities of one, three, five, seven, and ten years used in the estimation of the model. Data is monthly (end-of-month) and yields are in percent annualized.

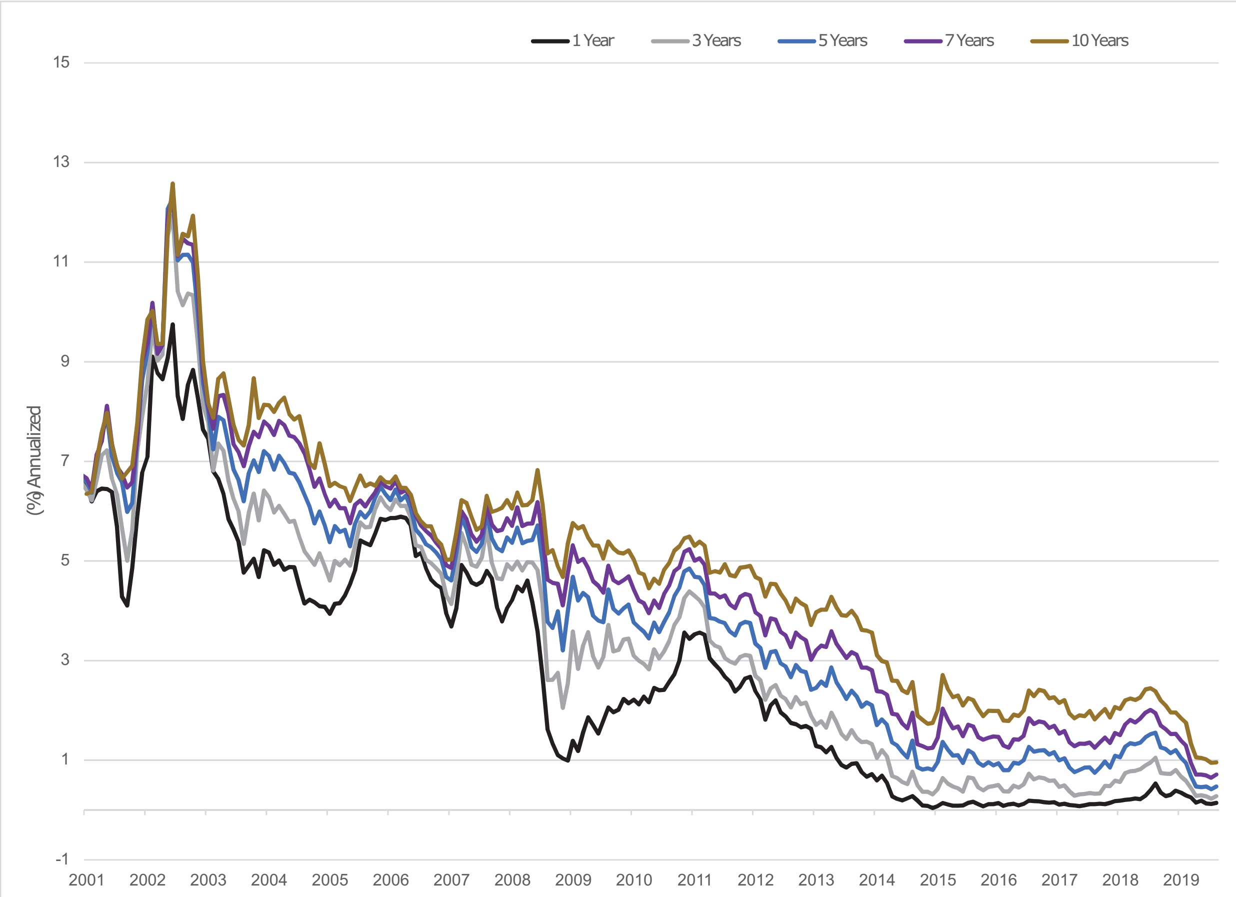

Figure 2: Time Series of Nominal Yields. This figure presents the time series of nominal yields with maturities of one, three, five, seven, and ten years used in the estimation of the model. Data is monthly (end-of-month) and yields are in percent annualized.

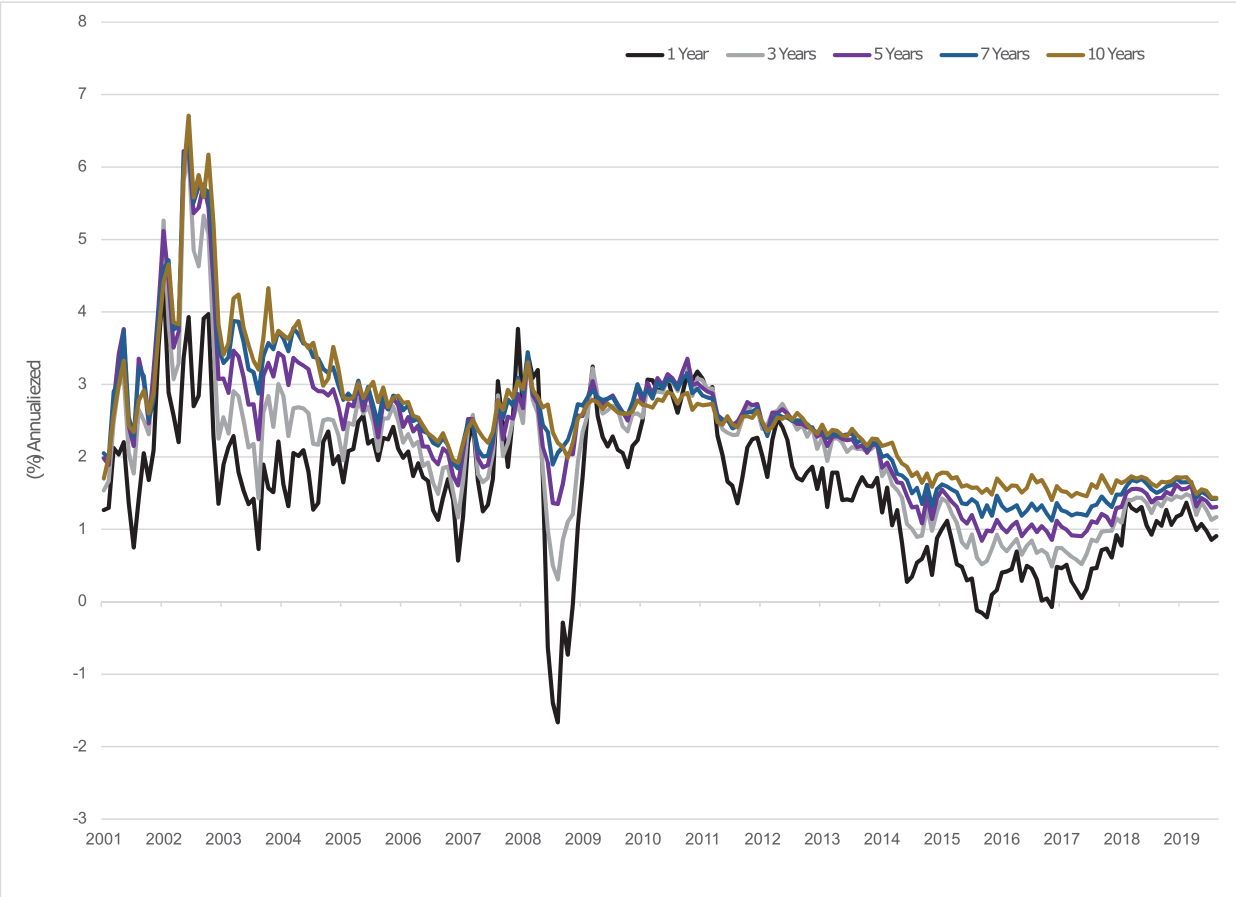

Figure 3: Time Series of Break-Even Inflation. This figure presents the time series of the break-even inflation (nominal minus real yield) with maturities of one, three, five, seven, and ten used in the estimation of the model. Data is monthly (end-of-month) and yields are in percent annualized.

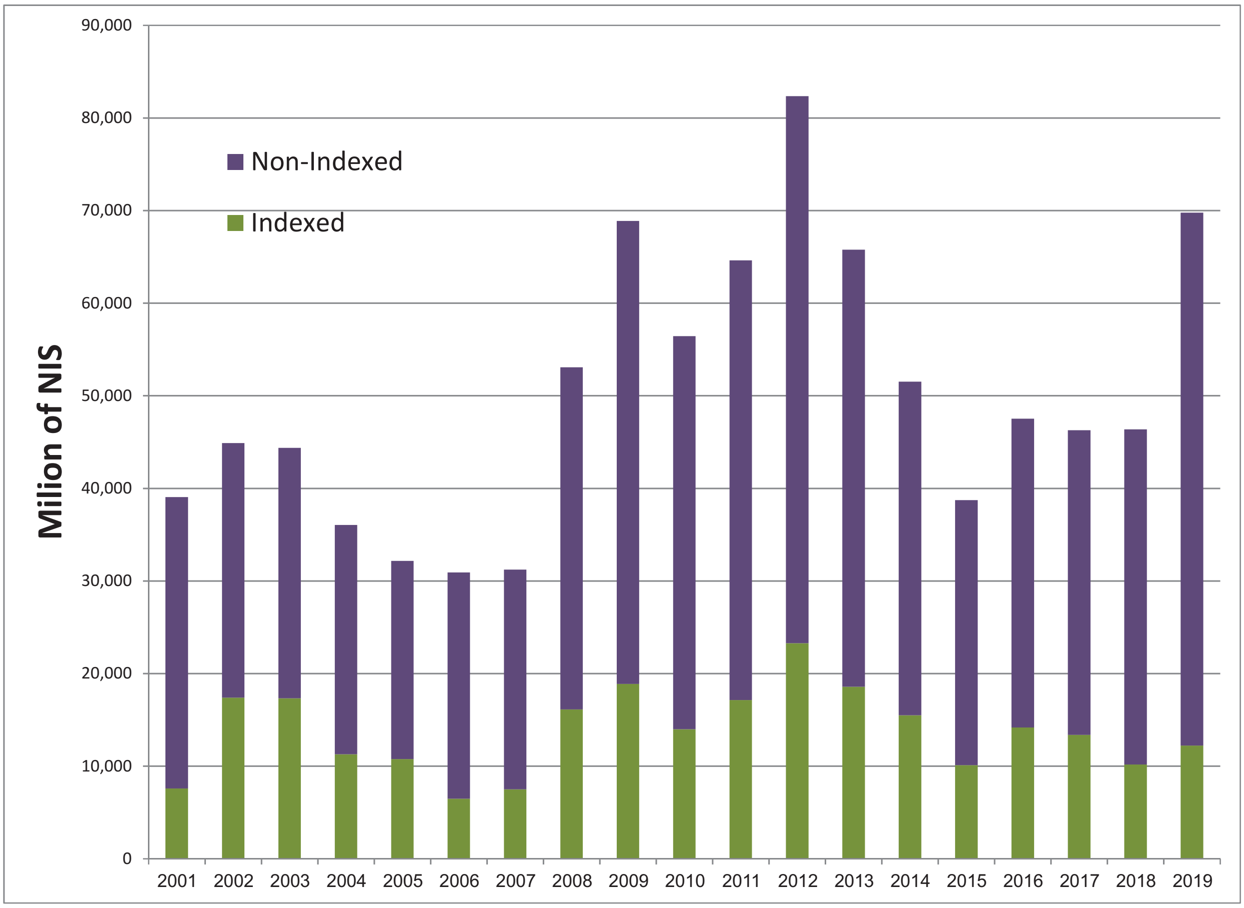

Figure 4: Issuance of indexed and non-indexed bonds. This figure plots yearly issuance of indexed and nominal bonds. Data is in millions of NIS.

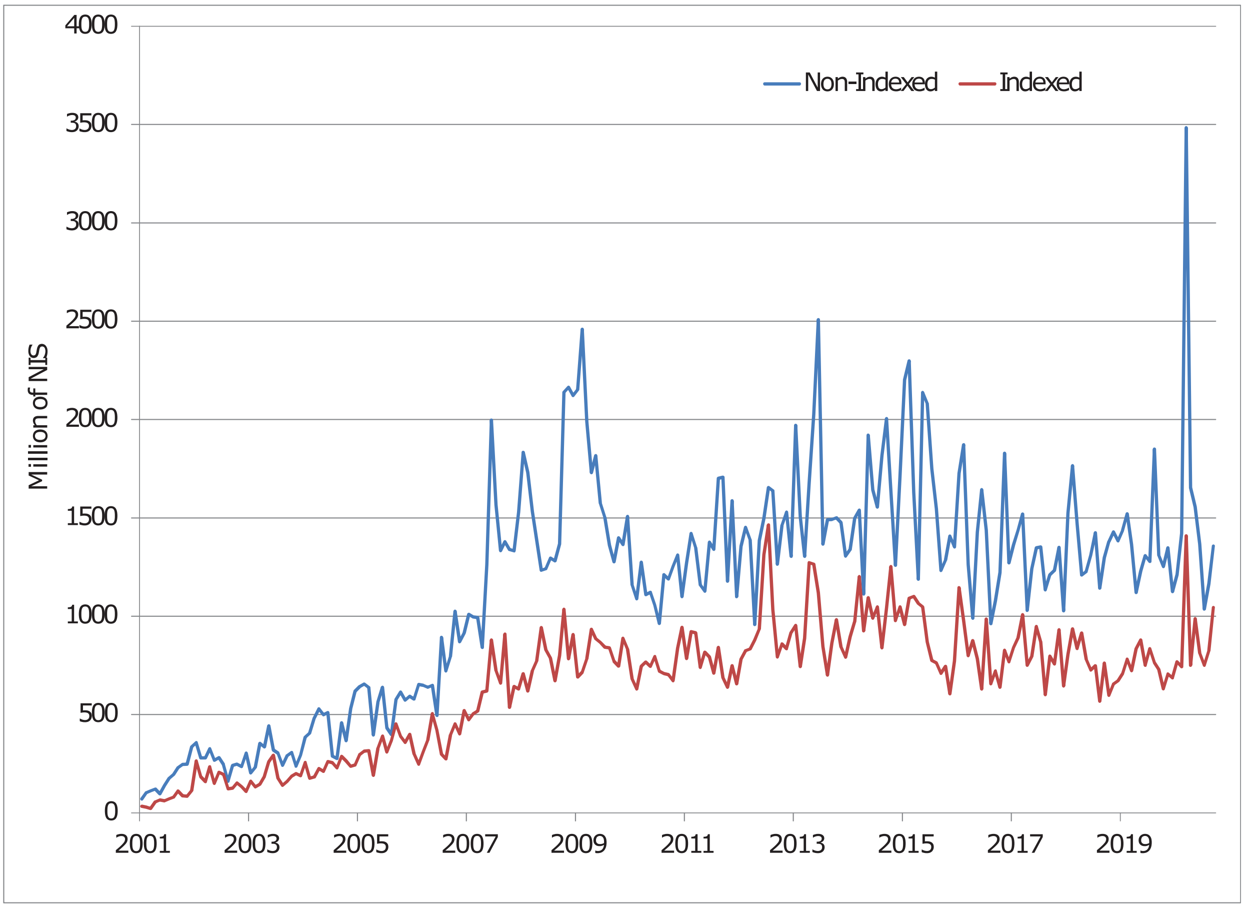

Figure 5: Volume (in millions of NIS) of bonds. The figure plots the trading volume of indexed and non-indexed bonds in the Israeli stock market.

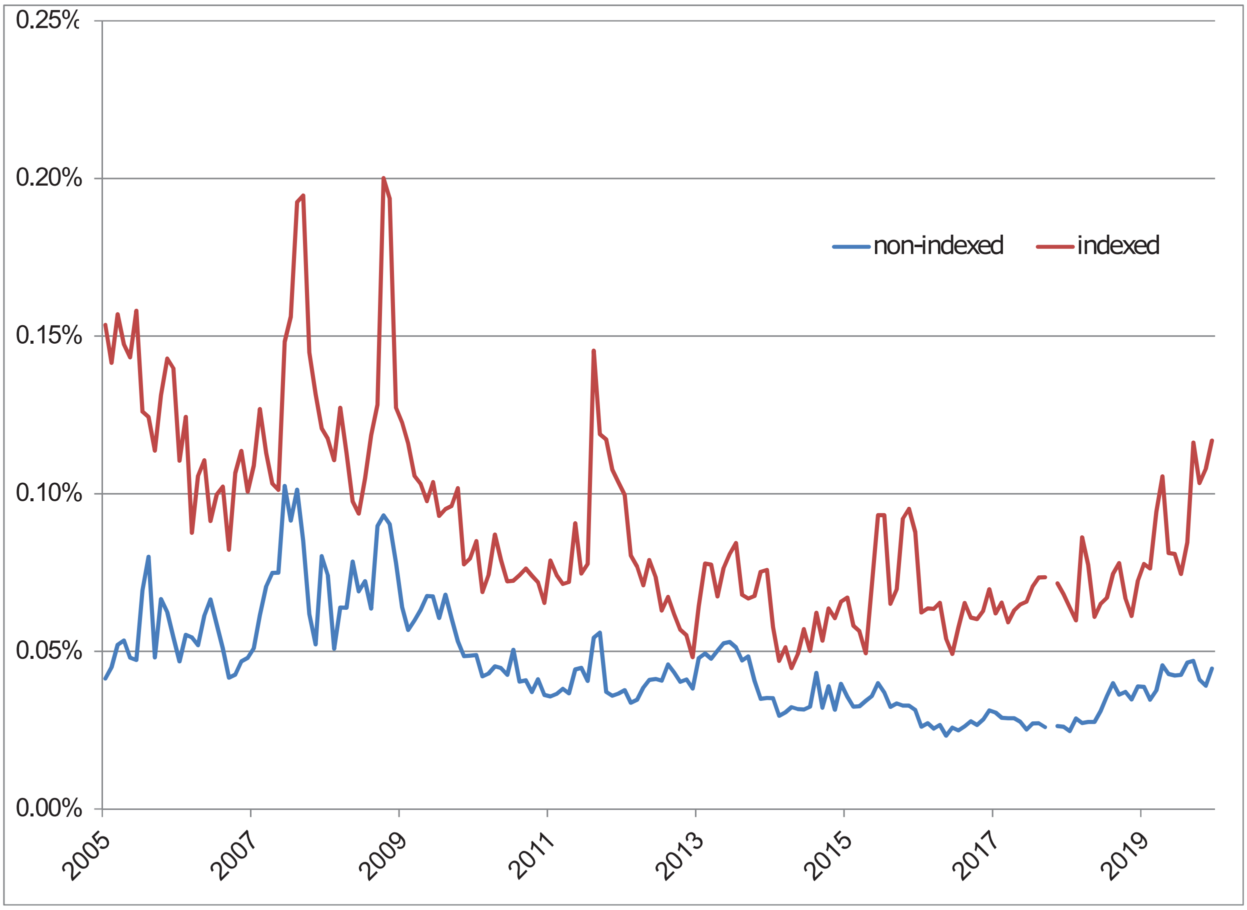

Figure 6: Bid-Ask Spreads. The figure plots the quoted spread divided by the midpoint price (in percentages) in the nominal and inflation-indexed bonds.

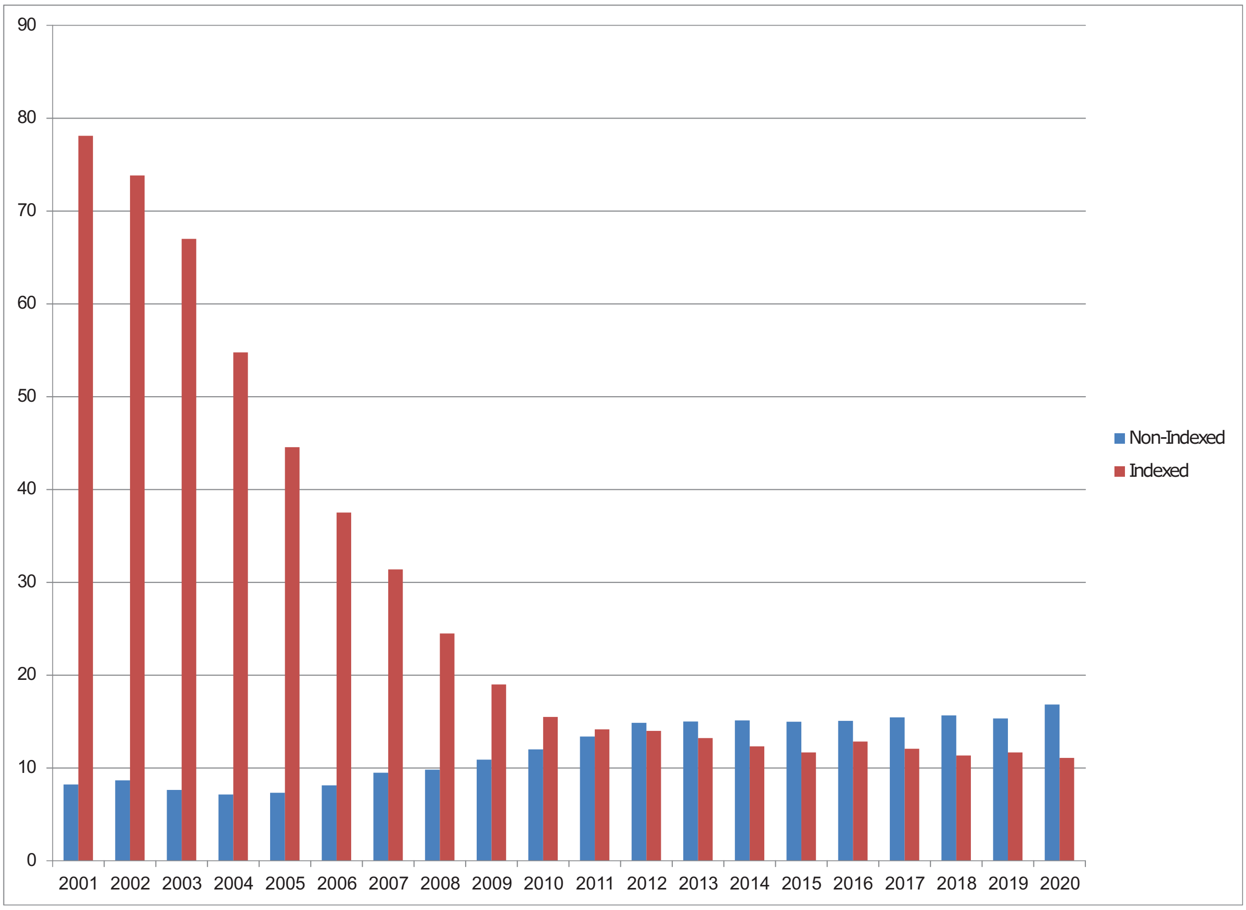

Figure 7: Number of Series. This figure plots the number of series of Indexed and non-indexed bonds.

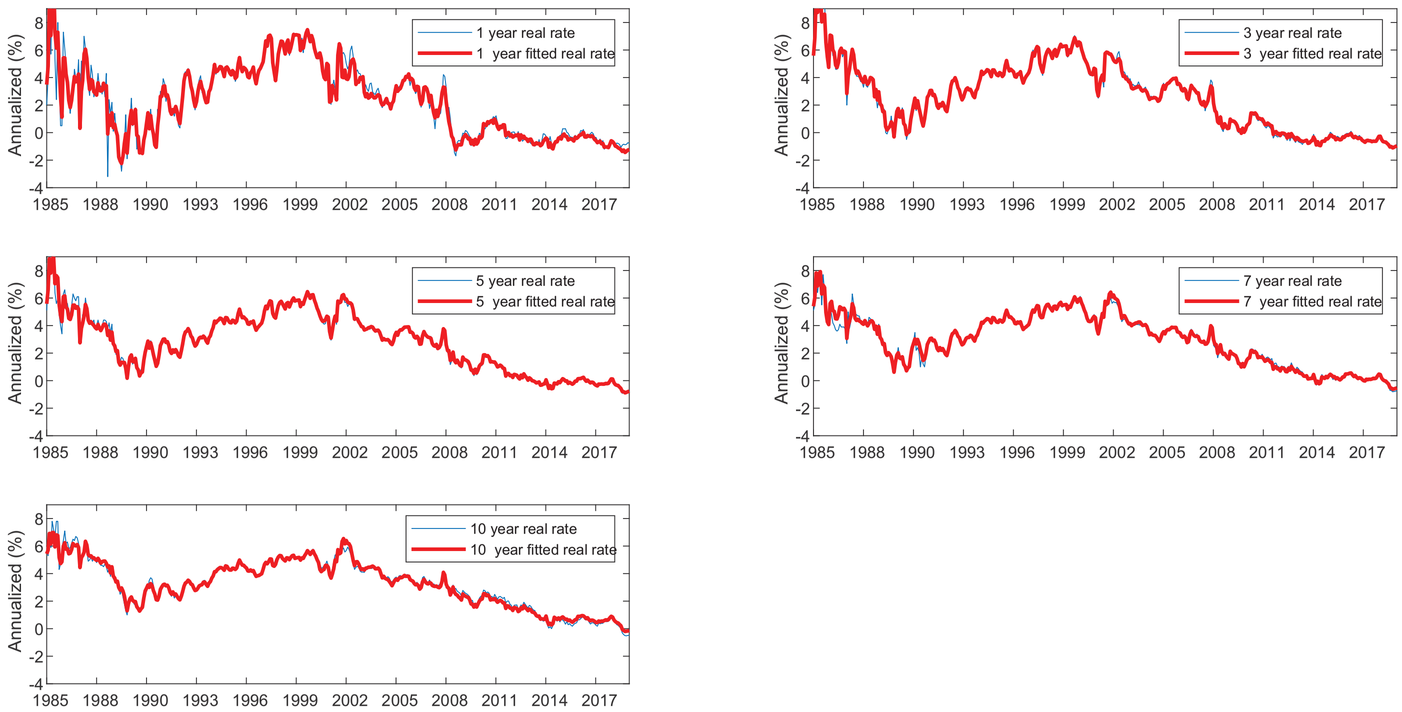

Figure 8: Real Yields Comparison. This figure plots fitted real yields from the three factor model and actual real yields for maturities of 1,3,5,7 and 10 years.

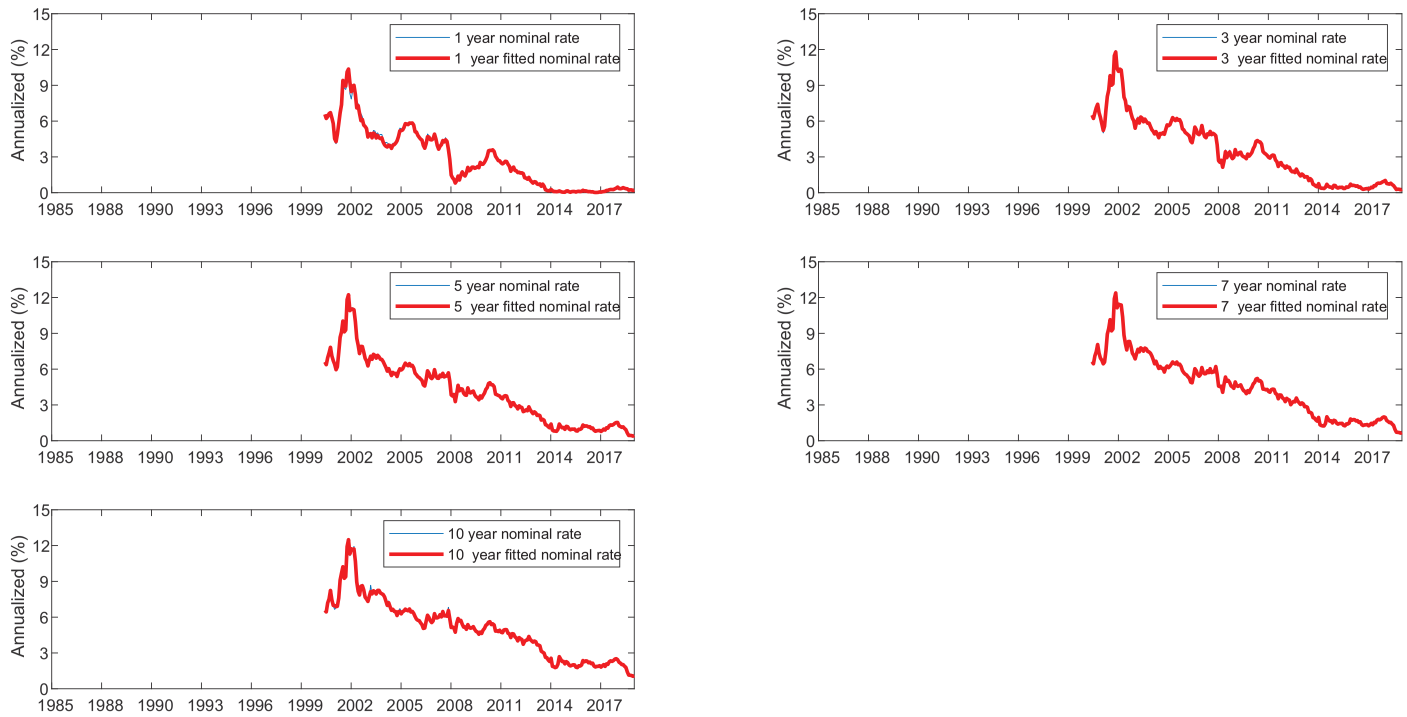

Figure 9: Nominal Yields Comparison. This figure plots fitted nominal yields from the three factor model and actual nominal yields for maturities of 1,3,5,7 and 10 years.

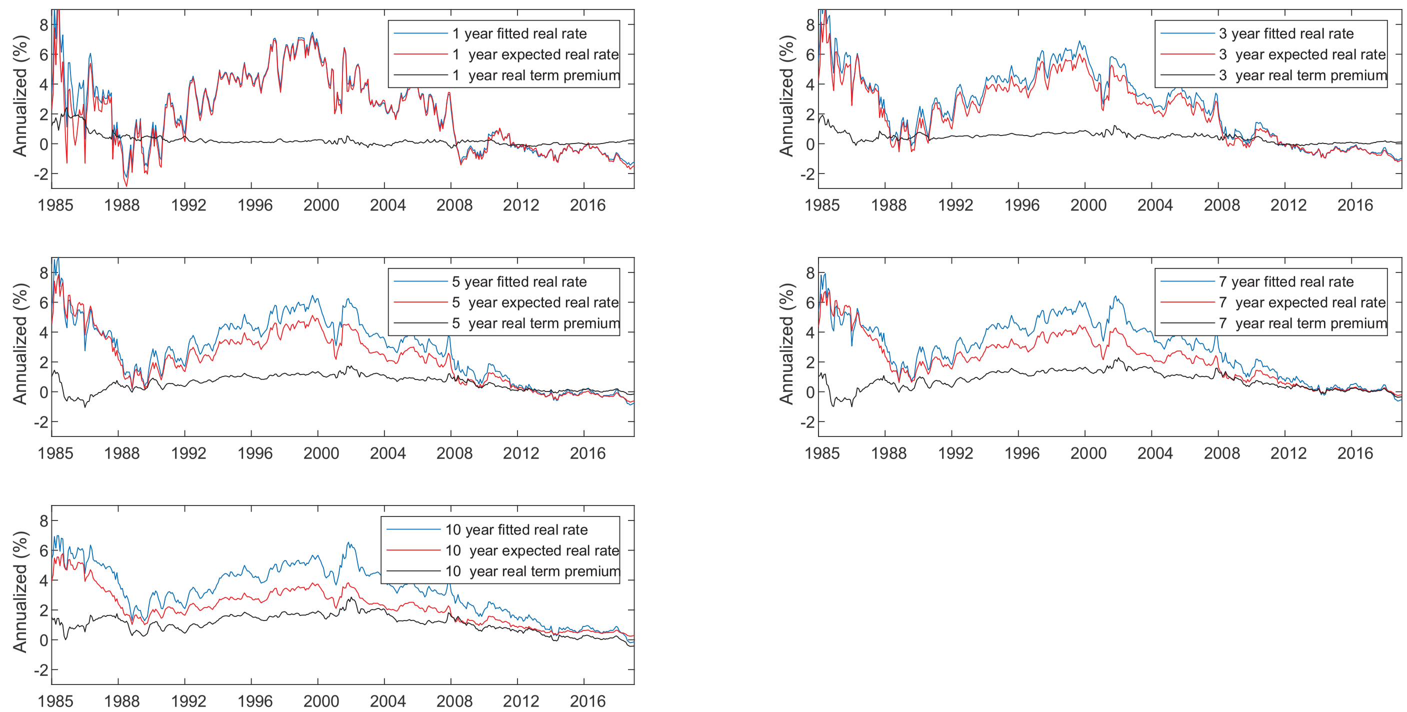

Figure 10: Fitted real rate Time Series Decomposition. This figure presents a decomposition of the fitted real rate rate to the expected real rate and the real term premium for the 01/1985-12/2019 period. The decomposition is based on a joint estimation of a four-factor model using real and nominal Israeli yields.

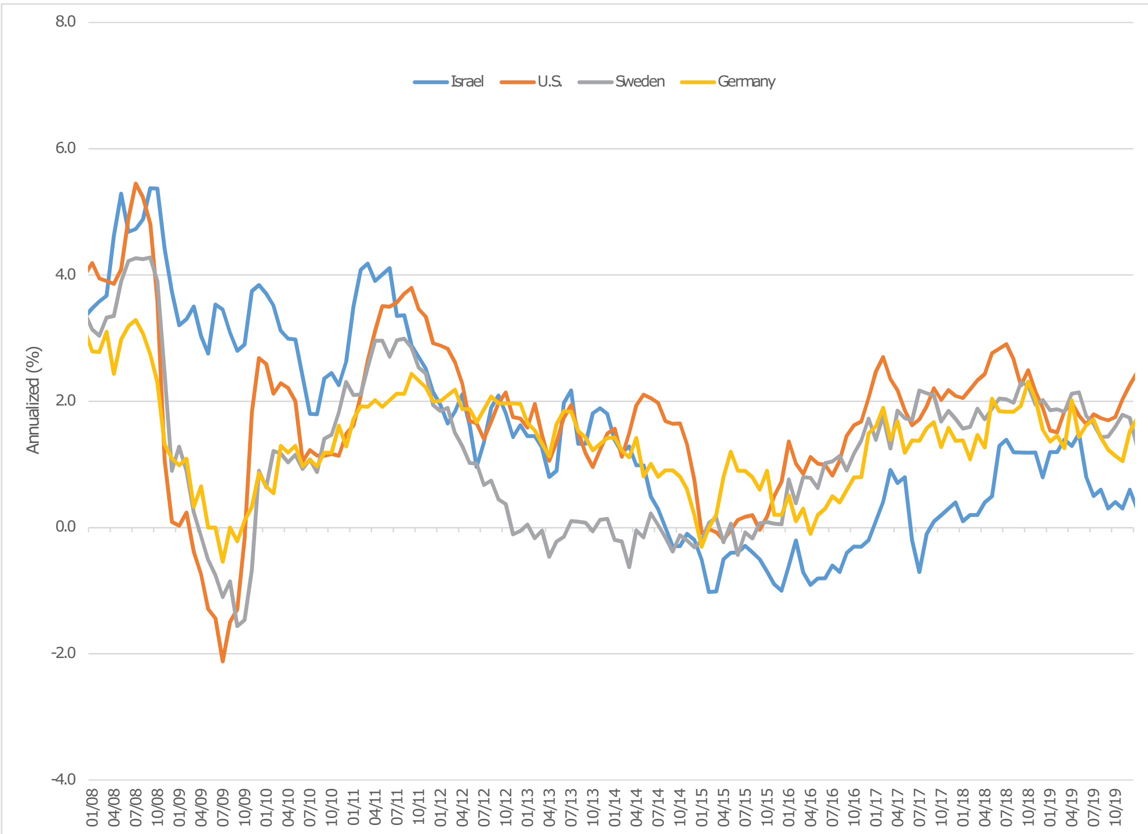

Figure 11: World Inflation. The figure plots the annual CPI inflation for selected countries.

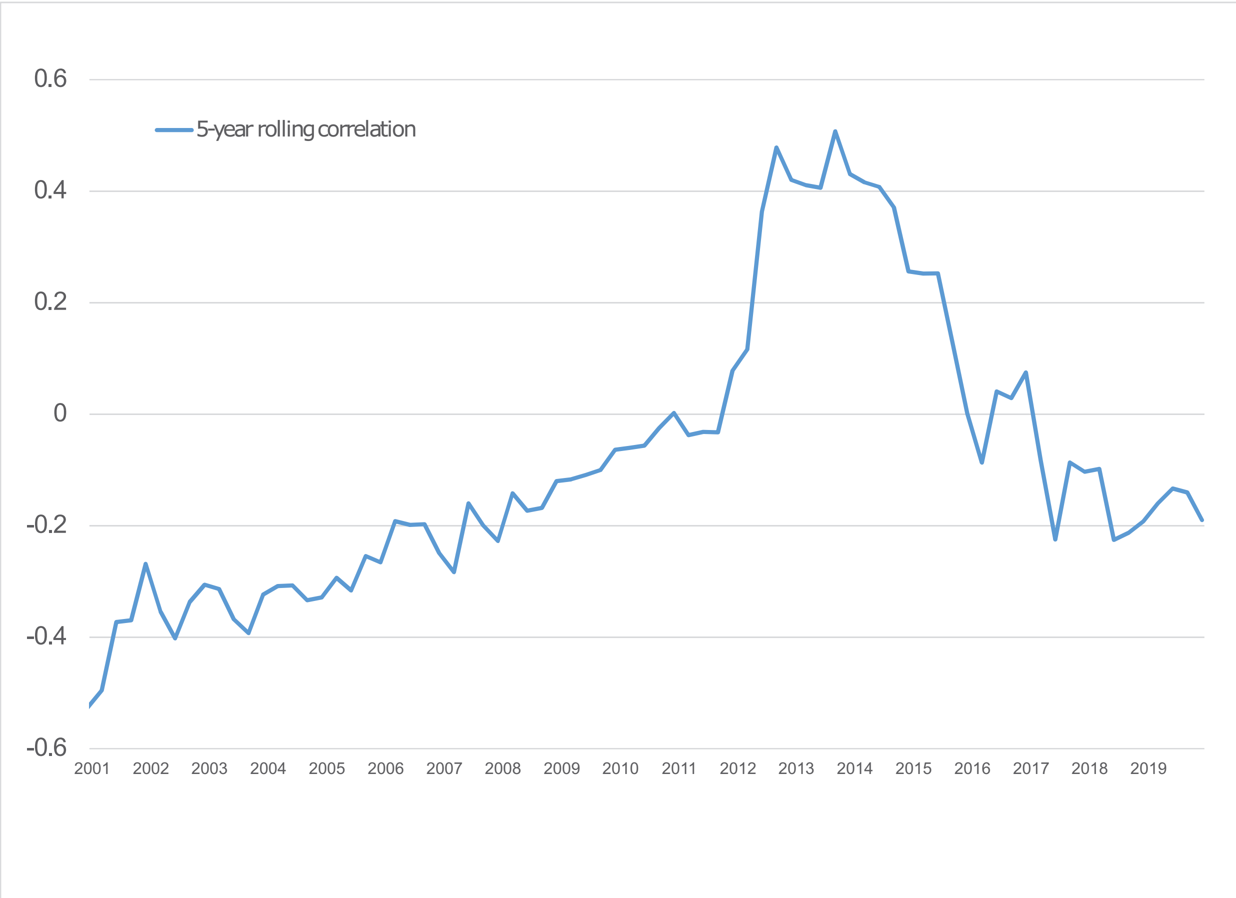

Figure 12: Rolling correlation between quarterly CPI and consumption growth. The figure plots the rolling correlation over five years between quarterly CPI and consumption growth from the first quarter of 2001 until the last quarter of 2019.

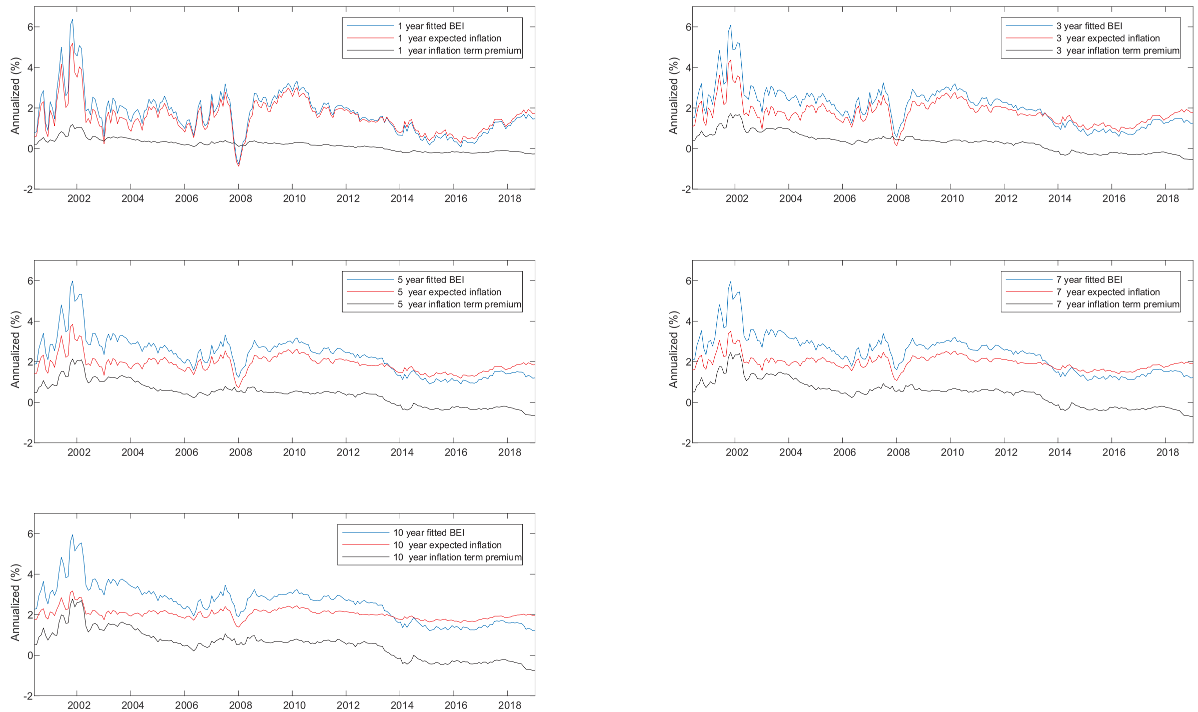

Figure 13: Fitted BEI Time Series Decomposition. This figure presents a decomposition of the fitted BEI rate rate to inflation expectations and the inflation term premium for the 05/2001-03/2018 period. The decomposition is based on a joint estimation of a four factor model using both real and nominal Israeli yields.

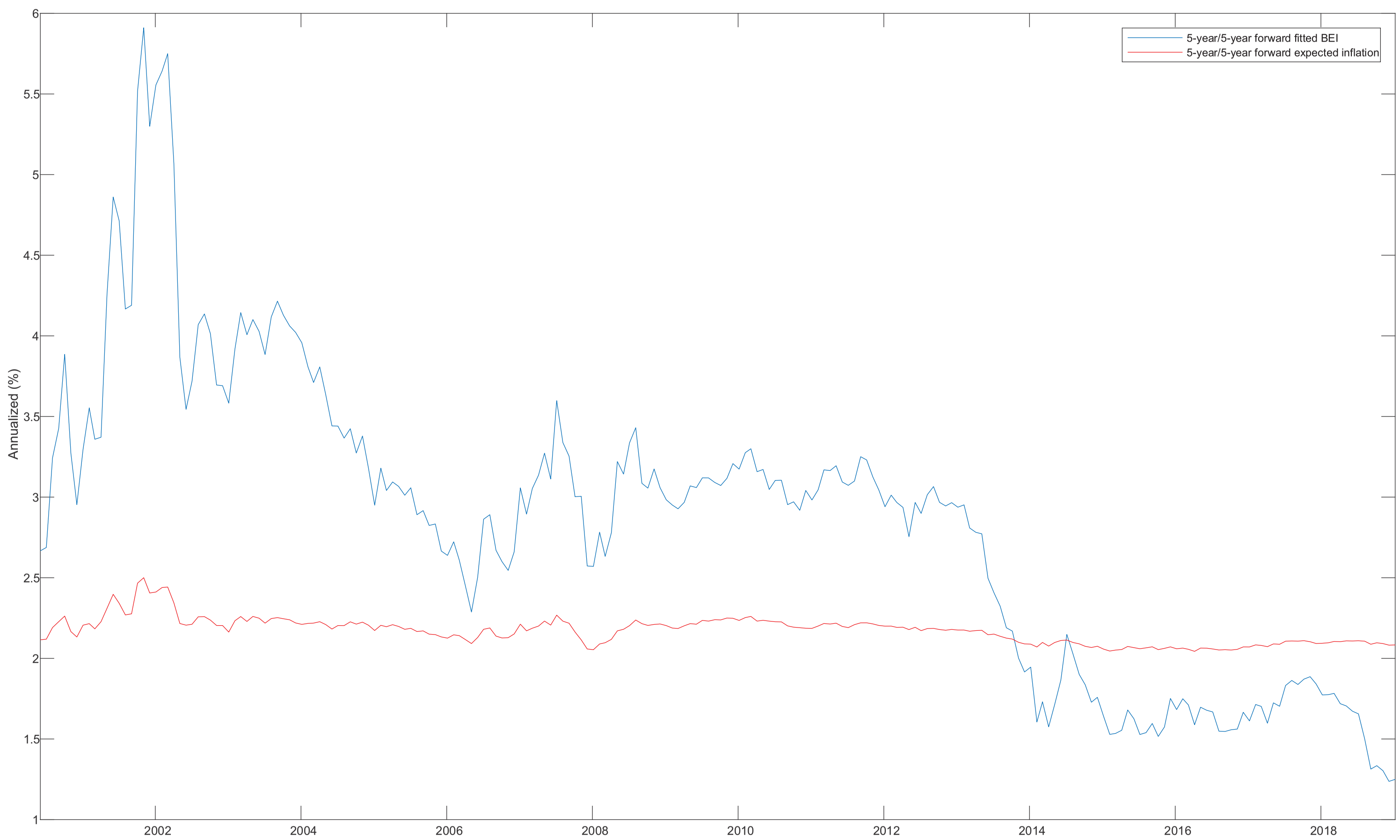

Figure 14: Fitted 5-year/5-year forward BEI Time Series Decomposition. This figure presents the fitted 5-year/5-year forward BEI rate and the 5-year/5-year inflation expectations for the 05/2001–12/2019 period. The decomposition is based on a joint estimation of a four factor model using both real and nominal Israeli yields.