Is The Market Pronatalist? Inequality, Differential Fertility, and Growth Revisited- PRELIMINARY AND INCOMPLETE: PLEASE DO NOT CIRCULATE, CITE, OR EVEN REMEMBER*

Michael Bar† *Moshe Hazan*‡ *Oksana Leukhina*§

David Weiss¶ *Hosny Zoabi*‖

April 26, 2018

Abstract

Recent public discussion has focused on inequality and social justice, while economists have looked at inequality's adverse effects on economic growth. One economic theory builds on the empirically negative relationship between income and fertility observed in the post demographic transition era. It argues that rising inequality leads to greater differential fertility – the fertility gap between rich and poor. In turn, greater differential fertility lowers the average education level, as the poor invest less in the education of their children. We show that the relationship between income and fertility has flattened between 1980 and 2010 in the US, a time of increasing inequality, as the rich increased their fertility. These facts challenge the standard theory. We propose that marketization of parental time costs can explain the changing relationship between income and fertility. We show this result both theoretically and quantitatively, after disciplining the model on US data. Without marketization, the impact of inequality on education through differential fertility is reversed. Policies, such as the minimum wage, that affect the cost of marketization, have a large effect on the fertility and labor supply of *high* income women.

*We thank Paul Beaudry, Alma Cohen, Mariacristina De Nardi, Oded Galor, Lutz Hendricks, Yishay Maoz, Marla Ripoll, Analia Schlosser, Itay Saporta-Eksten, Tom Vogl, David Weil, Alan Weiss, and seminar participants in various seminars, workshops, and conferences for helpful comments. We thank Yannay Shanan for excellent research assistance. Financial support provided by the Israel Science Foundation and the Falk Institute is greatly acknowledged.

†San Francisco State University. E-mail: mbar@sfsu.edu

‡Tel-Aviv University and CEPR. E-mail: moshehaz@post.tau.ac.il

§McMaster University. E-mail: oksana.m.leukhina@gmail.com

¶Tel Aviv University. E-mail: dacweiss@gmail.com

‖New Economic School, Moscow. E-mail: hosny.zoabi@gmail.com

We apply the insights of this theory to the literatures of the economics of childlessness and marital sorting.

Keywords: Income Inequality, Marketization, Differential Fertility, Human Capital, Economic Growth.

JEL Classification Numbers: E24, J13, J24, O40.

1 Introduction

A negative relationship between income and fertility has persisted for so long that its existence is often taken for granted in the literature (Jones & Tertilt 2008). This relationship has been typically explained by either the tradeoff between the quantity and quality of children, the opportunity cost of parental time, or both. Some of the many examples include Becker & Lewis (1973), Galor & Weil (1996), Galor & Weil (2000), and Doepke (2004). These mechanisms have led researchers to conclude that rising inequality would lead to more differential fertility, i.e. a greater gap in fertility between the poor and rich households (de la Croix & Doepke 2003, Moav 2005).[1] Hereafter, we refer to this as the standard theory. However, even as recent decades have seen a dramatic rise in income inequality in the US (Autor, Katz & Kearney 2008, Heathcote, Perri & Violante 2010), the relationship between income (or education) and fertility has flattened as high income families increased their fertility, and even become U-shaped (Hazan & Zoabi 2015), challenging the standard theory. We argue that the ability to outsource (marketize) parents time costs by purchasing babysitters, housekeepers, and prepared food, lessens children's opportunity cost of parental time. As inequality grows, the cost of marketization for the rich shrinks relative to their income, allowing them to have more kids without sacrificing time and careers.

In this paper, we show that changes in inequality, along with a decline in the price of market substitutes for parents' time with children, can quantitatively account for much of the changing relationship between income and fertility over time. We explore quantitatively and empirically the implications of our findings for aggregate human capital accumulation and policy (minimum wage).

In Section 2, we provide motivating evidence for the main facts we are looking to explain and provide cross-state panel analysis in support of our hypothesis.

Our point of departure is the standard model of fertility and educational investment in children, as in Galor & Weil (2000), applied to the case of inequality as

[1] Galor & Moav (2002) argue that the opposite is true before the demographic transition. Relatedly, Vogl (2016) indeed finds that the income-fertility gradient was positive in less developed countries before they experienced the demographic transition.

in de la Croix & Doepke (2003) and Moav (2005). This model features both a quantity-quality tradeoff with respect to children as well as an opportunity cost of parental time in childcare. We analyze this model under the assumption that the cost of children can be marketized. We show that the implication of this one assumption for the effects of inequality on differential fertility is crucial.

Turning towards our quantitative analysis, we calibrate the model to the US in 1980, when fertility and income had a negative relationship. We discipline the model by matching the salient features of cross-sectional US data. Namely, we match the income profiles of fertility rates, mother's labor supply, marketization expenditures, and college attainment rates.[2] The model successfully fits the empirical targets with 8 parameters chosen to match 40 moments.

We then feed into the calibrated model the observed cross-sectional wages for 2010 and a price decline of home production substitutes. The model predicts the 2010 relationships between income and fertility and between income and mother's time at home. In the model (data), the fertility of the top two deciles increases by 43.5% (40%) between 1980 and 2010. Our measure of differential fertility, which measures the average fertility of the top two deciles compared to decile two, increases by 41% (38.5%). An alternative measure of differential fertility, comparing the fertility of the top half of the income distribution to the bottom half, increases by 24.4% (18.6%). All of these results are untargeted.

Decomposing the mechanisms at work, we find that the change in the price of market substitutes relative to parental income, is crucially important for our findings. Furthermore, this result does not come just from a reduction in the price of home production substitutes, but also requires an increase in inequality in parental wages. Our results imply that a naïve modeler, working in 1980 under the view of the standard literature, which ignores marketization, would have predicted a significant decline in high income fertility over time if (s)he had been given perfect foresight over actual income distributions.

One implication of our theory is that rising inequality increase aggregate human capital, and thus growth. This is due to the fact that the rich tend to provide more

[2]The index of marketization is a measure of the relative expenditures on childcare, as measured in the Survey of Income and Program Participation (SIPP). See Appendix A for details.

human capital to their children, as represented by college graduation rates. Thus, as inequality increases, the average human capital of the next generation grows as *relatively* more kids are born to richer families.

One policy implication of our theory is that anything affecting the price of marketization should have an effect on the labor supply and fertility, especially high income women. One prevalent policy that may affect the price of marketization is the minimum wage. First, we show that, empirically, the minimum wage has a large pass-through effect on wages in the home production substitute sector. Evaluating this effect in the context of the calibrated model allows us to quantify the impact of min wage laws on fertility and labor supply of high income women. This analysis is done in Section 5.

Accordingly, we show that a disproportionately large number of workers in the home production substitute sectors receive the minimum wage.[3] Using cross state time series variation in the minimum wage from 1980–2010, we show that the minimum wage has a statistically significant and economically meaningful effect of about 58 cents in higher wages of home production substitute sector workers for every dollar increase in a state's minimum wage. We take an instrumental variables approach, as in Baskaya & Rubinstein (2012), as OLS may be biased as states tend to raise the minimum wage during good economic times.

We employ this estimated effect to perform a policy experiment, asking the model what the effects of a rise of the minimum wage to $15/hour, as per Bernie Sanders, would have on the labor supply and fertility of high income women.[4] Women reduce their labor supply and fertility, as marketizing becomes more difficult. We find this effect to be large.[5] We confirm this by estimating this elasticity using cross state time series variation in the minimum wage from 1980–2010 and the instrumental variable approach as discussed above. Our estimated elasticities are less than one-standard error apart from the one implied by the model.

[3]We define these sectors as in Mazzolari & Ragusa (2013).

[4]See `http://berniesanders.com/issues/a-living-wage`

[5]Doepke & Kindermann (2016) argue that policies that lower the childcare burden on mothers are significantly more effective at increasing fertility as compared to general child subsidies. We argue that the minimum wage is a policy that *increases* the childcare burden on mothers, and hence decreases fertility.

We conclude by discussing that explicit modeling of outsourcing of home production can also help us understand additional aspects of household behavior, focusing on two important phenomena: trends in childnessness rates (Baudin, de la Croix & Gobbi 2015) and marital sorting (Greenwood, Guner & Vandenbroucke 2017).

Hazan & Zoabi (2015) was the first paper to document a flattening of the fertility profile by mother's education, due to rising fertility rates among highly educated women. They qualitatively study a similar model to the one presented here to show theoretically the role of marketization. Furthermore, they exploit cross-state variation in wages and find that the wages of childcare workers, relative to mothers' wages, is negatively correlated with the propensity to have an extra child. This reduced-form evidence supports the quantitative analysis done in this paper. We differ in several critical ways. First, we document the flattening of the fertility-income profile. Second, we quantitatively evaluate the role of rising wage inequality and decreasing prices of home production substitutes in explaining this pattern. Finally, we expand on the theory presented to show theoretically and quantitatively the implications of inequality and marketization for human capital accumulation and minimum wage policy.

This paper is related to a large literature on motherhood and labor supply. Attanasio, Low & Sanchez-Marcos (2008) builds a life cycle model of fertility and labor force participation. They argue that reductions in child care costs can quantitatively account for the increase in labor supply of young mothers. Furtado (2016) finds that an increase in unskilled migration lowers wages in the child care services sector, and increases both fertility and labor supply.[6] Interestingly, she finds that native women with a graduate degree increase their labor supply and fertility much more than native women with just a college degree. Similarly, Cortés & Tessada (2011) exploit cross-city variation in immigration concentration, and show that an increase in low-skilled immigration increases labor supply, in particular of women in the top quartile of the wage distribution.[7] These women

[6] Notice that this attacks inequality from another direction. Rather than focus on a rise in inequality due to rising wages among high income households, she is studying an increase in the supply of low wage workers. Our mechanism is agnostic as to the source of rising inequality.

[7] Using data from Hong Kong, Cortés & Pan (2013) show that the ability to hire foreign workers

reduce time spent on housework and purchase more services as substitutes. Interestingly, Cortés & Pan (Forthcoming) show that increased marketization of household work allows women both to enter occupations that demand high levels of effort ("overwork"), and lowers the earnings gap in those occupations. While the importance of marketization of home production has been widely recognized (e.g. Greenwood, Seshadri & Vandenbroucke 2005, Greenwood, Seshadri & Yorukoglu 2005), the consequences of rising inequality on differential fertility in the presence of the possibility to outsource home production have not been widely studied.[8]

We continue as follows. Section 2 presents our motivating evidence. Section 3 describes the theoretical framework of our analysis. Section 4 provides details on the parameterization of the model, along with quantitative results. Section 5 analyzes the effects of the minimum wage on labor supply and fertility through the lens of the calibrated model. Section 6 discusses implications of marketization on the literatures on childlessness and marital sorting. We conclude in Section 7.

as live-in help increases labor force participation of mothers. They argue that child care cost reduction through immigration is a market alternative to child care subsidies.

[8] The literature on women's labor force participation is too vast to summarize here. However, a few papers showing how women's labor supply is related to structural transformation and taxes are worth noting, as they illuminate further potential effects of marketization on the macroeconomy. Akbulut (2011) argues that work at home, in which women have a comparative advantage, and work in services are quite similar. Thus, when demand for services rise, women's labor force participation rises as well. Buera, Kaboski & Zhao (2017) develop this argument further, with a quantitative model of sectoral reallocation and specialization between men and women, which they use to evaluate several hypotheses of the cause of structural transformation. Rendall (2018) argues that women's labor force participation, and thus the service sector of the economy, is strongly affected by taxes. Kaygusuz (2010) argues that tax changes in 1981 and 1986 in the US can explain much of the rise of married women's labor force participation, while Guner, Kaygusuz & Ventura (2012) argue that participation would be even higher if America moved to a system of taxing individuals rather than households. Finally, Duernecker & Herrendorf (2017) argue that labor productivity in home production in the US has stagnated in recent decades, while it has risen in other places such as Germany. They find this result based off the fact that wages of household workers, what we call HPS workers, have stagnated in the US but risen in Germany. Indeed, they argue that the reason for this stagnation in the US is cheap immigrant labor, which is used by richer Americans for home production services.

2 Motivating Evidence

In this section, we describe our motivating evidence. We first show data on cross-sectional fertility changes and inequality. We then use cross-state variation in the relative wage of high income women to home production substitute (HPS) sector workers, and show that states that had a larger increase in this ratio saw a larger increase in high income fertility.

Figure 1 shows fertility rates in the US in 1980 and 2010 for all native-born women, by education (years of schooling), broken into five categories: women with less than a high school degree (<12 years), with a high school degree (12 years), with some college (13-15 years), with a college degree (16 years), and with an advanced degree (>16 years).[9] Our fertility measure is "hybrid fertility rates" (HFR), which is a combination of the number of children ever born (CEB) to a woman by age a, and total fertility rates (TFR), the sum of age specific fertility rates from age $a+1$ till age 50.[10] Our HFR takes CEB of women at age 24, which is assumed to be after the decision about education has been made, and combines with TFR from 25 to 50. Fertility rates in 1980 are strongly negatively correlated with education, as has often been noted by the literature. However, in 2010, fertility rates are much flatter, and even rising between "some college" and "college plus".

In this paper, we are concerned with the impact of inequality and marketization on the relationship between *income* and fertility. As such, we measure inequality by 10 income deciles, rather than 5 education groups. Furthermore, we restrict attention to white, non-Hispanic Americans in order to abstract from changes in demographics over time. Additionally, we focus on married couples for two reasons. First, this allows us to abstract from differences between the fertility considerations of different types of households, and second it allows us to more easily calculate income deciles, without having to compare between single households and (potentially) dual income households. Figure 2 repeats Figure 1 for our

[9]Hazan & Zoabi (2015) show a very similar pattern when restricting the data to white non-Hispanic women.

[10]Formally, $HFR_t = n_{24,t} + \sum_{a=25}^{50} AFR_{at}$, where $n_{24,t}$ is the number of children ever born up to age 24 in year t and AFR_{at} are age specific fertility rates in year t. We estimate HFR separately for each educational group.

sample. In 1980 there was a clear negative relationship between income and fertility. Fertility rates in 2010 were little changed for the bottom half of the income distribution. However, fertility rates became disconnected from income starting at the 5th decile, representing a flat, or even somewhat U-shaped relationship between income and fertility. The difference between 1980 and 2010 is most pronounced for the top deciles. This change in fertility occurred as inequality has increased, as can be seen in Figures 3 and4, which show wages for wives and husbands, respectively, for each decile in each year in real 2010 dollars. Notice that the increase in fertility here is seemingly sharper than in Figure 1. However, the increase in fertility among the most educated women in Figure 1 corresponds to the increase in fertility among the higher deciles of Figure 2. In particular, 9th (10th) decile women saw an increase in fertility of 0.64 (0.83) children, while the highest education group saw an increase of 0.51 children.

The theory proposed in this paper suggests that women should increase their fertility when their wages relative to the price of home production substitutes (HPS), increases. Empirically, this pattern can be seen in the US cross state time series. Figure 5 shows that states that have seen greater percent change in the relative wage of high income (9th and 10th decile) women to workers in the home production substitute sector, between 1980 and 2010, have seen a greater percent increase in fertility of high income women. This supports the notion that, where market substitutes are relatively cheap (as measured by the wage of their workers), high income women have more children. In Appendix B we show the robustness of this relationship to controlling for changes in male wages and differential regional trends. Figure 6 shows this result to be true when using the full sample of women, as in Figure 1, and replacing high income women with women with advanced degrees. Indeed, the regression coefficient is virtually identical in these two figures.

3 Model

There is a unit measure of households composed of married females (f) and males (m) that are heterogenous on the wage offers that the members receive, de-noted w_f and w_m, respectively. The household derives utility from consumption c, number of children n, and their quality w_k (income per child). This approach is as in Galor & Weil (2000) and Moav (2005). The income per child is uncertain, and given by

$$w_k = \begin{cases} \omega \cdot w_{nc} & \text{w.p. } \pi(e) \\ w_{nc} & \text{w.p. } 1 - \pi(e) \end{cases}, \tag{1}$$

where w_{nc} is the income for non-college graduates, $\omega > 1$ is the college premium, and $\pi(e)$ is the probability of receiving a college degree as a function of their education good. The utility function, given the realization of the children's income, is assumed to be:

$$u = \ln(c) + \alpha \ln(n) + \tilde{\beta} \ln(w_k). \tag{2}$$

We assume that parents maximize their expected utility:[11]

$$E[u] = \ln(c) + \alpha \ln(n) + \tilde{\beta} \ln(w_{nc}) + \tilde{\beta} \ln(\omega) \pi(e). \tag{3}$$

Notice that the non-college income appears in the utility as a constant, and does not affect the household's decisions. Hence, we drop this constant in the analysis below.

We assume that π takes the form of:

$$\pi(e) = \ln\left(b(e + \eta)^{\theta}\right). \tag{4}$$

We choose this functional form for the probability of a child graduating college as it generates a negative relationship between fertility and income through a quantity-quality tradeoff.[12] Notice that plugging (4) into (3) and dropping the

[11] Notice that this formulation assumes that all siblings in a family have the same realization of college attainment uncertainty. An alternative formulation would allow for the uncertainty over college be to resolved child-by-child. The advantage to our approach is that it allows for a closed-form solution to the model.

[12] Notice that this function is not bounded between 0 and 1. However, this is not an issue in our calibration, as for any range of e chosen, it is possible to pick parameters such that $\pi(e) \in [0, 1]$. Jones, Schoonbroodt & Tertilt (2010) discuss conditions necessary on our π function such that it would yield a negative relationship between income and fertility, specifically that the elasticity of

constant term, $\tilde{\beta}\ln(w_{nc})$, yields:

$$E[u] = \ln(c) + \alpha\ln(n) + \beta\ln\left(b(e+\eta)^{\theta}\right), \tag{5}$$

where $\beta = \tilde{\beta}\ln(\omega)$, which is similar to the objective function used in de la Croix & Doepke (2003) and Moav (2005). We continue our analysis on the basis of (5).

Parents are required to spend the same amount of resources on the quality of each child. Thus, the budget constraint is given by:

$$c + p_n n + p_e e n = w_f + w_m, \tag{6}$$

where p_n, defined below, represents the cost associated with raising a child, regardless of quality, and p_e is the exogenously given price of a unit of education (quality).

We assume a technology for child rearing that includes marketization. Accordingly, we assume that kids require family resources combining mother's time, t_f, with market substitutes for home production, m, according to:

$$n = A\left(\phi t_f^{\rho} + (1-\phi)\, m^{\rho}\right)^{\frac{1}{\rho}}, \tag{7}$$

where $0 < \phi < 1$ controls the relative importance of mothers' time in the production of children, $\rho \leqslant 1$ controls the elasticity of substitution between the mother's time and home production substitutes, and A determines the total factor productivity (TFP) of child production. This production function explicitly takes into account the ability to marketize parental time in child rearing.[13]

Given a level of fertility, n, let $TC(n)$ be the total cost of n children. $TC(n)$ is then the solution to the cost minimization problem given by:

the human capital production function with respect to e is increasing. Our functional form both meets this criteria and yields a closed form solution.

[13]We model all household tasks as relating to child rearing, which requires a clean home, meals, activities with, and supervision of, children.

$$
\begin{aligned}
TC(n) \quad &= \quad \min_{t_f,m}\{t_f \cdot w_f + m \cdot p_m\} \\
\text{s.t.} \quad &\quad (7)
\end{aligned} \tag{8}
$$

where p_m is the price of the market substitutes.

The results, in terms of conditional factor demand and total cost function, are given by:

$$
t_f = \frac{(\phi/w_f)^{\frac{1}{1-\rho}}}{A\left[\phi^{\frac{1}{1-\rho}} w_f^{\frac{\rho}{\rho-1}} + (1-\phi)^{\frac{1}{1-\rho}}\, p_m^{\frac{\rho}{\rho-1}}\right]^{\frac{1}{\rho}}} n, \tag{9}
$$

$$
m = \frac{\left(\frac{1-\phi}{p_m}\right)^{\frac{1}{1-\rho}}}{A\left[\phi^{\frac{1}{1-\rho}} w_f^{\frac{\rho}{\rho-1}} + (1-\phi)^{\frac{1}{1-\rho}}\, p_m^{\frac{\rho}{\rho-1}}\right]^{\frac{1}{\rho}}} n, \tag{10}
$$

$$
TC\left(n, w_f, p_m\right) = \frac{1}{A}\left[\phi^{\frac{1}{1-\rho}} w_f^{\frac{\rho}{\rho-1}} + (1-\phi)^{\frac{1}{1-\rho}}\, p_m^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}} n \equiv p_n n. \tag{11}
$$

Using (5) and (6) to solve for the utility maximization problem gives the following optimal solutions for e and n:[14]

$$
e^* = \max\left\{\frac{\frac{p_n}{p_e}\frac{\beta\theta}{\alpha} - \eta}{1 - \frac{\beta\theta}{\alpha}}, 0\right\}, \tag{12}
$$

$$
n^* = \begin{cases} \left(1 - \frac{\beta\theta}{\alpha}\right)\left(\frac{\alpha}{1+\alpha}\right)\left(\frac{w_f + w_m}{p_n - \eta p_e}\right) & if\ \ e^* > 0 \\ \frac{\alpha}{1+\alpha}\left(\frac{w_f + w_m}{p_n}\right) & if\ \ e^* = 0 \end{cases} \tag{13}
$$

The solution for n, t_f, and m imply that an increase in $\frac{w_f}{p_m}$ yields a decrease in $\frac{t_f}{n}$

[14]We show the existence of a unique solution to the household problem in Appendix C.1.

and an increase in $\frac{m}{n}$, as families marketize the time costs of children more.[15] The ability of parents to substitute their own time with market goods and services leads to the following claim:

Claim 1 *When part of the time cost of children can be marketized, rising inequality may lead to the fraction of children born to high income families to rise.*

This follows from the fact that n^* is either increasing or U-shaped in w_f, in the interior solution, which we show formally in Appendix C.2. When the dispersion of w_f rises, differential fertility could change in either direction; there could be relatively more children born to poor households, if the downward sloping section of the U shape is dominant. However, there could also be relatively more children born to rich households. Changes in fertility patterns have implications for aggregate human capital levels.

Claim 2 *When part of the time cost of children can be marketized, rising inequality may lead to higher levels of average human capital in the next generation through differential fertility.*

This claim holds in the case where rising dispersion of w_f increases the fraction of children born to high income households, and thus increases the average human capital of the subsequent generation.

Much of the literature has abstracted from the assumption that some of the time cost of children can be marketized, and assumed that p_n is proportional to w_f. If one makes such an assumption, and substitutes into Equations (12) and (13), then fertility is strictly decreasing in w_f in the interior solution. We refer to the "Standard Theory".[16]

We show in Figures 2, 3, and 4 that, empirically, rising inequality was associated with an increase in fertility among richer households between 1980 and 2010.

[15] Additionally, if $\rho > 0$, there is an increase in relative spending on market substitutes, i.e. $\frac{p_m m}{w_f t_f}$ rises.

[16] Notice that if p_n is proportional to w_f, and $w_m = 0$, then (13) collapses to the optimal fertility solution as in de la Croix & Doepke (2003) and Moav (2005). To be precise, this is the exact objective function in de la Croix & Doepke (2003) when $\alpha = \beta$.

Moreover, we show in Figure 5 that richer households increased their fertility more in states that experienced larger increases in income inequality. We will show in Section 4.4 that the model with marketization can account for relationship between income and fertility both in 1980 and 2010. Accordingly, rising inequality, through marketization, led to differential fertility favoring more children in richer households and thus more human capital. Counterfactuals show that abstracting from marketization would yield the opposite result.

Finally, a word must be said about two ways of modeling men and fertility. First, if men do not spend time raising children, as in our benchmark, then we say that there are "traditional gender roles". Men's wages under traditional gender roles act as any other form of wealth. A higher male wage yields more fertility, as can be seen directly in (13), through an income effect. Under this framework, it is possible that the changing fertility patterns in US data, where now high income households are likely to have relatively more children, can be explained by rising inequality among men, regardless of the ability to marketize. This is the assumption we make in our quantitative analysis below, as it allows for an alternative explanation for the emergence of the U-shape seen in the data.

Alternatively, we could assume "modern gender roles", in which men do engage in child care. Thus, p_n does depend on w_m. Clearly, this could be modeled in a large number of ways.[17] To understand the intuition of how modern gender roles interact with inequality and marketization, consider the extreme example of a Leontief function that aggregates time that husbands and wives spend in childcare into one "parental services" variable. Under this assumption, men are required to spend one hour of time in child care for every hour that their wife spends in child care. In this model couples can be seen as one person with $w = w_m + w_f$ with all the same implications for the interaction between inequality and marketization. This assertion applies more generally when men and women are not perfect complements in the production of children (Siegel 2017).

[17] For an analysis on how parents allocate time to childcare, see Gobbi (2016).

4 Quantitative Exercise

In this section, we discuss the calibration of the model, the model fit, and breakdown of the mechanisms driving changing fertility patterns over time. We calibrate the model to 1980, and then study its implications under the 2010 wages and prices of home production substitutes. We begin by discussing the parameterization of the model and the model fit. We then test the model predictions for 2010, and then break down quantitatively the various forces at work.

Throughout the quantitative exercise, we assume 10 representative couples that we map to income deciles, as described in Appendix A.

4.1 Parameterization

This model has 10 parameters, $\Omega \equiv \{\alpha, \beta, \eta, \theta, b, \phi, \rho, p_e, A, p_{m,1980}\}$. We now describe how we pick these parameter values, which are reported in Table 1.

p_e and $p_{m,1980}$ are normalized to one without loss of generality.[18] The remaining 8 parameters are picked to match model moments to data moments from 1980. In particular, we match the profile of fertility, the profile of mother's time at home, the profile of college attainment rates of children born to different income deciles in 1980 , and the index of relative expenditures on home good substitutes.[19] Each profile contains 10 moments, representing the 10 deciles, yielding 40 moments. See Appendix A for a description of the empirical moments. The model has a closed form solution which can be inverted to infer parameter values from the data. Due to the high number of moments relative to parameters, we minimize the distance between the model moments and the data moments in order to obtain the best fit.

[18]We show this formally in Appendix E.

[19]Regarding the index of marketization, we use the childcare module of the Survey of Program Participation and Income (SIPP) to estimate relative uses of market substitutes. Our index measures based off of expenditures on childcare hours purchased in the marketplace. Since this is only one aspect of marketization, we use this to target the relative use of marketization across deciles, rather than taking the absolute expenditure levels literally. The implicit assumption is that there is a strong correlation between the use of childcare and other market substitutes for parents' time. See Appendix A for more details.

Formally, we pick parameters to minimize the mean squared error of the loss function:

$$\{\alpha, \beta, \eta, \theta, b, \phi, \rho, A\} = \arg\min \sum_i \left(\frac{M_i(\Omega) - D_i}{D_i} \right)^2, \tag{14}$$

where $M_i(\Omega)$ is the value of the model moment i when evaluated at parameter values Ω. D_i is the data value of moment i.

While all of these 8 parameters are picked together, certain moments inform on them more than others. At abuse of language, we describe a parameter as being picked to match a target, while it is understood that all parameters are jointly determined against the empirical moments. Table 1 shows the results of our identification strategy described below.

We begin by discussing α, β, and η which are picked to match fertility rates by decile. As can be seen in Equation (13), α plays a large role in determining the level of fertility, and can thus be thought of as being identified off the level of the fertility profile. The slope of fertility with respect to income depends on both an income effect, as kids are a normal good, and a substitution effect, as higher wages imply a higher opportunity cost of time with kids. In this model, fathers' wages, w_m, are purely an income effect, while mothers' wages contain both effects. β is important in the income effect of extra income. η controls the strength of the substitution effect. Thus, these three parameters are identified off the level and slope of the fertility profile with respect to both parent's wage offers.

Turning to θ and b, these parameters are closely related to education. First, however, notice that β and θ are inseparable in the utility function. However, θ affects the mapping between education expenditures, e, and college attainment, $\pi(e)$, while β does not. Thus, θ can be thought of as being identified off the slope of the profile of college attainment by decile, while β is identified off of the slope of the fertility profile, as described above. As seen in Equations (12) and (13), b does not affect the amount invested in children or quantity of children. It does, however, impact the education obtained. Therefore, it can be identified by the level of the profile of college attainment.

ϕ, ρ, and A are the parameters of the production function for kids. ϕ, ρ control the tradeoff between mother's time and home production substitutes, m, in the production of children. ϕ controls the relative importance of the mother's time in child care, while ρ controls the substitutability between mother's time and market goods. A controls how much resources are needed for childcare, in particular the amount of market goods needed. These three variables thus determine how many resources of each type are needed and available, per child, across the income distribution. As such, they can be thought of being identified off both the level and slope of the profile of mother's time at home and the index of marketization.

4.2 Parameters and Model Fit

Table 1 shows the calibrated parameter values. Notice that the parameter values found here are consistent with much of the literature. For instance, the calibrated value of α suggests that $\frac{\alpha}{1+\alpha} = 31\%$ of household resources are dedicated towards children. Lino, Kuczynski, Rodriguez & Schap (2017) find that families with 2-3 children, as is in the norm in the model, spend 37–57% of their expenditures on their children. Assuming that households have children at home for half of their adult life (de la Croix & Doepke 2004), our number of 31% is roughly consistent with these estimates. While ϕ is somewhat high, this actually is conservative, as it reduces the importance of marketization in the calibration. Our value for ρ implies an elasticity of substitution between mother's time and market goods of 2.5, is along the lines of the upper range of estimates reported in Aguiar & Hurst (2007).

Figure 7 shows the model fit, matching 40 moments with 8 parameters. The model successfully fits empirical targets for 1980, by decile, despite its parsimonious nature. The top left panel shows the model and data for mother's time at home. The top right panel shows the model fit for fertility. The bottom left figure shows the model fit for college attainment rates of children born to families in different deciles in 1980. Finally, the bottom right shows the model fit for the index of marketization.

Overall, the model fit is excellent. Beginning with women's time at home, the match between the model and data is close to perfect. Turning towards fertility, both the model and data exhibit a strongly negative relationship between income decile and fertility rates, with the exception of the first decile.[20] The model is also able to capture the level of college attainment, by decile, almost perfectly. Finally, the index of relative marketization is well matched, showing that relative marketization rates in the model are similar to those in the data.

The average fraction of household income spent on market substitutes is 4.7%. This seems quite reasonable; expenditures on market substitutes are a relatively small fraction of total household income, and growing over time with increased use of market substitutes.

4.3 Change in p_m

We next turn towards the calculation of the change in p_m between 1980 and 2010. There is no consensus in the literature what this price change is, or even exactly comprises m. We consider m to be composed of two types of market substitutes for home production: home production durables, d, such as dishwashers and washing machines, and time of home production substitute (HPS) workers, t. We first discuss the price change that we take of each type of input, and then our choice of a 2% annual price reduction.

Greenwood, Guner, Kocharkov & Santos (2016), report a range of estimates from the literature of 2-13% annual price declines of home production durables, and in turn use 5%. We use 4% in order to be more conservative. The price of HPS workers is harder to measure. On one hand, measured wages in the Current Population Survey (CPS) have remained roughly constant. On the other hand, if the productivity of these workers has increased, then the price of their output should decline even if wages remain constant. Attanasio et al. (2008) argue that there was a 25% drop in the relative cost of childcare services for kids under age 4, and a 5% drop for kids age 4-7. We take the price decline to be 10% over this whole period, or 0.35% per year between 1980 and 2010. These price declines are not enough to calculate a change in the price of m, as we need some indication

[20] The imperfect fit results from a corner solution in education for the first two deciles.

the substitutability (or complementarity) between d and t, as well as their relative importance. However, they do indicate that a price decline of m of roughly 2% a year is reasonable; a 4% decline in durables and a 0.35% decline in HPS worker costs suggests 2% as a midpoint. We next do a more formal analysis of the interaction of durables and HPS workers in home production in order to explore changes in p_m.

We begin by assuming a function form for the aggregation of d and t. In particular, we assume that m is composed of a CES production function of t and d:

$$m = (\phi_m d^{\rho_m} + (1 - \phi_m) t^{\rho_m})^{\frac{1}{\rho_m}}, \tag{15}$$

where ρ_m controls the elasticity of substitution and ϕ_m the relative weight. We denote the price of durables to be p_d, and the price of HPS worker's time to be w. The price of this composite good p_m is thus given by:

$$p_m = \left(\phi^{\frac{1}{1-\rho_m}} p_d^{\frac{\rho_m}{\rho_m - 1}} + (1 - \phi_m)^{\frac{1}{1-\rho_m}} w^{\frac{\rho_m}{\rho_m - 1}} \right)^{\frac{\rho_m - 1}{\rho_m}} \tag{16}$$

Minimizing costs yields expenditures durables relative to expenditures on HPS workers, which are given by:

$$\frac{p_d d}{wt} = \left(\frac{p_d}{w} \right)^{-\frac{\rho_m}{1-\rho_m}} \left(\frac{1 - \phi_m}{\phi_m} \right)^{\frac{1}{1-\rho_m}}. \tag{17}$$

In order to calculate the empirical counterpart to (17), we take the Survey of Consumer Expenditures (CEX) in 1980 and 2010. Our sample is married white households ages 25-55.[21] For durables, we calculate expenditures using house furnishing and equipment this quarter ("houseeqcq"). For demand for HPS workers, we use babysitters and housekeepers ("domsrvcq" in 2010, and "housopcq" in 1980). We find that expenditures on durables relative to HPS workers is 3.61 in 1980 and 1.45 in 2010.

[21]There is well known bias in CEX data, such that comparing the CEX and the National Income and Product Accounts (NIPA) over time show huge divergences. Attanasio, Hurst & Pistaferri (2012) surveys some of the literature on this subject. As a result, we only use CEX to examine *relative* expenditures on different types of goods, rather than absolute expenditures.

We then take (17) in 2010 divided by the same in 1980, yielding:

$$\frac{\frac{pm}{wt}_{2010}}{\frac{pm}{wt}_{1980}} = \left(\frac{\frac{p}{w}_{2010}}{\frac{p}{w}_{1980}}\right)^{-\frac{\rho}{1-\rho}}. \tag{18}$$

Using (18), and the fact that the ratio of relative expenditures is, from the data, $\frac{1.45}{3.60}$, we can infer that $\rho = -4.13$. This implies strong complementarity between the two inputs, with an elasticity of substitution of approximately -0.2. While we do not know $\frac{p}{w}$ in either year, the change in prices of durables and HPS worker services described above imply the change in the price ratio, namely that it went down by approximately 67%.

We are still missing two unknowns necessary to calculate the change in p_m over time: $\frac{p}{w}_{1980}$ and ϕ_m. Using our normalization of $p_{m,1980} = 1$, as well as (17) being equal to 3.6 in 1980, allows us one more equation to use, however this not sufficient for identification. If we assume $\phi = .05$, then we can solve for $\frac{p}{w}_{1980} = 2.42$ and the implied change in p_m to be 1.91% per year between 1980 and 2010. If we assume $\phi = 0.95$, we can solve for $\frac{p}{w}_{1980} = 10.04$ then the implied annual change in p_m is 3.55%. We take as a benchmark $\phi = .064$, which implies a 2% annual price reduction between 1980 and 2010 and $\frac{p}{w}_{1980} = 2.57$. We refer to this exercise as our benchmark case. We perform robustness tests with $\phi = 0.1$ and $\phi = 0.028$, which have the same deviation of 0.036 from our benchmark. We refer to these robustness exercises as "high ϕ" and "low ϕ", respectively.[22]

4.4 Results

We next analyze the results of the model by comparing the model prediction for 2010, and then break down the mechanisms in the model.

Figure 8 shows the data for fertility and labor supply in 1980 and 2010, as well as the model using the benchmark, high ϕ, and low ϕ.[23] Our benchmark produces

[22] Both robustness tests include a constant deviation from the benchmark ϕ.

[23] There is one point worth discussing about time allocations. Our model focuses on understanding time allocation between home production and work, implicitly assuming that the total

the best fit, as can be seen in Figure 8. The left panel shows the model fit for women's time at home. The fit is quite good, though the model somewhat understates time spent at home for the first decile, and somewhat overstates for the top two deciles. The right panel shows the model fit for fertility. With the exception of the rise in fertility between the first and second deciles, which is due to corner solutions in the model, the model accurately captures the declining fertility rates through the fifth decile, and the subsequent flattening/rising fertility rates. The rise in fertility of the top decile is overstated, with fertility in the model being higher than that of the data by approximately 0.4 children. Overall, the model does a good job matching the changes in fertility rates and labor supply of married women. Notice that these moments are not targeted.

Turning towards another out of sample moment, Mazzolari & Ragusa (2013) study the effects of inequality on demand for home production substitutes. They look at cross-city variation in US employment growth in the home production substitutes sector between 1980 and 2005. Thus, they are estimating changes in demand for home production substitutes during our time period. They find that a one standard deviation (four percentage points) increase in a city's top decile wage bill is associated with a 8-16% growth in the number of hours in the home services sector.[24] Our model generates a rise of slightly over 11% in 2010. The model's sensitivity of marketization to wages is thus in the middle of the range of estimates in their paper.

We next break down the results of the model, explore the implications of differential fertility on human capital, and the mechanisms driving the change in differential fertility in the data.

We use the following measures in the model and data. First, we look specifically at average fertility of the top two deciles, which we call "High Income Fertility".[25]

time on non-leisure activities has not revealed a systematic trend. American Time Use Survey (ATUS) data, however, suggests that leisure may have slightly declined between 1975 and 2003 for the group of married females that we consider: by 6 hours per week for the top deciles and 3.5 hours for the bottom deciles. These are based on our own calculation, and we note that the 1975 ATUS gets reduced to a very small sample once we apply our sample restrictions. If this extra time is devoted towards *quantity* of children, rather than *quality* (time spent reading to children, other education), then our results may be slightly biased for 2010.

[24] This is the range of their IV estimates. See their Table 2.

[25] Mathematically, this is expressed as $\frac{n(10)+n(9)}{2}$, where $n(i)$ is the fertility rate of decile i.

We study how this measure changes between 1980 and 2010. Second, we look at the difference between high income fertility and low income (second decile) fertility.[26] We refer to this as "Measured Differential Fertility" (MDF), and study its changes over time.[27] Notice that this term differs from "differential fertility", as the later measures *all* fertility across the income distribution. We include a second version of MDF, "MDF Top/Bottom", in which the numerator is fertility in the top half of the income distribution and the denominator is fertility in the bottom half of the income distribution. Finally, our measure of the effects of differential fertility on human capital is the average college attainment of the next generation. We measure this using the fertility rates in the model against the actual college attainment rates by decile in the 1980 data.[28] We use the 1980 data for two reasons. First the data on college attainment rates for children born in 2010 will not be available for decades to come. Second, this allows us to disentangle the effects of differential fertility from generally rising trends in college attainment.

Table 2 summarizes the data, the main model results, and the breakdown of model mechanisms. Recall that none of the data moments reported here are targeted. The first column shows the percentage change in high income fertility, the percentage change in MDF, the percentage change in MDF Top/Bottom, and the percentage point (p.p.) change in the fraction of the next generation attaining a college degree in the data. High income fertility rose by 40%. Low income (second decile) fertility remained constant. These two facts combine to imply that MDF increased by about 40% as well. In the data, MDF top/bottom increased by 18.6%. Overall, changes in differential fertility imply a 1.70 p.p. increase in college attainment rates of the next generation. The second column follows the same pattern for the benchmark model. In the model, high income fertility rises by 43.5%, while MDF increases by 41%, MDF top/bottom increases by 24.4%,

[26]Mathematically, this is expressed as $\frac{n(10)+n(9)}{2}/n(2)$, where $n(i)$ is the fertility rate of decile i.

[27] We use the second decile as a measure of poor fertility rather than the bottom decile, as we abstract from changes in welfare over time which may have had a disproportionate effect on the poorest households.

[28] To be precise, we measure change in aggregate college attainment as $CG^S_{2010} - CG^S_{1980}$, where $CG^S_t = \sum_d \frac{n^S_t(d)}{\sum_d n^S_t(d)} \pi^{data}_{1980}$. Here we take S to be one of the possible scenarios (data, model, counterfactual), d is an income decile, and $\pi^{1980}_D(i)$ is the empirical college graduation rate of decile i in 1980.

and college attainment rates of the next generation rise by 2.4 p.p.[29] These measures show how well the model predicts the empirical changes in fertility over time. Columns 3 and 4 repeat this pattern for our exercises breaking down model mechanisms, described next.

There are two mechanisms in which inequality leads to changing differential fertility in the model. The first is increased marketization, as measured by changes in $\frac{w_f}{p_m}$. The second is the income effect on demand for children, as measured by changes in w_m. We now address each in turn.

In Figure 9 (left panel) we recalculate the model results in 2010 holding constant $\frac{w_f}{p_m}$, by decile, between 1980 and 2010.[30] We do so by varying p_m by decile in 2010. This maintains the same relative cost of marketization in the 2010 model as was in the 1980 model, allowing us to explore the importance of marketization for our results. As can be clearly seen, without a decreasing relative price of marketization, high income fertility falls drastically, and the income profile of fertility becomes steeper. This is the exact opposite of what happened in the data.[31] This is directly along the lines of the standard theory; without marketization, increases in inequality decrease fertility among high income families, as opposed to the increase seen in the data.

Consistent with Figure 9 (left panel), Column 3 of Table 2 compares this counterfactual model, which we refer to as "No Δ Marketization", with the benchmark model and data. Without a lower relative price of marketization, high income fertility counterfactually falls by 34%. More importantly, and consistent with the standard model, MDF counterfactually falls, using both measures. This in turn decreases the fraction of the next generation that attains a college education by 1.23 p.p. as opposed to the 1.70 p.p. *increase* the data measures. This is despite the

[29] When calculating the college attainment rates in the model using the model's prediction for graduation by decile, the number rises from 38.3% to 42.8%, an even larger increase.

[30] While we leave out the first decile, as the model indicates a corner solution, this is not crucial. The effects of marketization on the first decile are minimal. Furthermore, because in this decile qualify for various welfare programs that our model does not capture, complicating the analysis.

[31] Notice that the *level* of fertility is lower for all deciles. This is due to the higher wage growth for women than for men across all deciles. Specifically, as can be seen from Equation (13), the small income effect generated by the growth in men's wage is counterbalanced by a larger increase in the price of the market substitute goods. What matters for our quantitative results, however, is the fraction of children representing each decile in the next generation.

fact that rising male inequality may have led to a flattening of the fertility profile due to the income effect. Thus, as we observe above, a naïve modeler, working in 1980 and ignoring marketization, would have predicted a widening of differential fertility and thus a decline in college attainment rates over time if (s)he had been given perfect foresight over actual income distributions.[32] Adding this counterfactual decrease implied by the standard theory to the increase seen in the data, the bias from not including marketization is a little under 3 percentage points of college attainment. As noted above, this estimate implies that differential fertility's impact on education is comparable to more than one-quarter of the general rise in education between these two cohorts of white, non-Hispanic non-immigrant Americans born in 1950 and 1980. Thus, the bias induced by ignoring marketization is both quantitatively large and changes the sign of the estimated implications of inequality on differential fertility, and thus education.

In Figure 9 (right panel) we recalculate the model results in 2010 holding constant w_m at its 1980 value. This allows us to measure the income effect on fertility in the model due to men's rising wages and increased marital sorting. As can be seen, counterfactual model for 2010 is quite similar to the actual model results for 2010, with somewhat lower fertility rates for high income households. The intuition is clear; those households saw a great rise in male income which, through the income effect, should increase fertility. Shutting down this mechanism leads to less fertility.[33]

Consistent with Figure 9 (right panel), Column 4 of Table 2 compares this counterfactual model, which we refer to as "No Δ w_m", with the benchmark model and data. When abstracting from changes in male income, high income fertility still rises 30%, which is 69% of the 43.5% increase in the benchmark model. This means that the income effect can explain at most 21% of the increase in high income fertility. MDF (top/bottom) increases 24% (15.1%), which is 59% (62%) of the 41% (24.4%) increase in the benchmark model, implying that the income effect can explain at most 41% (38%) of increased MDF. Finally, the college attain-

[32]Notice that the standard theory does not allow for any marketization, while in our counterfactual exercise we do not allow for cheaper marketization over time. Thus, while the two exercises are not perfectly comparable, the basic idea of no changing marketization is explored.

[33]The opposite happens for the low end of the distribution where male real incomes actually fell over time.

ment rates of the next generation rise by 1.60 p.p., which is 67% of the increase in the benchmark model, implying that the income effect can explain at most 33% of the increase in human capital attributed to changing differential fertility. In each case, we wrote that the income effect can explain "at most" a certain amount, since we are making modeling assumptions that favor the strength of the income effect.[34]

We note two more interesting facts about this exercise. The first is that the findings are under the extreme assumption of traditional gender roles. If men bore a time cost of children as well, then marketization would presumably be an even stronger force for differential fertility in the model. Thus, our findings are conservative. Second, we note that this measure of the impact of the income effect on differential fertility captures all of the empirical mechanisms causing an increase in male wages by decile, including sorting. To see this point, imagine that sorting increases, with no other change in inequality. Then the higher deciles would begin to measure higher male wages. Thus, this exercise captures the maximum effect of rising differential fertility in the data due to increased marital sorting and an income effect through men.

Delving deeper into our results, we perform two more exercises in order to disentangle the role of marketization and inequality on fertility. First, we expand on the exercise described above as "No Δ Marketization" by separately analyzing the effects of changing w_f and p_m. Figure 10 (left panel) shows the model fertility rates, by decile, in 1980 and 2010. It then adds two curves. "1980 with 2010 w_f" shows the 1980 model with women's wages from 2010. "1980 with 2010 p_m" shows the 1980 model with marketization prices from 2010. That is, we separately analyze the effects of the changes in the numerator and denominator of $\frac{w_f}{p_m}$. As can be seen, simply changing w_f lowers fertility rates. However, the relationship between income and fertility flattens greatly after the 5th decile, as in the data, and even increases between the 9th and 10th deciles. As opposed to this, if only p_m changes, fertility increases. Here, the first deciles have a positive relationship between income and fertility due to the corner solution in e. However, by

[34]Notice that these two exercises do not show that marketization and the income effect add up to the total effect. This is as there is an interaction between the two mechanisms; when p_m decreases, real wages increase, yielding higher demand for children.

the 4th decile, the relationship becomes negative, only flattening out after the 8th decile. We conclude two things from this exercise. First, inequality in women's wages was a significant force for the flattening relationship between income and fertility. Second, the interaction between w_f and p_m changing is what allows the model to match both the level and shape of the fertility profile in 2010.

The second exercise is to show, mechanically, what is causing the change in fertility patterns between the two inputs into child production, viz. mother's time (t_f) and market substitutes (m), as in (7). Figure 10 (right panel) shows the relationship in the model between income and fertility in 1980 and 2010. The curve "1980 m with 2010 t_f" shows what fertility would have looked like had the 1980 levels of m been combined with the 2010 levels of t_f. Since mother's time at home is decreasing for all deciles between 1980 and 2010, the level of fertility is lower. However, for our purposes, it is important to note that fertility would still have been negatively correlated with income. The curve "1980 t_f with 2010 m" shows what fertility would have looked like had the 1980 levels of t_f been combined with the 2010 levels of m. Since all deciles purchase more market substitutes in 2010, fertility is higher. However, it is clear that market substitutes changes are what led to a flat, or even increasing, relationship between income and fertility. This exercise supports the claim that it is changes in marketization that led to the changing relationship between income and fertility.

5 The Minimum Wage, Revisited

In this section, we first discuss the theory as to why the price of marketization has an effect on the higher income part of the income distribution. We then show empirically, using cross state variation, that the minimum wage does indeed have a large effect on the wages in the home production substitutes sector. We then ask the model how large the effects of a minimum wage increase are on labor supply and fertility. We end by turning to a reduced form empirical analysis to estimate the effect of the minimum wage on the labor supply of high income women and find elasticities that are higher than implied elasticity by the model.

5.1 Minimum Wage: Theory

The effects of the minimum wage have been widely studied, but focus on the effects of policy changes on people at the lower end of the income distribution (Manning 2016). The theory presented thus far makes a stark prediction; anything that changes the price of home production substitutes (i.e., the price of marketization), such as caretakers for children, should affect the labor supply and fertility of *all* households. Thus, the minimum wage has an effect on the labor supply even of women whose wages are not directly impacted by the minimum wage. Since we are interested in the effect of the minimum wage on labor supply through the price of marketization, we abstract from the direct impact of the minimum wage on the wage offers households receive, and focus instead on the indirect impact of the minimum wage on the price of market substitutes for home production, as represented by p_m in the model.

Claim 3 *When mother's time, t_f, and other inputs, m, are gross substitutes, $\rho \in (0, 1)$, an increase in the minimum wage decreases labor supply, when fertility cannot adjust, that is, $\frac{\partial t_f}{\partial p_m}|_{n=n_0} > 0$. Moreover the effect is differential across the income distribution. A sufficient condition for the effect to be increasing with wages is $\rho > \frac{1}{2}$. That is, $\frac{\partial^2 t_f}{\partial p_m \partial w_f}|_{n=n_0} > 0$ if $\rho > \frac{1}{2}$.*

Proof. Follows directly from differentiating (9) with respect to p_m, and then again with respect to w_f, holding n constant. ■

One can think of the effect of the minimum wage on labor supply holding fertility constant as a short run effect. That is, fertility decisions have already been completed, then labor supply changes as described by Claim 3. However, the minimum wage affects fertility as well for families that can still adjust their fertility choices.

Claim 4 *Increases in the minimum wage decrease fertility. That is, $\frac{\partial n}{\partial p_m} < 0$.*

Proof. Follows directly from differentiating (13) with respect to p_m. ■

The magnitude of the effects of the minimum wage on fertility are differential across the income distribution, but it is theoretically ambiguous whether the magnitude increases or decreases with income. We show below that, quantitatively, the richer households see the greatest decline in fertility. Notice that an increase in the minimum wage increases the mother's time allocated per child, but decreases overall fertility. Therefore, the net effect on labor supply is ambiguous. Again, we show the net effect to be lower labor supply, especially among higher wage households, following a minimum wage increase.

5.2 Minimum Wage: Quantitative Analysis

What are the effects of minimum wage changes on marketization? To answer this question, we first estimate the passthrough rate of the minimum wage on HPS sector wages by exploiting cross-state variation in the minimum wage over time. We show that the minimum wage has a strong impact on average wages of workers producing home production substitutes. We then use our estimates to conduct a policy experiment in the model by calculating a change in the price of these goods following an increase of the federal minimum wage to $15/hour, as suggested by Bernie Sanders during the 2016 presidential election. We ask the model how a change in p_m in line with this minimum wage increase would affect labor supply, fertility, and investment in children differentially across the income distribution. We end with a further comparison of the model predictions with our own estimates from the US cross-state time series data.

Using CPS data from 1980-2010, we compute the real wage of workers in the industries of the economy associated with home production substitutes.[35] Figure 11 shows the distribution of the real wage, relative to the minimum wage, both for the industries of the economy associated with home production substitutes and other sectors of the economy. The figure clearly shows that workers in industries of the economy associated with home production substitutes are much more likely to earn wages that are close to the minimum wage.

[35] The selection of these industries follows Mazzolari & Ragusa (2013).

In order to infer the effect of the minimum wage on the wages of home production substitute sector workers, we would like to estimate regressions of the following structure:

$$w_{ist}^{\text{HPS}} = \alpha + \beta w_{st}^{\min} + \gamma \bar{w}_{st} + \delta_{below} + \delta_t + \delta_s + \delta_{age} + \delta_{educ} + \delta_{Hispan} + \delta_{race} + \delta_{occ} + \epsilon_{ist}, \tag{19}$$

where w_{ist}^{HPS} is the real wage of individual i working in the home production substitute (HPS) sector , living in state s in year t, $w_{st}^{\min}$ is the real minimum wage in state s in year t. This is computed as the maximum between the state and the Federal minimum wage.[36] $\bar{w}_{st}$ is the average wage of workers outside of the HPS sector in year t and state s. This allows us to control for state level economic fluctuations that may affect wages in the HPS sector.[37] $\delta_t, \delta_s, \delta_{age}, \delta_{educ}, \delta_{Hispan}, \delta_{race}$, and δ_{occ} are year dummies, state dummies, and demographic controls including age dummies, educational dummies, a dummy for being Hispanic, race dummies, and occupational dummies, respectively. δ_{below} is an indicator that is equal to one if that person is making at least the minimum wage and zero otherwise. We include this variable to control for the fact that there are many workers, roughly 30%, for whom the minimum wage does not seem to be binding. While we are not proposing a theory as to why these workers are paid less, we want to include them separately in our regression.[38] ϵ_{ist} is an error term.

Estimating (19) may yield an upward biased estimate of β if states tend to raise the minimum wage during good economic conditions, when wages in general are rising. We take two approaches to address this issue. First, we estimate (19) including on the right hand side the average wage in state s and year t.[39] The idea is that if HPS sector workers' wages have similar cyclicality as the rest of the workers in the economy, then the estimate of the relative wage implicitly controls for economic conditions. Second, we take an instrumental variables ap-

[36]The data source for the minimum wage by state and year is Vaghul & Zipperer (2016).

[37]Our results below show that this variable is not important quantitatively or statistically for our findings.

[38]For example, about 9 percent of workers in this sector are in managerial occupations, of whom 90 percent earn wages above the minimum wage with an average of 2.5 times the minimum wage.

[39]We calculate this average wage without workers in the home production substitute sector in order to avoid the reflection problem (Manski 1993).

proach along the lines of Baskaya & Rubinstein (2012). The approach relies on two assumptions. The first is that the federal minimum wage is exogenous to local economic conditions, and therefore exempt from the critique above. However, whether or not the federal minimum wage binds is endogenous to the state. Accordingly, the second assumption is that the level of liberalism in the state determines how likely the federal minimum wage is to bind. Thus, our instrument for the minimum wage in state s and year t is the interaction between the federal minimum wage in year t and an index of state s liberalism from before the sample time period (Berry, Ringquist, Fording & Hanson 1998, Berry, Fording, Ringquist, Hanson & Klarner 2010).[40]

The coefficient of interest is β, which shows the dollar change in HPS sector wages when the minimum wage increases by a dollar. Table 3 reports the results of the estimation. Column 1 only controls for year and state fixed effects and for having a wage that is below or above the minimum wage. Column 2 adds the average real wage in the state. Column 3 repeats Column 1 but replaces year fixed effects with region-year fixed effects. Column 4 adds to Column 1 demographic controls, again switching year fixed effects with region-year fixed effects. Column 5 adds to Column 4 the average wage in the state. As can be seen by comparing these columns, the estimate of the impact of the minimum wage on the wages in the HPS sector is relatively stable, declining slightly only when adding the demographic controls. The OLS estimates thus imply that a \$1 increase in the minimum wage yields approximately a 65-77 cent increase in wages in the HPS sector. Columns 6–10 repeat Columns 1–5, but instruments for the effective minimum wage in the state using the interaction of state liberalism and the federal minimum wage as described above. The IV estimates indicate that a \$1 increase in the minimum wage yields approximately a 55-75 cent increase in wages in the HPS sector.[41]

[40] We use the average of their nominate measure of state government ideology from 1960–1980. The index of state liberalism has a range of 1 to 100, with more liberal states receiving a higher score, with an average (standard deviation) of 62.3 (11.3).

[41] We also estimated (19) in log-log specifications which follow Table 3. In all specifications we obtain estimates that are highly significant and approximately 0.5, with no clear difference between the OLS and the 2SLS estimates. An elasticity of 0.5 would imply a somewhat larger effect of changing the minimum wage on p_m than the one implied by the level regressions reported in Table 3.

To calculate how a change in the minimum wage to $15/hour affects average wages in these sectors, we proceed as follows, using observations from 2010. First, we calculate the average wage in the HPS sectors. Then, we create a counterfactual wage for everyone. This wage is equal to the actual wage if the person earned less than the minimum wage. That is, we assume that people who earn less than the minimum wage are unaffected by changes in the minimum wage.[42] For everyone else, their counterfactual wage is equal to their old wage + (15-minimum wage)*0.58. That is, we increase their wages by the estimated β from Column 10, our most demanding specification, in Table 3 multiplied by $15 less the minimum wage in that individual's state in 2010. We then compare the average of this counterfactual wage to the average observed wage, and find it to be 21.1 percentage higher. Using the price of m, as given by (16), along with the inferred parameter values described in Section 4, we find that a 21.1% increase in HPS wages would imply a 13.8% increase in p_m. Thus, for our exercise, we increase p_m by 13.8%. Note that we do not assume that this minimum wage change affects the wages of mothers or fathers in the model. That is, we are only asking how it affects people's ability to marketize. Accordingly, we only analyze the effects on deciles 5–10, and ignore the left tail of the distribution. Also notice that our approach assumes perfect pass-through of the HPS sector wages on the price of home production substitutes. We do this for simplicity, and note that it will cause an upward bias in the magnitude of our quantitative exercise.

The results are shown in Figure 12. The top panel shows the fertility by decile with the higher minimum wage in 2010 relative to the benchmark model in 2010. The bottom panel shows the relative mother's time at home. The minimum wage decreases fertility, differentially more for higher income households, and increases mother's time at home, differentially for higher income households. The magnitudes are large. A 10^{th} (5^{th}) decile household decreases fertility by 12.8% (9.4%), while the mother spends 9.7% (2.5%) more time at home. Notice

[42] We are unsure why a person in our sample is earning less than minimum wage. It could that this is a result of misreported data, lack of enforcement of the minimum wage, or an uncovered sector (waiters). To be conservative, we assume these people are unaffected by the minimum wage. Had we assumed them to be affected, then the counterfactual wage estimated here would be even higher, yielding a greater estimated impact of the minimum wage on home production substitute sector wages.

that these numbers are for women under the assumption that they can adjust fertility. What about those who are "locked in" their fertility choice? We recalculate changes in mother's time at home for these mothers using the model's fertility in 2010 with the increased cost of marketization. A 10^{th} decile mother increases time at home by 25.9%, while a 5^{th} decile mother increases it by 13.1%. These numbers are larger as the family has not had a chance to scale back fertility. The short run effect on labor supply is also very large. The average reduction in labor supply by women in the 9^{th} and 10^{th} deciles is 3.5%.

In order to verify this prediction, we estimate directly from the data the effect of the minimum wage on the labor supply of high income women. Specifically, we estimate regressions of the following structure:

$$\log Hours_{ist} = \alpha + \beta \log w_{st}^{\min} + \delta_t + \delta_s + \delta_{age} + \delta_{educ} + \delta_{Ind} + \delta_{occ} + \epsilon_{ist}, \qquad (20)$$

where $\log Hours_{ist}$ is the log of yearly hours supplied by woman i, living in state s, in year t. All other variables have been described in (19). Notice that β is the elasticity of labor supply with respect to the minimum wage. We use CPS data for the years 1980–2010. Our sample comprises white non-Hispanic married women aged 25-54, whose real hourly wage is in the 9^{th} and 10^{th} decile. Again, like in the estimation of β in Equation (19), estimating (20) with OLS might induce an upward bias if hours of high income women and the state minimum wage are procyclical. To overcome this issue we estimate (20) using OLS and 2SLS when, again, state s minimum wage in year t is instrumented with the interaction between the federal minimum wage in year t and an index of state s liberalism from before the sample period.

Table 4 reports estimates of β. Column 1 only controls for year and state fixed effects. Column 2 repeats column 1 but replaces year fixed effects with region-year fixed effects. Column 3 adds to Column 2 age and education fixed effects. Column 4 adds to Column 3 industry fixed effects, Column 5 replaces the industry fixed effects in Column 4 with occupation fixed effects. Finally, Column 6 includes both industry and occupation fixed effects. As can be seen from the table, all of the OLS estimates are very close to 0 and none are even remotely significant. Columns 7–12 repeat Columns 1–6, but instrument for the state minimum wage. All of the estimates are statistically significant and economically meaningful. They imply that the elasticity of labor supply of high income women with respect to the minimum wage is in the range of -0.65 to -0.38.

Finally, Table 5 repeats Table 4 for men. As can be seen from the table, all the OLS and the 2SLS estimates are close to 0 and none are even remotely significant. This is exactly what was expected under the assumption of traditional gender roles.

6 Additional Implications of the Rise in Marketization

In this section, we discuss additional implications of the rise in marketization. We first discuss how marketization affects childlessness rates of women by education (or income), and then ask how marketization affects the endogenous incentives for marital sorting on education (or income).

6.1 Childlessness

How does the ability to marketize the cost of children affect fertility on the extensive margin (childlessness rates) among educated women?

Baudin et al. (2015) estimate childlessness by woman's education, for those over 45, in the 1990 US census. They find that highly educated women ($>$ 16 years of schooling) have relatively high rates of childlessness. In particular, they show that childlessness rates among married women with a college degree or less range between 6 to 10 percent, while childlessness rates among married women with Master degrees and Doctoral degrees are 13.7 and 19.1 percent, respectively. Baudin et al. (2015) attribute these high rates of childlessness to the high opportunity cost of these women raising children. According to our theory, this opportunity cost should be decreasing over time, as women marketize the cost of children more and more. Indeed, in Figure 13, we show that the rates of childlessness for women with advanced degrees relative to other women is decreasing over time.[43] Indeed, this ratio falls from over two to almost 1, yielding no difference in childlessness rates by 2014. The change is driven by decreasing childlessness among educated women, as in our sample, the childlessness rates of other women remained stable.

A natural question to ask is, how much of the changes in fertility rates of these highly educated women can be accounted for by changes along the extensive margin, or childlessness, versus the intensive margin, or the number of children born to a mother conditional on having at least one? Figure 14 (left panel) shows the average number of children ever born to all married women with advanced degrees between 1990 and 2014. It is increasing over time, at a rate of about 0.01 children per year. Figure 14 (left panel) shows the average number of children born to these women, conditional on them having at least one child, over our sample time frame. That is, the intensive margin of fertility. This intensive margin has remained flat. Thus, the increase in fertility shown in the left panel is explained by the decrease in childlessness, or increased fertility along the extensive margin, rather than an increase in fertility along the intensive margin.

Our model is not equipped to differentiate between fertility changes along the intensive and extensive margins. We leave this as a promising path for future research.

6.2 Endogenous Sorting

We discuss how the rise in marketization can help explain the rise in marital sorting.

Greenwood et al. (2016) show how a narrowing gender wage gap, rising skill premium, and technological improvement in home goods (cheaper marketization) lead to, among other things, a rise in sorting. The intuition is as follows.

[43] We are using more than college educated women relative to other women in a sample that is not restricted to white non-Hispanics in order to be consistent with Baudin et al. (2015). The results are not qualitatively sensitive to this sample selection.

When the gender gap is narrow, women's wages are relatively more important for the household, increasing the desire for men to marry higher wage women. The same is true as the skill premium rises. They find that cheaper marketization leads to a rise in married women's labor force participation, which they argue is important for the desire to sort. "A skilled man is indifferent on economic grounds between a skilled and unskilled woman if neither of them works, assuming that skill doesn't affect a woman's production value at home. When both work, however, the skilled woman becomes the more attractive partner, at least from an economic point of view" (Greenwood et al. 2016, p. 35). Fertility is not discussed in Greenwood et al. (2016). However, if children comprise an additional benefit to marriage, the mechanism proposed in this paper would reinforce the mechanisms they study.

To see this point, consider a man who is choosing between two women, one with a high wage and the other with a low wage. In 1980, the man would face a tradeoff. The high wage woman would provide more income, and thus consumption, but at a cost of fewer children. In 2010, the high wage women could marketize her time with children, such that there is no more tradeoff. That is, the man would not have to choose between high wages and a large family, yielding more of an incentive to marry a high wage woman. This argument is consistent with the fact that marriage outcomes for college educated women have improved relative to non college educated women, measured by the fraction of those ever married or currently married (Figure 15, for data on white non-Hispanic women, ages 35–44).

While these data are not conclusive, they are suggestive of a path for promising future research.

7 Conclusions

In this paper we have shown that the relationship between income and fertility has flattened between 1980 and 2010 in the US, a time of increasing inequality, as the rich increased their fertility. These facts challenge the standard theory according to which rising inequality should steepen this relationship. We propose that marketization of parental time costs can explain the changing relationship between income and fertility. We show this result both theoretically and quantitatively, after disciplining the model on US data. Without marketization the model yields a quantitatively significant biased estimate of rising inequality's impact on differential fertility and thus education. Going from the standard theory to the one with marketization implies an increase of just under 3 percentage points of college attainment. This is equivalent to more than one-quarter of the rise in college completion between white non-Hispanic non-immigrant Americans born in 1950 and 1980.

We have used the calibrated model to shed new light on the effects of changes in the minimum wage. Specifically, we have shown that an increase in the minimum wage to $15/hour, as per Bernie Sanders, would imply an increase in the cost of market goods of about 14 percent. This increase would have a significant detrimental effect on the labor supply and fertility of women, with high income women responding much more than lower income women.

We ended with a discussion on the insights our theory offers for the literatures of the economics of childlessness and marital sorting. These are promising avenues for future research.

References

Aguiar, M. & Hurst, E. (2007), 'Life-cycle prices and production', *The American Economic Review* **97**(5), 1533–1559.

Akbulut, R. (2011), 'Sectoral changes and the increase in women's labor force participation', *Macroeconomic Dynamics* **15**(2), 240–264.

Attanasio, O., Hurst, E. & Pistaferri, L. (2012), The evolution of income, consumption, and leisure inequality in the us, 1980-2010. NBER WP 17982.

Attanasio, O., Low, H. & Sanchez-Marcos, V. (2008), 'Explaining changes in female labour supply in a life-cycle model', *The American Economic Review* **98**(4), 1517–1552.

Autor, D. H., Katz, L. F. & Kearney, M. S. (2008), 'Trends in u.s. wage inequality: Revising the revisionists', *Review of Economics and Statistics* **90**, 300–323.

Baskaya, Y. S. & Rubinstein, Y. (2012), Using federal minimum wages to identify the impact of minimum wages on employment and earnings across the u.s. states. Unpublished Manuscript.

Baudin, T., de la Croix, D. & Gobbi, P. E. (2015), 'Fertility and childlessness in the united states', *The American Economic Review* **105**(6), 1852–1882.

Becker, G. S. & Lewis, G. H. (1973), 'On the interaction between the quantity and quality of chldren', *Journal of Political Economy* **81**, S279–S288.

Berry, W. D., Fording, R. C., Ringquist, E. J., Hanson, R. L. & Klarner, C. (2010), 'Measuring citizen and government ideology in the american states: A reappraisal', *State Politics and Policy Quarterly* **10**, 117–135.

Berry, W. D., Ringquist, E. J., Fording, R. C. & Hanson, R. L. (1998), 'Measuring citizen and government ideology in the american states, 1960-93', *American Journal of Political Science* **42**, 327–348.

Buera, F. J., Kaboski, J. P. & Zhao, M. Q. (2017), 'The rise of services: the role of skills, scale, and female labor supply', *Journal of Human Capital* . forthcoming.

Cortés, P. & Pan, J. (2013), 'Household production: Foreign domestic workers and native labor supply in hong kong', *Journal of Labor Economics* **31**(2), 327–371.

Cortés, P. & Pan, J. (Forthcoming), 'When time binds: Substitutes to household production, returns to working long hours and the gender wage gap among the highly skilled', *Journal of Labor Economics* .

Cortés, P. & Tessada, J. (2011), 'Low-skilled immigration and the labor supply of highly skilled women', *American Economic Journal: Applied Economics* **3**(1), 88–123.

de la Croix, D. & Doepke, M. (2003), 'Inequality and growth: Why differential fertility matters', *The American Economic Review* **93**(4), 1091–1113.

de la Croix, D. & Doepke, M. (2004), 'Public versus private education when differential fertility matters', *Journal of Development Economics* **73**, 607–629.

Doepke, M. (2004), 'Accounting for fertility decline during the transition to growth', *Journal of Economic Growth* **9**(3), 347–383.

Doepke, M. & Kindermann, F. (2016), Bargaining over babies: Theory, evidence, and policy implications. NBER Working Paper w22072.

Duernecker, G. & Herrendorf, B. (2017), 'On the allocation of time Ű a quantitative analysis of the roles of taxes and productivities', *European Economic Review* . forthcoming.

Furtado, D. (2016), 'Fertility responses of high-skilled native women to immigrant inflows', *Demography* **53**, 27–53.

Galor, O. & Moav, O. (2002), 'Natural selection and the origin of economic growth', *Quarterly Journal of Economics* **117**(4), 1113–1191.

Galor, O. & Weil, D. N. (1996), 'The gender gap, fertility, and growth', *The American Economic Review* **86**(3), 374–387.

Galor, O. & Weil, D. N. (2000), 'Population, technology, and growth: From malthusian stagnation to the demographic transition and beyond', *The American Economic Review* **90**(4), 806–828.

Gobbi, P. (2016), Childcare and commitment within households. IRES Discussion Paper 2013/19.

Greenwood, J., Guner, N., Kocharkov, G. & Santos, C. (2016), 'Technology and the changing family', *American Economic Journal: Macroeconomics* **8**(1), 1–41.

Greenwood, J., Guner, N. & Vandenbroucke, G. (2017), 'Family economics writ large', *Journal of Economic Literature* . forthcoming.

Greenwood, J., Seshadri, A. & Vandenbroucke, G. (2005), 'The baby boom and baby bust', *The American Economic Review* **95**(1), 183–207.

Greenwood, J., Seshadri, A. & Yorukoglu, M. (2005), 'Engines of liberation', *Review of Economic Studies* **72**(1), 109–133.

Guner, N., Kaygusuz, R. & Ventura, G. (2012), 'Taxation and household labour supply', *The Review of Economic Studies* **79**, 1113–1149.

Hazan, M. & Zoabi, H. (2015), 'Do highly educated women choose smaller families?', *The Economic Journal* **125**(587), 1191–1226.

Heathcote, J., Perri, F. & Violante, G. (2010), 'Unequal we stand: An empirical analysis of economic inequality in the united states 1967-2006', *Review of Economic Dynamics* **13**(1), 15–50.

Hendricks, L. & Leukhina, O. (2017), 'The return to college: Selection bias and dropout risk', *International Economic Review* . forthcoming.

Jones, L. E., Schoonbroodt, A. & Tertilt, M. (2010), Fertility theories: Can they explain the negative fertility-income relationship?, *in* J. Shoven, ed., 'Demography and the Economy', University of Chicago Press, pp. 43–100.

Jones, L. E. & Tertilt, M. (2008), An economic history of fertility in the u.s.: 1826-1960, *in* P. Rupert, ed., 'Frontiers of Family Economics', Emerald, pp. 165 – 230.

Kaygusuz, R. (2010), 'Taxes and female labor supply', *Review of Economic Dynamics* **13**, 725–741.

Lino, M., Kuczynski, K., Rodriguez, N. & Schap, T. (2017), Expenditures on children by families, 2015, Technical report, United States Department of Agriculture.

Manning, A. (2016), The elusive employment of the minimum wage. CEP Discussion Paper No 1428.

Manski, C. F. (1993), 'Identification of endogenous social effects: The reflection problem', *The Review of Economic Studies* **60**(3), 531–542.

Mazzolari, F. & Ragusa, G. (2013), 'Spillovers from high-skill consumption to low-skill labor markets', *The Review of Economics and Statistics* **95**(1), 74–86.

Moav, O. (2005), 'Cheap children and the persistence of poverty', *The Economic Journal* **115**(500), 88–110.

Rendall, M. (2018), 'Female market work, tax regimes, and the rise of the service sector', *Review of Economic Dynamics* **28**, 269–289.

Ruggles, S. J., Alexander, T., Genadek, K., Goeken, R., Schroeder, M. B. & Sobek, M. (2010), *Integrated Public Use Microdata Series: Version 5.0 [Machine-readable database]*, Minneapolis, MN.

Shang, Q. & Weinberg, B. A. (2013), 'Opting for families: Recent trends in the fertility of highly educated women', *Journal of Population Economics* **26**(1), 5–32.

Siegel, C. (2017), 'Female relative wages, household specialization and fertility', *Review of Economic Dynamics* **24**, 152–174.

Vaghul, K. & Zipperer, B. (2016), Historical state and sub-state minimum wage data. Washington Center for Equitable Growth.

Vogl, T. (2016), 'Differential fertility, human capital, and development', *Review of Economic Studies* **83**(1), 365–401.

Table 1: Calibrated Parameter Values

Parameter	Interpretation	Value	Indentification
α	Weight on # children	0.45	Fertility
β	Weight on quality of children	0.67	Fertility
η	Basic edu.	2.06	Fertility
θ	Exponent π	0.43	College Attainment
b	Scaling	0.87	College Attainment
ρ	Elasticity wife/m	0.59	Labor Supply
ϕ	TFP child production	3.77	Labor Supply
A	Share of mother's time	0.90	Index of Marketization
$p_{m,1980}$	Price of market substitutes 1980	1	Normalization
p_e	Cost of education	1	Normalization

Table 2: Results: Model Mechanisms

	Data	Model	No Δ Marketization	No Δw_m
	(1)	(2)	(3)	(4)
%Δ High Income Fert	40%	43.5%	-34%	30%
%Δ MDF	38.5%	41%	-14%	24%
%Δ MDF Top/Bottom	18.6%	24.4%	-11.1%	15.1%
Δ Fraction College (pp)	1.70	2.4	-1.23	1.60

Notes: "%Δ High Income Fert." is the percentage change in the number of children born to the top 2 deciles. "%Δ MDF" is the percentage change in the fertility of the top two deciles relative to the fertility of the 2nd decile. "%Δ MDF Top/Bottom" is the percentage change in the fertility of the top half of the income distribution relative to the fertility of the bottom half of the income distribution. "Δ Fraction College (pp)" is the the change in the fraction of children who receive a college education. Here, the fraction of children who receive a college education is calculated by summing the fraction of children born to each decile multiplied by the fraction of children from that decile born in 1980 who graduated college. All changes refer to between 1980 and 2010. The "Data" column reports each of these variables in the data. "Model" does so for the benchmark model. "No Δ Marketization" does so for the counterfactual 2010 where $\frac{w_f}{p_m}$ is held constant, by decile, at the 1980 level. "No Δw_m" does so for the counterfactual 2010 where w_m is held constant at the 1980 level.

Table 3: The Effect of the Minimum Wage on the Wage in Industries Associated with Home Production Substitutes

Dependent Variable: The Real Wage										
	OLS					2SLS				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Minimum Real Wage	0.764***	0.771***	0.770***	0.665***	0.648***	0.747***	0.645***	0.550**	0.632**	0.582**
	(0.059)	(0.053)	(0.063)	(0.058)	(0.056)	(0.169)	(0.133)	(0.267)	(0.248)	(0.247)
State FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	No	No	Yes	Yes	Yes	No	No	No
Region × Year FE	No	No	Yes	Yes	Yes	No	No	Yes	Yes	Yes
Average State Wages	No	Yes	No	No	Yes	No	Yes	No	No	Yes
Demographic Controls	No	No	No	Yes	Yes	No	No	No	Yes	Yes
1^{st} Stage F-Statistic	–	–	–	–	–	16.47	15.90	26.72	26.93	26.08
Obs.	228,197	228,197	228,197	228,197	228,197	228,197	228,197	228,197	228,197	228,197
R^2	0.258	0.259	0.259	0.372	0.372	0.258	0.258	0.259	0.372	0.372

Notes: Standard errors in parentheses are clustered at the state level.* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Sample comprises workers in industries of the economy associated with home production substitutes for the years 1980 to 2010 using CPS data. Demographic controls include age fixed effects, education fixed effects, occupation fixed effects, Hispanic and race fixed effects. The instrument for Columns 6–10 is the interaction between average state liberalism between 1960 and 1980 and the real federal minimum wage.

Table 4: The Effect of the Minimum Wage on the Labor Supply of High Income Women

	Dependent Variable: Log Yearly Hours											
	OLS						2SLS					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Log min. wage	-0.026	-0.006	-0.020	0.039	0.022	0.040	-0.523***	-0.655***	-0.608***	-0.478**	-0.378*	-0.398*
	(0.087)	(0.070)	(0.066)	(0.049)	(0.054)	(0.053)	(0.183)	(0.252)	(0.232)	(0.214)	(0.222)	(0.240)
Year FE	Yes	No	No	No	No	No	Yes	No	No	No	No	No
Region× Year FE	No	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes
State FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Age FE	No	No	Yes	Yes	Yes	Yes	No	No	Yes	Yes	Yes	Yes
Education FE	No	No	Yes	Yes	Yes	Yes	No	No	Yes	Yes	Yes	Yes
Industry FE	No	No	No	Yes	No	Yes	No	No	No	Yes	No	Yes
Occupation FE	No	No	No	No	Yes	Yes	No	No	No	No	Yes	Yes
1^{st} stage F statistic	–	–	–	–	–	–	15.73	24.42	24.55	24.68	24.76	24.92
Obs.	86,414	86,414	86,414	86,414	86,414	86,414	86,414	86,414	86,414	86,414	86,414	86,414
R^2	0.013	0.015	0.046	0.256	0.291	0.309	0.012	0.014	0.046	0.255	0.290	0.309

Notes: Standard errors clustered at the state level are in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. The dependent variable is the log of yearly hours worked. Sample of White non-Hispanic married women aged 25-54, whose real hourly wage is in the 9th and 10th deciles. Women are assigned to hourly wage decile by state, year and 5-year age group.

Table 5: The Effect of the Minimum Wage on the Labor Supply of High Income **Men**

	Dependent Variable: Log Yearly Hours											
	OLS						2SLS					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Log min. wage	0.043	0.010	0.005	0.002	-0.008	-0.011	-0.124	-0.124	-0.045	0.022	-0.071	-0.041
	(0.034)	(0.030)	(0.027)	(0.026)	(0.026)	(0.026)	(0.115)	(0.148)	(0.122)	(0.121)	(0.122)	(0.119)
Year FE	Yes	No	No	No	No	No	Yes	No	No	No	No	No
Region× Year FE	No	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes
State FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Age FE	No	No	Yes	Yes	Yes	Yes	No	No	Yes	Yes	Yes	Yes
Education FE	No	No	Yes	Yes	Yes	Yes	No	No	Yes	Yes	Yes	Yes
Industry FE	No	No	No	Yes	No	Yes	No	No	No	Yes	No	Yes
Occupation FE	No	No	No	No	Yes	Yes	No	No	No	No	Yes	Yes
1^{st} stage F statistic	–	–	–	–	–	–	15.28	25.11	25.19	25.44	25.32	25.63
Obs.	100,927	100,927	100,927	100,927	100,927	100,927	100,927	100,927	100,927	100,927	100,927	100,927
R^2	0.014	0.015	0.067	0.159	0.201	0.210	0.013	0.015	0.067	0.159	0.201	0.210

Notes: Standard errors clustered at the state level are in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. The dependent variable is the log of yearly hours worked. Sample of White non-Hispanic married **men** aged 25-54, whose real hourly wage is in the 9th and 10th deciles. **Men** are assigned to hourly wage decile by state, year and 5-year age group.

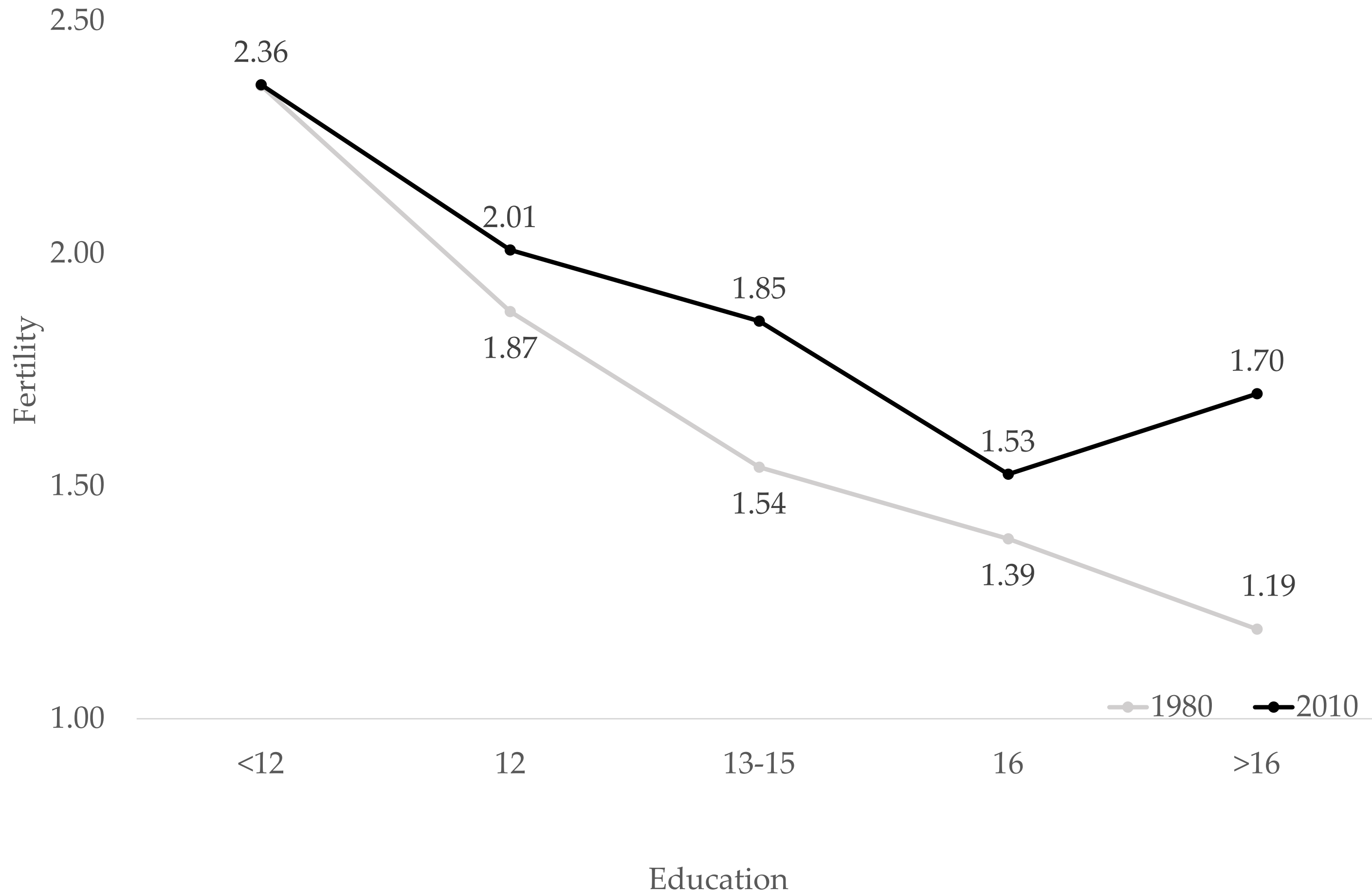

Figure 1: Fertility by Mother's Education 1980 & 2010. Authors calculations using Census and American Community Survey Data, using all native-born American women. Fertility rates are hybrid fertility rates. "< 12" refers to women with less than a high school degree. "12" refers to women who graduated high school. "13-15" refers to women with some college. "16" refers to college graduates. "> 16" refers to women with advanced degrees.

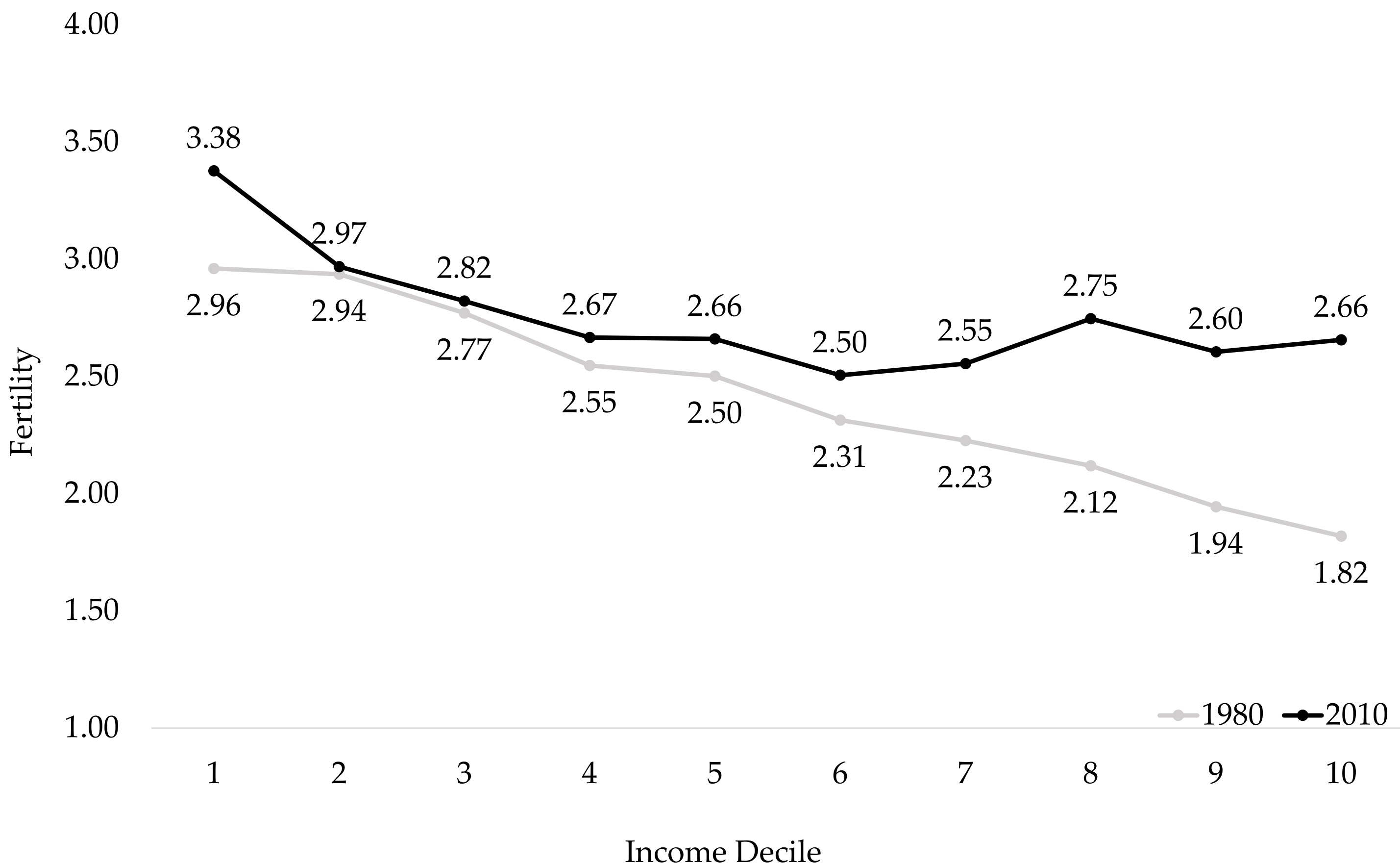

Figure 2: Fertility by Income Decile 1980 & 2010. Authors calculations using Census and American Community Survey Data. The sample is restricted to white, non-Hispanic married women. Fertility rates are hybrid fertility rates, constructed by age-specific deciles. Deciles are constructed using total household income.

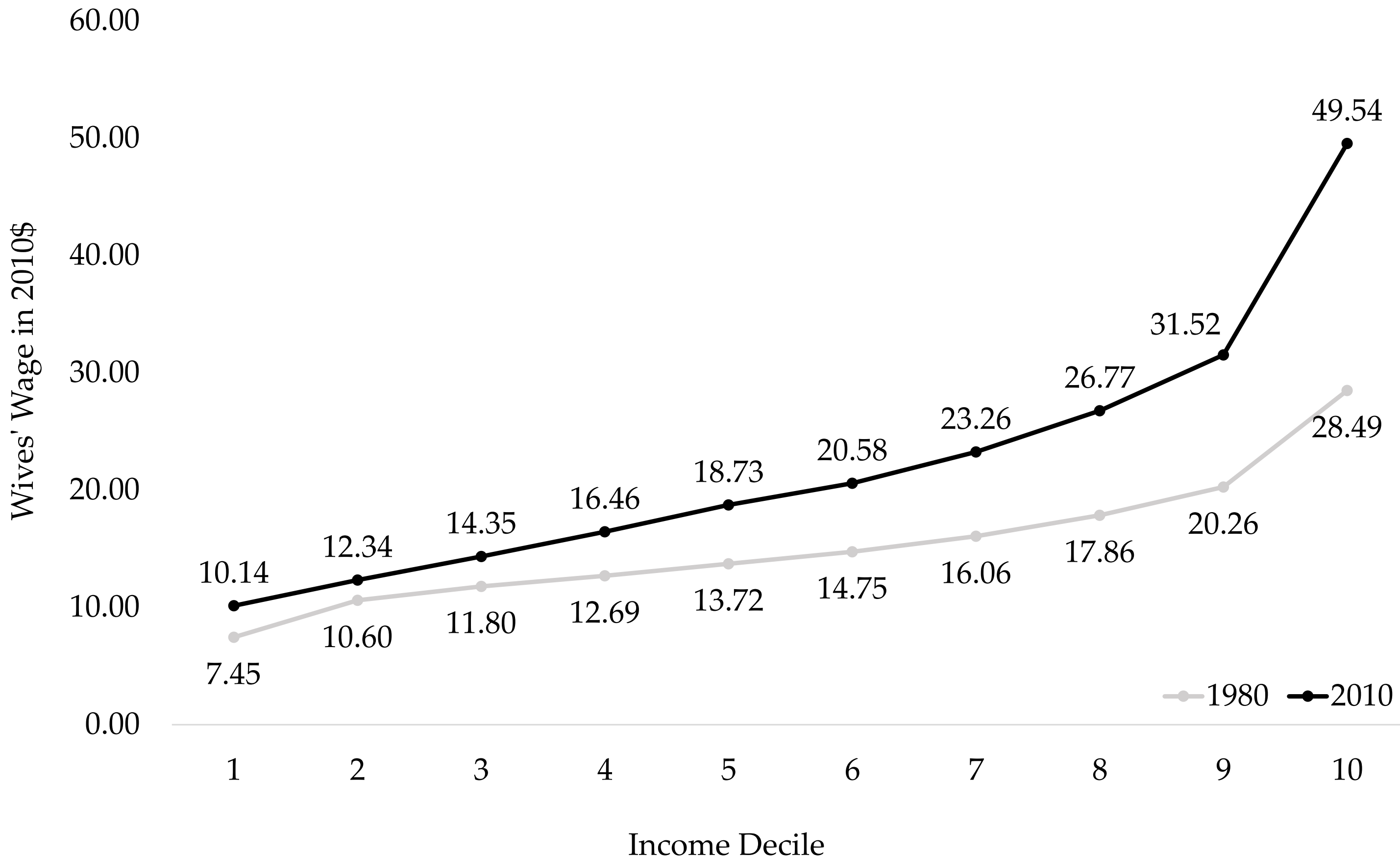

Figure 3: Wives' Wage by Income Decile 1980 & 2010. Authors calculations using Census and American Community Survey Data. The sample is restricted to white, non-Hispanic married women. Deciles are constructed age-by-age, using total household income. Representative wages for each decile is the average of these decile-specific wages from ages 25 to 50. See Appendix A for more details.

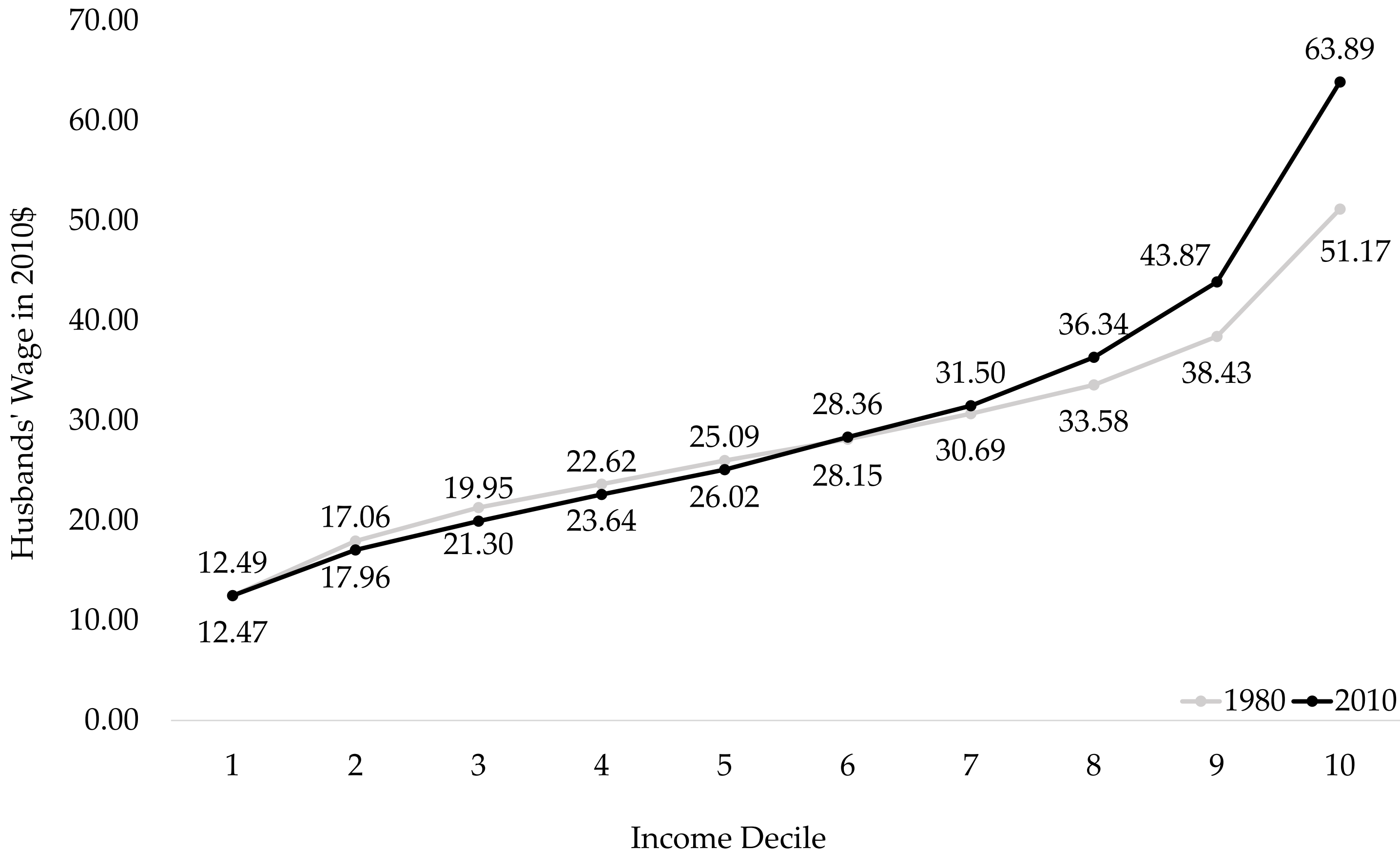

Figure 4: Husbands' Wage by Income Decile 1980 & 2010. Authors calculations using Census and American Community Survey Data. The sample is restricted to white, non-Hispanic married men. Deciles are constructed age-by-age, using total household income. Representative wages for each decile is the average of these decile-specific wages from ages 25 to 50. See Appendix A for more details.

Figure 5: The change in the relative wage of high income women to workers in HPS at the state level is defined as the percent change in the ratio of the average wage of women in the top two deciles to the average wage in the home production substitute sector. The change in fertility is defined as the percentage change in hybrid fertility rates for the top two decile women. Changes from 1980 to 2010. Deciles are constructed age-by-age by total household income, and wages of women are averaged over ages. Wages of HPS workers, as defined in Appendix A, are constructed by state-year. See Appendix A for more details on the exact definition of these variables. Data for high income women are restricted to white, non Hispanic, married women. Data on HPS sector workers are not restricted.

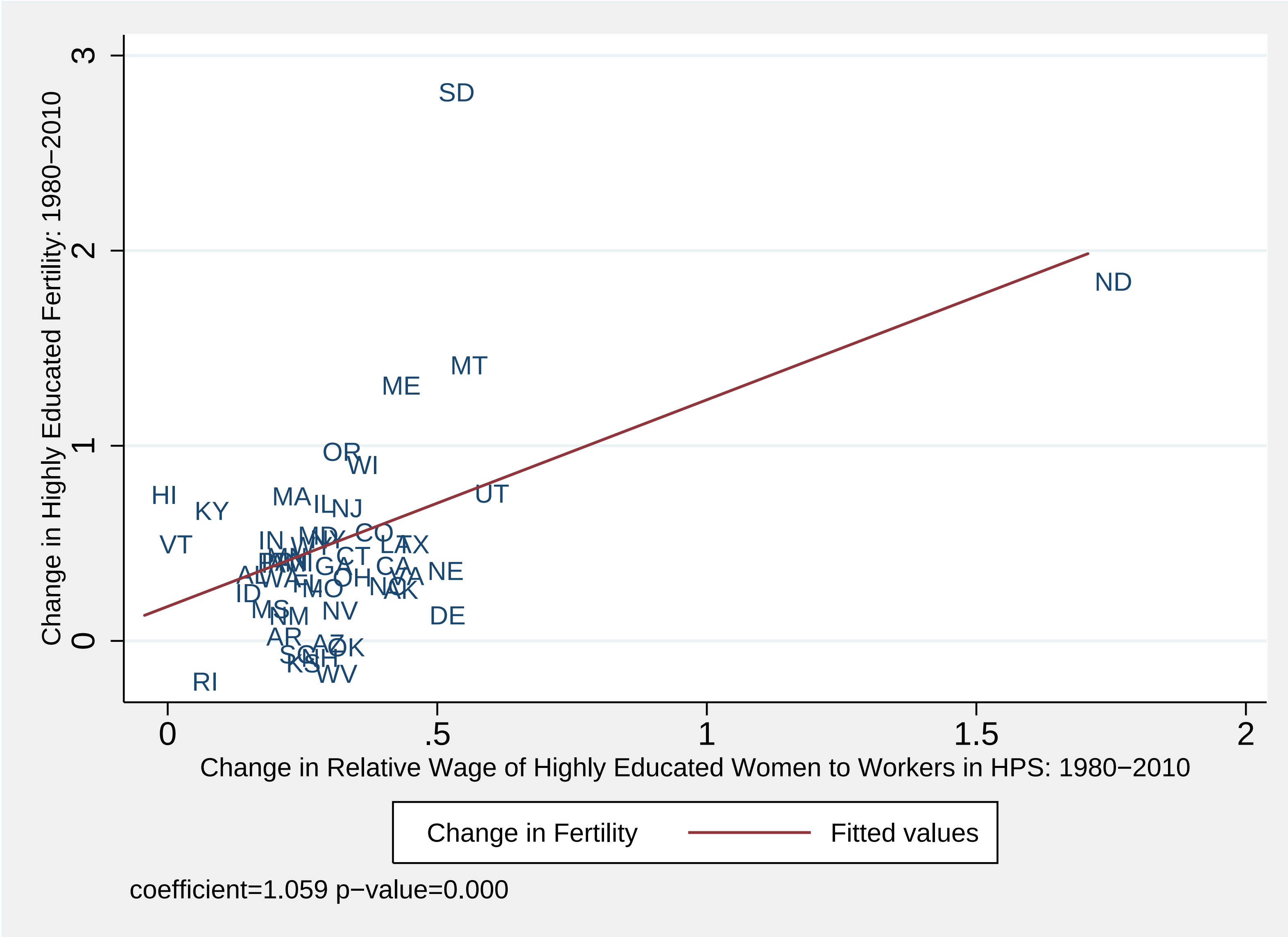

Figure 6: The change in the relative wage of high education women to workers in HPS at the state level is defined as the percent change in the ratio of the average wage of women who have advanced degrees to the average wage in the home production substitute sector. The change in fertility is defined as the percentage change in hybrid fertility rates for women with advanced degrees. Changes from 1980–2010. Wages of women with advanced degrees are averaged over ages. Wages of HPS workers, as defined in Appendix A, are constructed by state-year. Data for women with advanced degrees are restricted to native born American women. Data on HPS sector workers are not restricted. See Appendix A for more details on the exact definition of these variables.

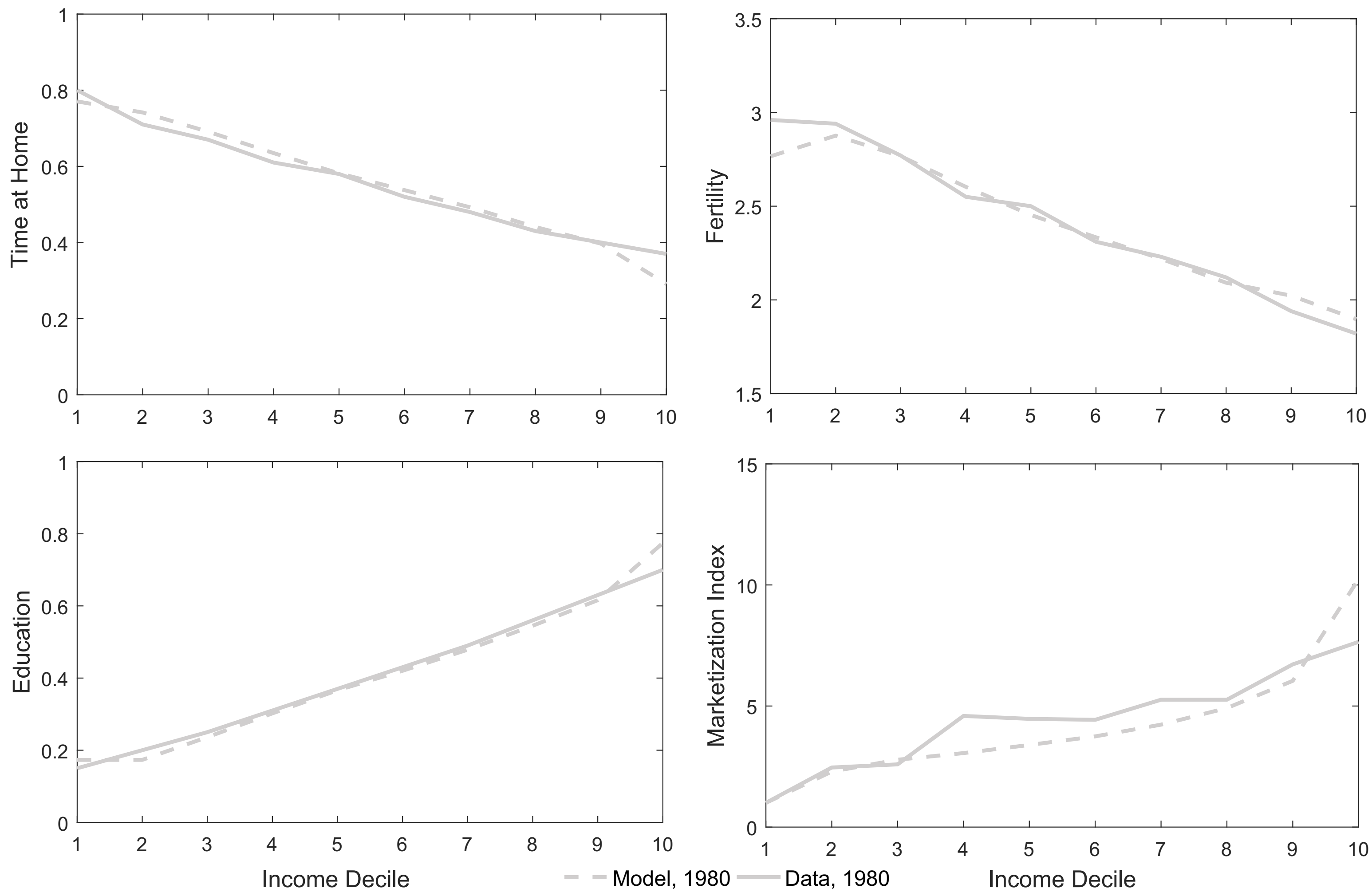

Figure 7: Model Fit Notes: The top left panel, "Time at Home", is mother's time at home as measured by women's time not working in the data, and t_f in the model. The top right panel, "Fertility", is n in the model and hybrid fertility rates in data. "Education" is the fraction of children born to each decile who graduate college in the data and $\pi(e)$ in the model. "Marketization index" is the expenditures on babysitters, by decile, relative to the 1st decile in the data, and $\frac{p_m m(d)}{p_m m(1)}$ in the model, where $m(d)$ is the amount of market goods m purchased by decile d. "Model, 1980" refers to the calibrated model in 1980. "Data, 1980" refers to the relevant data described in this note and the text.

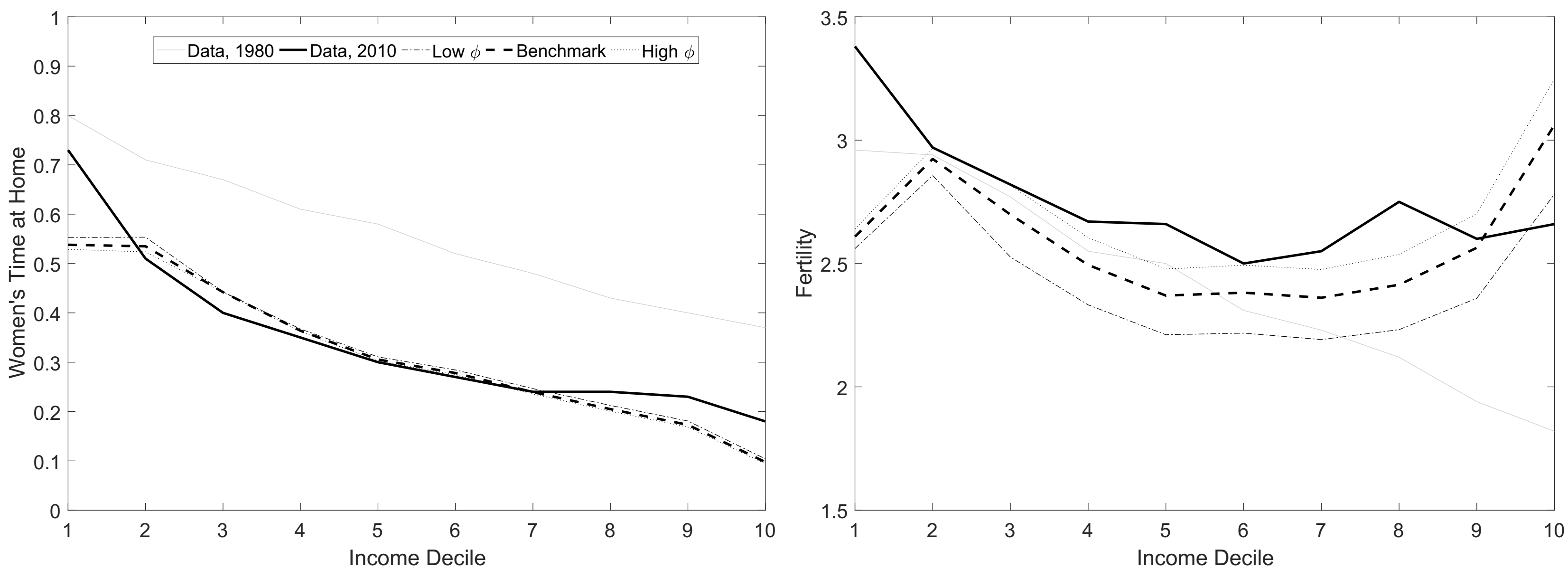

Figure 8: Model Fit, 2010. Notes: "Time at Home", measures t_f in the model, and the fraction of time a woman is not working, by decile, in 2010. "Fertility" is hybrid fertility rate in the data and n in the model. "Data 2010" refers to the data, as described in the text. "Benchmark", "Low ϕ", and "High ϕ" refer to the calibrated model's prediction for 2010, for the benchmark case, the "low ϕ" case, and the "high ϕ" case, respectively. When simulating the model in 2010, we only change w_f, w_m, and p_m from the 1980 values. w_f, w_m are taken from the data as described in Appendix A.

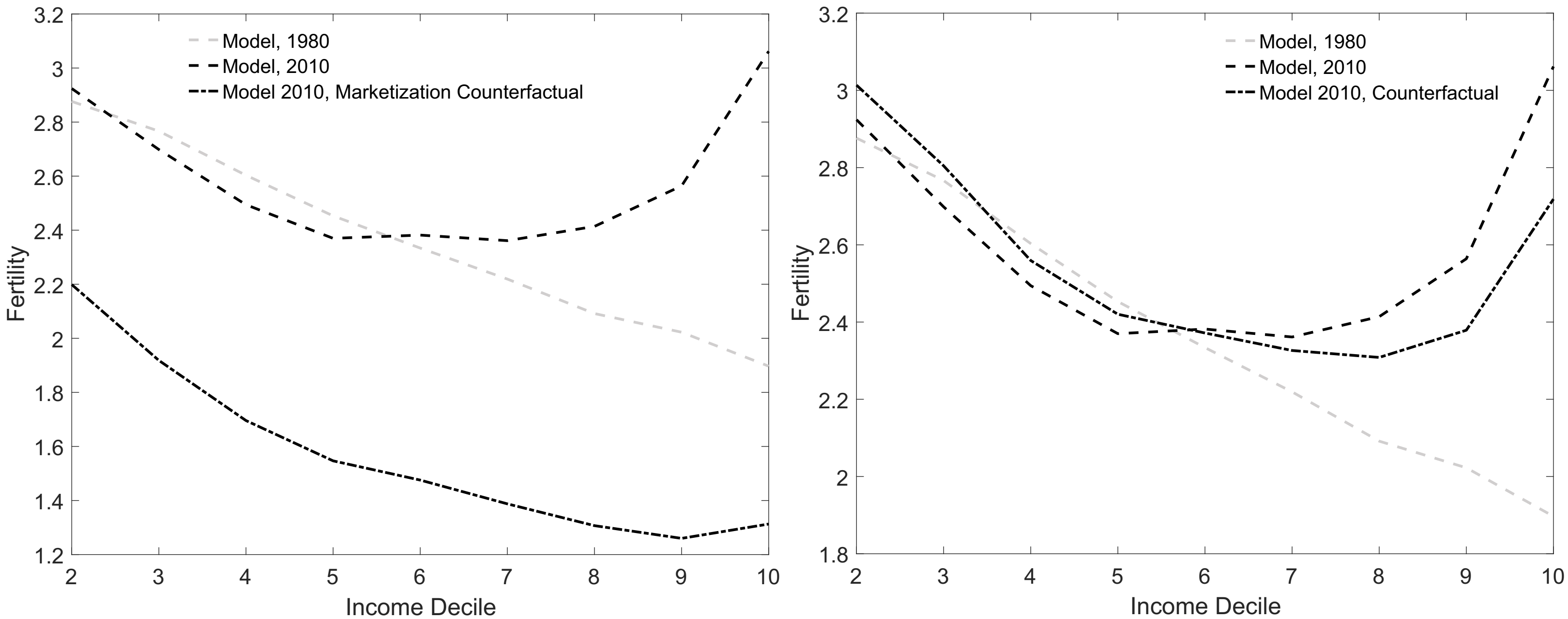

Figure 9: Counterfactuals. Notes: Fertility is n in the model. “Model 1980” is the model calibrated to 1980, while “Model 2010” is the benchmark model in 2010 in both panels. Left panel: The curve labeled “Marketization Counterfactual” is fertility in the 2010 model using the same relative price of market substitutes ($\frac{w_f}{p_m}$), by decile, as in 1980. Right Panel: The curve labeled “Model 2010 Counterfactual” is fertility in the 2010 model using the male wages from 1980.

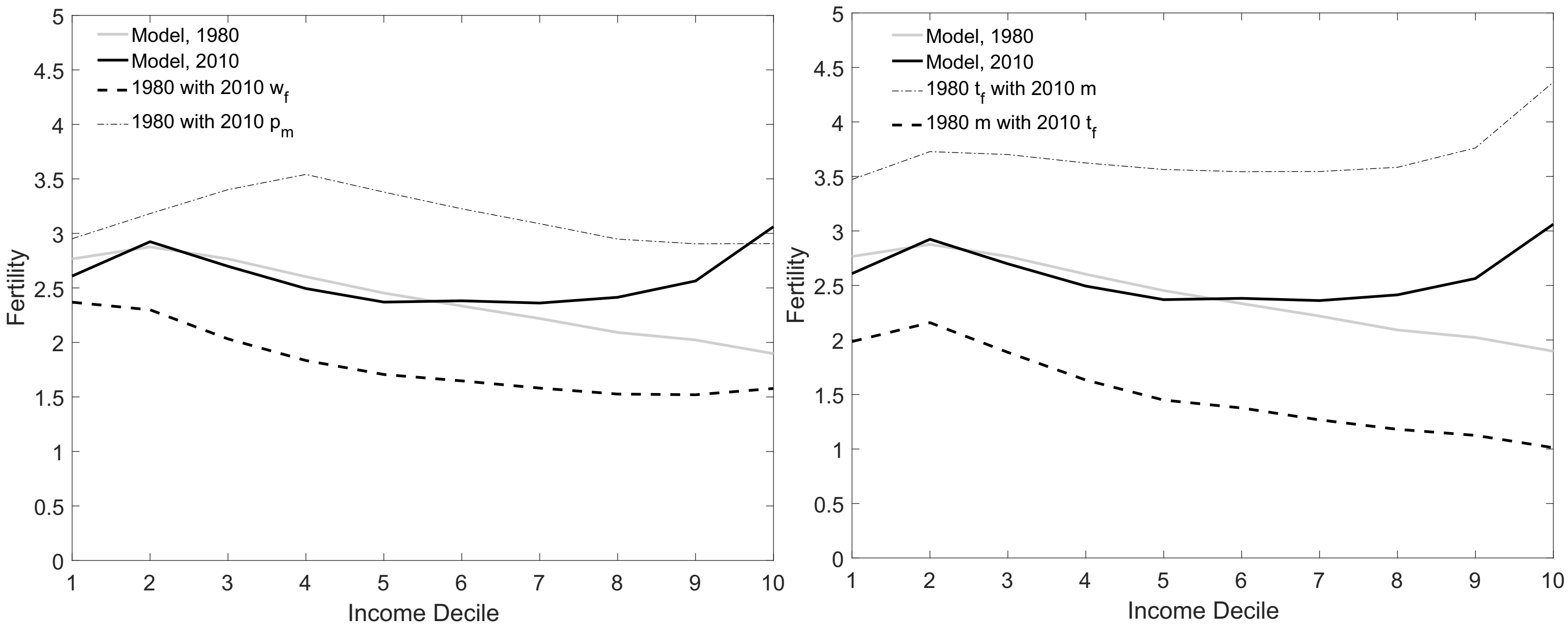

Figure 10: Disentangling Results. Notes: Fertility is n in the model. "Model 1980" is the model calibrated to 1980, while "Model 2010" is the benchmark model in 2010 in both panels. Left panel: The curve labeled "1980 with 2010 w_f" shows fertility in the 1980 model with women's wages from 2010. The curve labeled"1980 with 2010 p_m" shows fertility in the 1980 model with marketization prices from 2010. Right Panel: The curve "1980 m with 2010 t_f" shows fertility with the 1980 levels of m been combined with the 2010 levels of t_f. The curve "1980 t_f with 2010 m" fertility with the 1980 levels of t_f been combined with the 2010 levels of m.

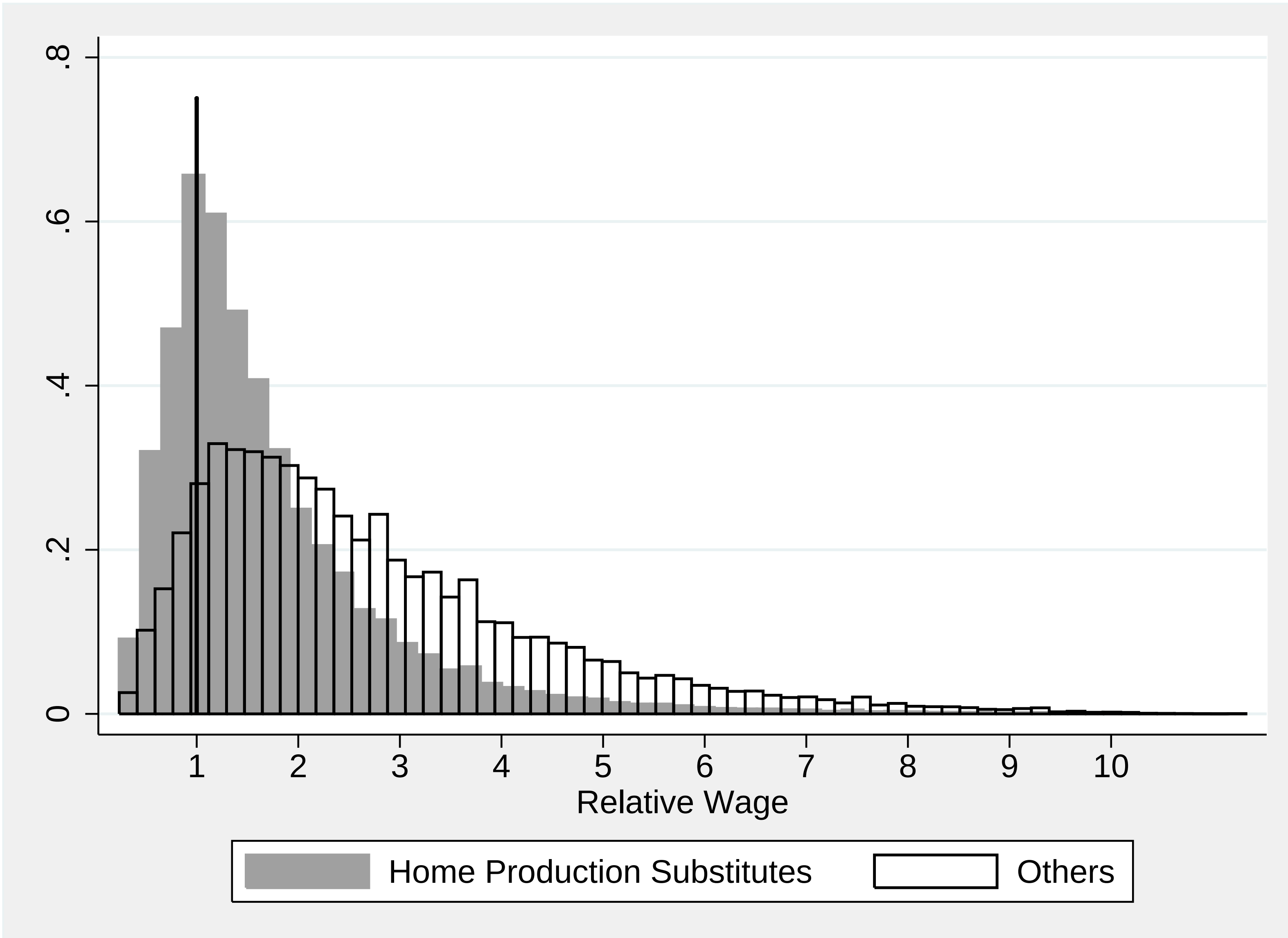

Figure 11: The distribution of real wages, relative to the effective real minimum wage in each state and year, by sector of the economy. Data from Current Population Survey, 1980–2010, using all workers. Home Production Substitute sector workers as defined in Appendix A.

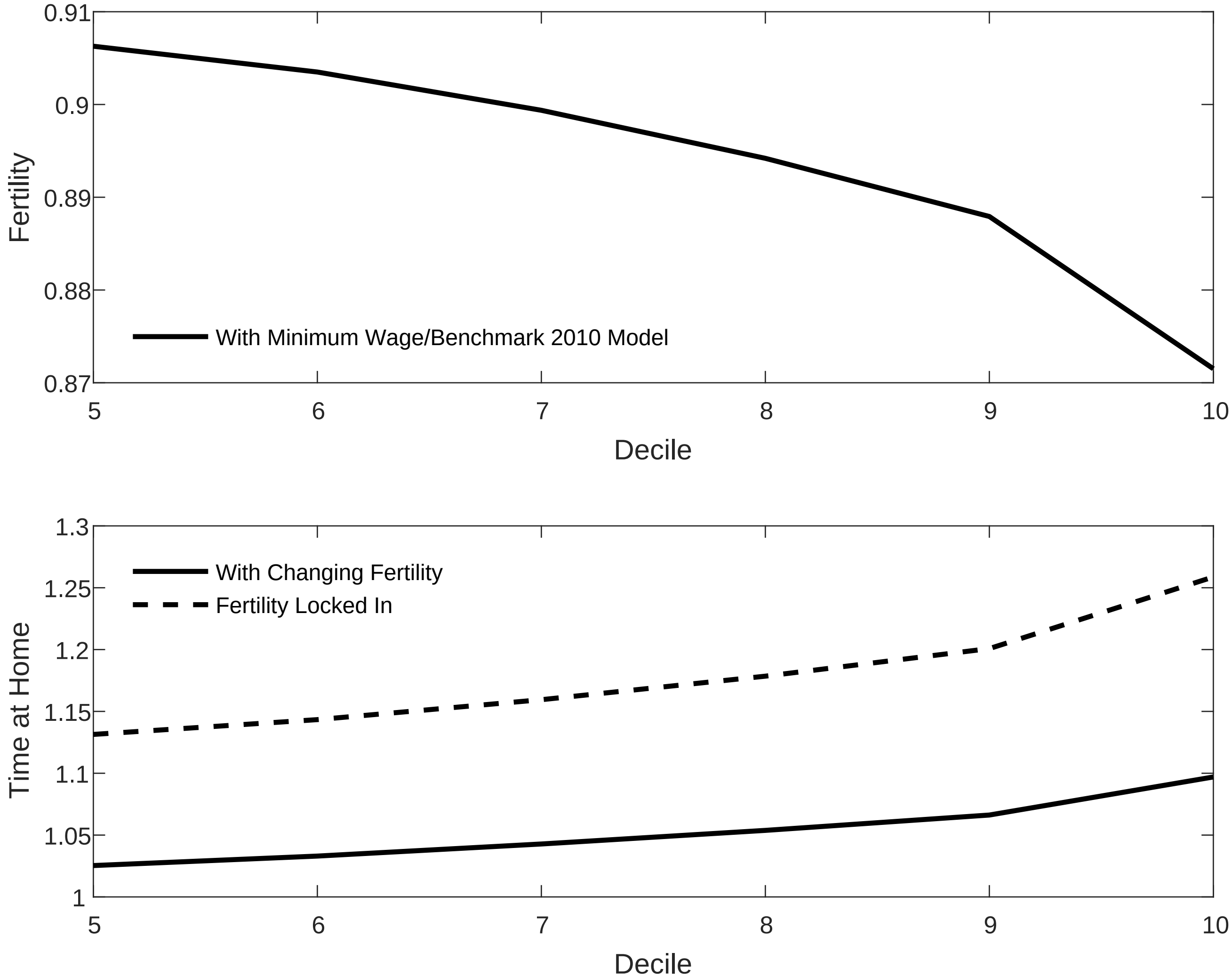

Figure 12: The top panel shows fertility (n) in the 2010 version of the model, with a \$15 minimum wage, divided by fertility in the benchmark 2010 model. The bottom panel shows mother's time at home (t_f) in the 2010 version of the model, with a \$15 minimum wage, divided by mother's time at home in the benchmark 2010 model. The curve "With Changing Fertility" reports this ratio when fertility is allowed to change with the increased minimum wage, while the curve "Fertility Locked In" reports this ratio when households are forced to maintain the same fertility rate as in the benchmark 2010 model.

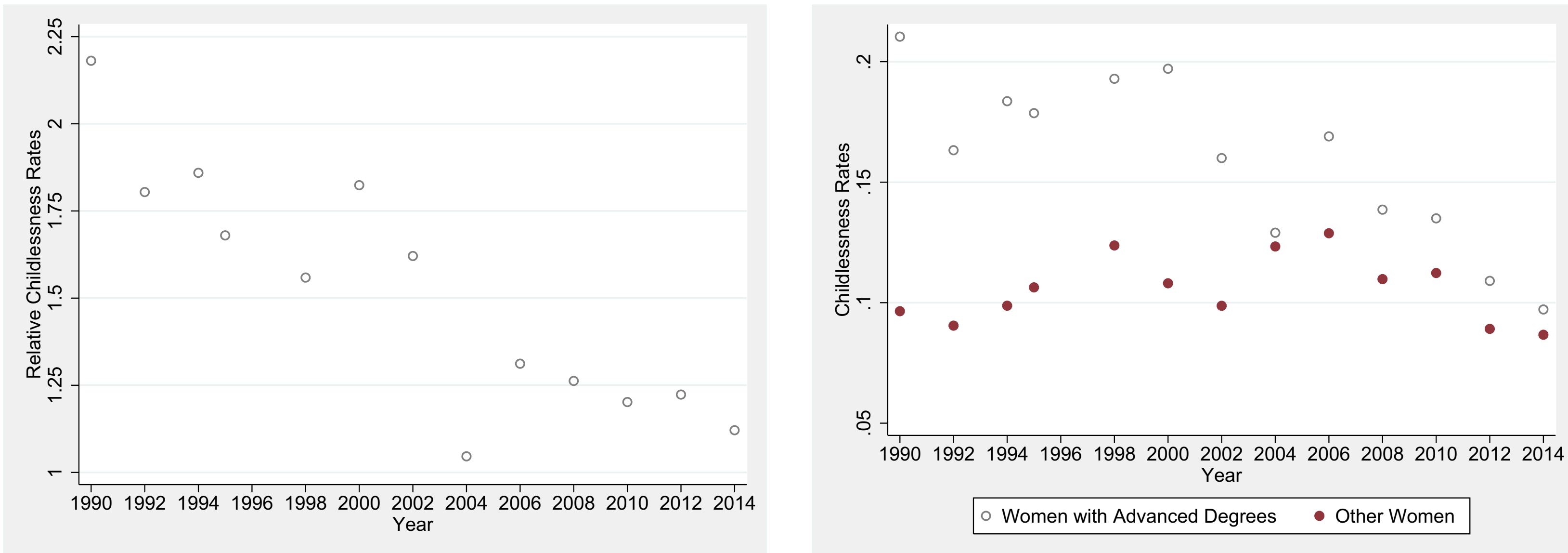

Figure 13: The left panel shows the childlessness rates of married women with advanced degrees (>16 years of school) relative to other women. The right panel shows the childlessness rates (the "extensive" margin of fertility) of married women with more than a college education labelled "Women with Advanced Degrees" and of women with up to and including a college education labelled "Other Women". Data is from the Fertility and Marriage supplement of the Current Population Survey (CPS) from 1990–2014, married women ages 40–44. Women over 45 are not asked about their fertility history in this survey.

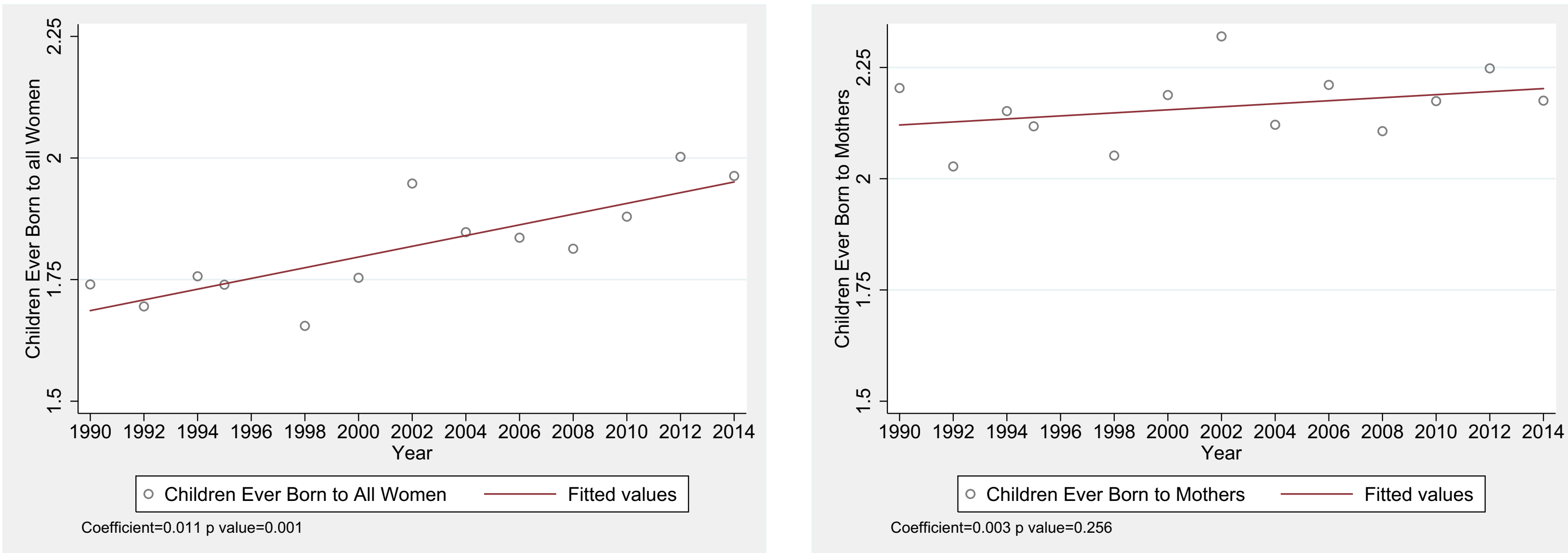

Figure 14: The left panel shows the average number of children ever born for all married women with advanced degrees (>16 years of school). The right panel shows the number of children ever born to women with advanced degrees, conditional on having at least one child (the "intensive" margin of fertility). Data is from the Fertility and Marriage supplement of the Current Population Survey (CPS) from 1990–2014, married women ages 40–44. Women over 45 are not asked about their fertility history in this survey.

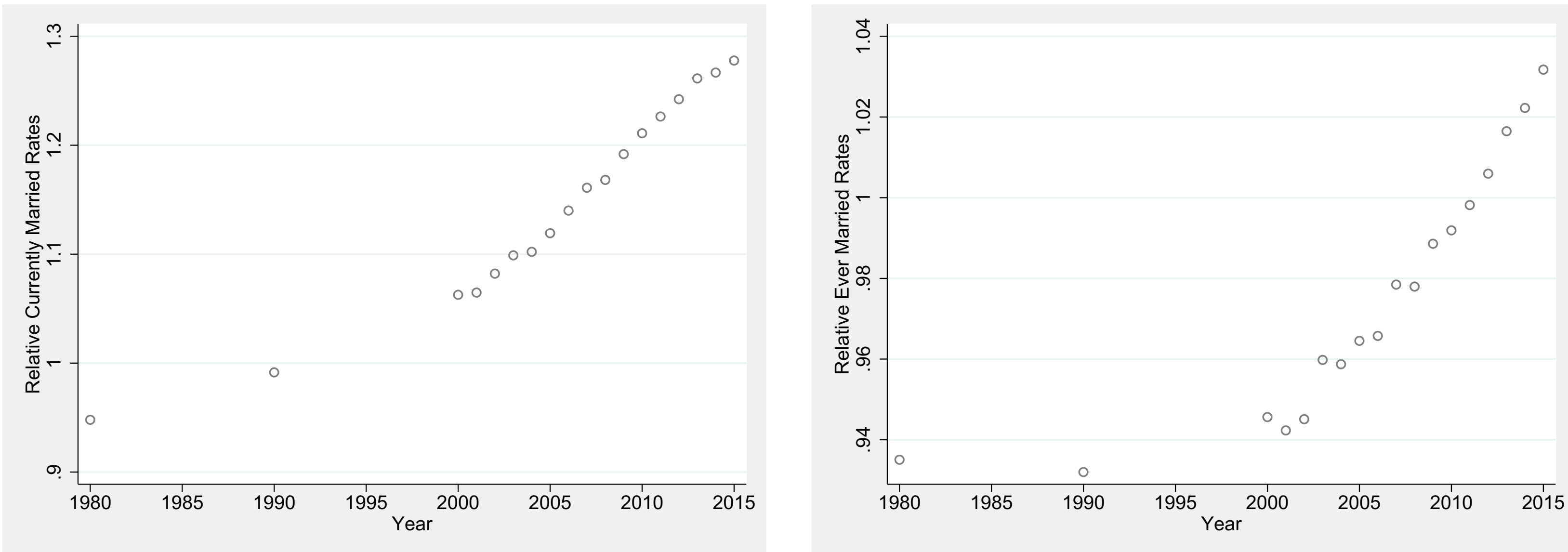

Figure 15: The left panel shows the fraction of women with at least a college degree who are currently married divided by the fraction of other women who are currently married. The right panel shows the fraction of women with at least a college degree who have ever been married divided by fraction of other women who have ever been married. The data is from the US census and ACS. The sample is comprised of white, non-Hispanic women ages 35–44.

A Data

We employ the 1980 Census and the American Community Survey (ACS) 2010 (Ruggles, Alexander, Genadek, Goeken, Schroeder & Sobek 2010) for measuring incomes, fertility and work hours of each spouse and inferring wages for non-working females. Additionally, we use the National Longitudinal Study of Youth 1997 (NLSY 97) for measuring educational attainment of children born around 1980, by family income. Finally, we employ the Survey of Program Participation and Income for measuring childcare expenditure by family income. In this study, we focus on the growth of inequality between 1980 and 2010. These years are chosen to allow us to follow the cohort from the NLSY 97 (born around 1980) for measuring their educational attainment by their parental income, while still studying the period of rising income inequality as defined by Autor et al. (2008).

A.1 Mapping of Model Objects to the Data

The mapping between the model and the data is not trivial. In the model, there is one period of adult life which aims to capture the entire working-age lifecycle. In the data, we observe choices of various couples of different age (fertility, work hours, etc) for a period of one year. To map the model to the data, we take the view that a model couple goes through its lifecycle by behaving according to the average age-specific behavior of those couples in the data that it represents.

There are ten types of couples in the model, each of measure 0.1. Each type of couple stands in for exactly 10% of the entire population of married couples of working age. Married couples in the data are allocated into these deciles according to their observed income. We do so based on the ranking of the couples' observed annual income in their group, defined by the wife's age.

From the 1980 Census and 2010 ACS data, we need to derive decile-specific empirical moments for household lifetime income, male lifetime income, male and female wages, male and female lifetime work hours, and couple's lifetime fertility, $I_{f,i}^{year}, I_{m,i}^{year}, w_{f,i}^{year}, w_{m,i}^{year}, hours_{f,i}^{year}, n_i^{year}, hours_m^{1980}$ for each decile $i \in [1, 2, ...10]$

and $year = 1980,\ 2010$. We state income and hours moments in annualized terms and report wages in hourly terms. This is done for clarity.

We restrict attention to white non-Hispanic married couples, aged 25-55, with the husband working for wages and working at least 35 hours per week and at least 40 weeks per year, following Autor et al. (2008). We also drop the couples in the bottom and top 2% of the income distribution.

All data couples assigned to a particular income decile are used to derive the average statistics for the model couple representing that decile. To compute the decile-specific lifetime income and hours moments for men, we first average the appropriate quantity within the decile-age cells. For each decile, we then sum across ages.

In the model, all men work full time throughout their life cycle, which is normalized to be 1. This corresponds to the average lifetime hours of full-time male workers in 1980, $hours_m^{1980}$ (~2,300 hours in annualized terms). We infer the data counterpart of $w_{m,i}^{year}$ as $I_{m,i}^{year}/hours_m^{1980}$. Note that the 1980 average hours are used to derive $w_{m,i}^{year}$ in each year. Because our model does not allow for male hours variation across time or deciles, this method ensures that any such variation is reflected in the purchasing power of couples.

We infer the data counterpart of $w_{f,i}$ as $I_{f,i}^{year}/hours_{f,i}^{year}$.[44]

Note that when we consider say a 37 year old woman in 1980 in a given decile, we observe her work hours, which partly reflect her number of children and their age distribution. Our goal here, however, is to derive average working hours for a *hypothetical* woman that experiences her lifecycle according to the cross-sectional profile. We need to proxy the hours each woman would work if she were to follow the 1980s cross-sectional fertility profile, not that of her own cohort. To this end, we regress female work hours in a given year on the actual age distribution of her children (i.e. number of children under 2, 2-3, 4-6, 7-10, 11 to 17), income decile and age dummies. We then predict the average adjusted female hours in each decile and for each age using the children's age distribution implied by the

[44]Note that if we were to impute wages for non-working females via a Heckman procedure and then take average wages for each decile, our model would not be able to accurately match both female income and female hours. Both of these quantities are critical to our analysis.

cross-sectional fertility profile. For each decile, we sum these average adjusted hours across age groups to obtain $hours_{f,i}^{year}$ and infer the data counterpart of time spent in home production $t_{f,i}^{year}$ as

$$1 - hours_{f,i}^{year} / hours_m^{1980}.$$

We infer the empirical counterpart of n_i as a decile-specific hybrid Total Fertility Rate (TFR), as in Shang & Weinberg (2013). We first compute the average age-specific-birth-rate, based on all women in decile i. We then sum across all ages to compute decile-specific TFR. To obtain decile-specific hybrid TFR, we add on the average lifetime fertility among the 25 year-old women in the appropriate decile.

We estimate college attainment for 1980 from NLSY97. Specifically, using the 2011 wave, we observe non-black non-Hispanic individuals, born between 1980 and 1982, and assign them into income deciles according to their parental household income in 1996. We assume that individuals with at least four years of college are college graduates. We measure college attainment π_i^{1980} as the fraction of children with a college degree among all children in the appropriate decile.

Finally, we use the childcare module of the Survey of Program Participation and Income (SIPP) to estimate relative uses of market substitutes.[45] Our index measures based off of expenditures on childcare hours purchased in the marketplace. Since this is only one aspect of marketization, we use this to target the relative use of marketization across deciles, rather than taking the absolute expenditure levels literally. The implicit assumption is that there is a strong correlation between the use of childcare and other market substitutes for parents' time. To calculate childcare expenditures across deciles, we break households into 5-year age groups from 25–30 until 50–55. Within each group, we divide households into deciles according to their income. We then sum the childcare expenditures for each decile over the lifecycle. The index is this measure relative to the expenditures on childcare used by decile 1. As before, our sample is married, white, non-Hispanic households.

[45]We use the 1990 childcare module as a proxy for the 1980 index of marketization, as this is the earliest available data.

B Cross State Relationship Between the Relative Price of Marketization and High Income Fertility

In this appendix, we explore further the cross-state relationship between changes in high income fertility between 1980 and 2010 and the change in the relative price of marketization, as first introduced in Figure 5.

We estimate regressions of the following structure:

$$\Delta\% n_s = \Delta\% \left(\frac{w_{fs}}{w_s^{\text{HPS}}} \right) + \Delta\% w_{ms} + \delta_r + \epsilon_s, \tag{B.1}$$

where the dependent variable $\Delta\% n_s$ is the percentage change in hybrid fertility rates for the top two decile of white non-Hispanic married women in state s. The main explanatory variable of interest, $\Delta\% \left(\frac{w_{fs}}{w_s^{\text{HPS}}} \right)$, is the percentage change in the ratio of the average wage of white non-Hispanic married women in the top two deciles to the average wage in the home production substitute sector in state s. $\Delta\% w_{ms}$ is the percentage change of the average wage of white non-Hispanic married men in the top two deciles in state s. δ_r is a set of region fixed effects for each region $r \in$ {Northeast, South, Midwest, West}. ϵ_s is an error term. These variables are described in detail in Appendix A. All regressions are estimated with robust standard errors.

$\Delta\% \left(\frac{w_{fs}}{w_s^{\text{HPS}}} \right)$ captures the change, over our time period, in the relative price of marketizing a woman's time. Quantitatively, this variable is shown to be crucial for explaining changing fertility patterns in Section 4.4. $\Delta\% w_{ms}$ captures changes in the demand for children induced by increases in male wages and quantitatively evaluated in Section 4.4. The regional fixed effects are implicitly interacting differentially with time, as all our variables are changes between 1980 and 2010. This allows us to control for differential regional trends.

Table B.1 describes the results. Column 1 regresses changes in fertility only on changes in the relative price of marketization. Notice that this regression describes the results shown graphically in Figure 5. Column 2 adds the change in male wages over time, while Column 3 adds region fixed effects. Columns 4–6

Table B.1: The Effect of Marketization on High-Income Fertility

	Dependent Variable: Percent Change in High-Income Fertility					
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta\% \left(\frac{w_{fs}}{w_s^{\text{HPS}}}\right)$	1.064***	1.068***	0.952***	1.211***	1.247***	0.966*
	(0.279)	(0.294)	(0.332)	(0.433)	(0.424)	(0.486)
$\Delta\% w_{ms}$		-0.916	-1.186		0.450	0.187
		(1.453)	(1.511)		(0.779)	(0.876)
Region FE	No	No	Yes	No	No	Yes
Obs.	50	50	50	48	48	48
R^2	0.154	0.171	0.199	0.148	0.154	0.178

Notes: Robust standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

repeat Columns 1–3, but drop outlying observations, namely North Dakota and Wyoming.

All specifications show a statistically significant and economically meaningful elasticity between fertility and the relative price of marketization, of around 1. Controlling for changes in men's wages or regional fixed effects do not have meaningful effects on the point estimates or standard errors. Dropping outliers strengthens all specifications, but also increases the standard errors somewhat. In particular, the estimate in Column 6 has a point estimate that is basically the same as its counterpart in Column 3, but has a higher p-value of 0.053. All other specifications are significant at the 1% level. Changes in men's wages do not have a meaningful impact on changes in fertility rates, consistent with their relatively weak effect in the model, documented in Section 4.4

C Proofs

C.1 Existence and Uniqueness of the Solution to the Household Problem

Proposition 1 *The necessary and sufficient condition for existence of a unique solution to the household's problem is* $\frac{\beta\theta}{\alpha} < 1$.

Proof. The household's optimization problem can be written as follows:

$$\max_{e \geqslant 0} U(e) = -\ln\left(\frac{p_n}{p_e} + e\right) + \frac{\beta\theta}{\alpha}\ln(e+\eta)$$

There is a possibility that $U(e)$ is unbounded above, and therefore the household's problem has no solution. We can write the objective function as follows:

$$U(e) = \ln\left(\frac{(e+\eta)^{\frac{\beta\theta}{\alpha}}}{\frac{p_n}{p_e} + e}\right)$$

Taking the limit as $e \to \infty$,

$$\begin{aligned}
\lim_{e\to\infty} U(e) &= \ln\left(\lim_{e\to\infty}\frac{(e+\eta)^{\frac{\beta\theta}{\alpha}}}{\frac{p_n}{p_e}+e}\right) \\
&= \ln\left(\lim_{e\to\infty}\frac{\frac{\beta\theta}{\alpha}(e+\eta)^{\frac{\beta\theta}{\alpha}-1}}{1}\right) = \begin{cases} \infty & \frac{\beta\theta}{\alpha} > 1 \\ 1 & \frac{\beta\theta}{\alpha} = 1 \\ -\infty & \frac{\beta\theta}{\alpha} < 1 \end{cases}
\end{aligned}$$

The first step used chain rule of limits, and the second step used L'Hospital's rule since we have a limit of the form $\frac{\infty}{\infty}$. Intuitively, $\frac{\beta\theta}{\alpha}$ is the weight on quality in the utility function. When this weight is very high, it is possible that the household would like to choose $e \to \infty$ and $n \to 0$, which makes the problem unsolvable. Thus, in order to make the objective function bounded above, we have to impose the restriction $\frac{\beta\theta}{\alpha} \leqslant 1$.

Case 1: $\frac{\beta\theta}{\alpha} = 1$

$$\begin{aligned} U(e) &= -\ln\left(\frac{p_n}{p_e} + e\right) + \ln(e + \eta) \\ U'(e) &= -\frac{1}{\frac{p_n}{p_e} + e} + \frac{1}{e + \eta} \end{aligned}$$

In this case, the solution to the household's problem is as follows:

$$\begin{aligned} \frac{p_n}{p_e} &> \eta \Rightarrow U'(e) > 0 \ \ \forall e, \text{i.e. } U(e) \ \text{ is monotone increasing, } e^* \to \infty \\ \frac{p_n}{p_e} &< \eta \Rightarrow U'(e) < 0 \ \ \forall e, \text{i.e. } U(e) \ \text{ is monotone decreasing, } e^* = 0 \\ \frac{p_n}{p_e} &= \eta \Rightarrow U'(e) = 0 \ \ \forall e, \text{i.e. } U(e) \ \text{ is constant, } e^* \in (-\infty, \infty) \end{aligned}$$

Case 2: $\frac{\beta\theta}{\alpha} < 1$

In this case, the first order necessary condition for interior maximum is $U'(e^*) = 0$:

$$\begin{aligned} U(e) &= -\ln\left(\frac{p_n}{p_e} + e\right) + \frac{\beta\theta}{\alpha}\ln(e + \eta) \\ U'(e) &= -\frac{1}{\frac{p_n}{p_e} + e} + \frac{\frac{\beta\theta}{\alpha}}{\eta + e} = 0 \\ \frac{\eta + e}{\frac{p_n}{p_e} + e} &= \frac{\beta\theta}{\alpha} \\ e\left(1 - \frac{\beta\theta}{\alpha}\right) &= \frac{\beta\theta}{\alpha}\frac{p_n}{p_e} - \eta \\ e^* &= \frac{\frac{\beta\theta}{\alpha}\frac{p_n}{p_e} - \eta}{1 - \frac{\beta\theta}{\alpha}} \end{aligned}$$

The second order sufficient condition for e^* to be a local maximizer is:

$$\begin{aligned} U''(e^*) &< 0 \\ \frac{1}{\left(\frac{p_n}{p_e}+e^*\right)^2} - \frac{\frac{\beta\theta}{\alpha}}{(\eta+e^*)^2} &< 0 \\ \left(\frac{\eta+e^*}{\frac{p_n}{p_e}+e^*}\right)^2 &< \frac{\beta\theta}{\alpha} \end{aligned}$$

Using the first order condition:

$$\begin{aligned} \left(\frac{\beta\theta}{\alpha}\right)^2 &< \frac{\beta\theta}{\alpha} \\ \frac{\beta\theta}{\alpha} &< 1 \end{aligned}$$

Thus, $\frac{\beta\theta}{\alpha} < 1$ guarantees that a solution to the household's problem exists, and the first order necessary condition is a local maximum. Moreover, since the critical point is unique, the local maximum must also be the unique global maximizer. ∎

C.2 Shape of fertility across income deciles

We start with preliminary derivations needed for the proofs below. We focus on the region of parameter values where the solution is interior, i.e. $e^* > 0$. In this case, optimal fertility is given by

$$e^* = \max\left\{\frac{\frac{p_n}{p_e}\frac{\beta\theta}{\alpha}-\eta}{1-\frac{\beta\theta}{\alpha}}, 0\right\} \tag{C.2}$$

$$n^* = \left(1-\frac{\beta\theta}{\alpha}\right)\frac{\alpha}{1+\alpha}\left(\frac{w_f+w_m}{p_n-\eta p_e}\right) \tag{C.3}$$

$$\text{where } p_n = \frac{1}{A}\left[\phi^{\frac{1}{1-\rho}} w_f^{\frac{\rho}{\rho-1}} + (1-\phi)^{\frac{1}{1-\rho}} p_m^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}}. \tag{C.4}$$

Notice that $e^* > 0$ and existence of solution to household's problem, $\frac{\beta\theta}{\alpha} < 1$, imply that $n^* > 0$ and $p_n - \eta p_e > 0$. To analyze the effects on fertility, it suffices to ignore the constant term and focus on the ratio term $\frac{w_f + w_m}{p_n - \eta p_e}$. Clearly, n^* is increasing in w_m as male wages work purely through the positive income effect appearing in the numerator. Female wages, however, affect both the numerator (the positive income effect) and the denominator (the negative price effect). Let $\mathcal{E}_{Y,X}$ denote the elasticity of Y with respect to X. It follows that, for small changes in w_m and w_f, the approximate implied change in n^* is given by

$$\Delta n^* \approx \mathcal{E}_{num,w_m}\Delta w_m + \mathcal{E}_{num,wf}\Delta w_f - \mathcal{E}_{denom,w_f}\Delta w_f \tag{C.5}$$

where the elasticity terms are computed as follows:

$$\begin{aligned}
\mathcal{E}_{num,w_f} &= \frac{\partial\left(w_f + w_m\right)}{\partial w_f}\frac{w_f}{w_f + w_m} = \frac{w_f}{w_f + w_m} \\
\mathcal{E}_{num,w_m} &= \frac{\partial\left(w_f + w_m\right)}{\partial w_m}\frac{w_m}{w_f + w_m} = \frac{w_m}{w_f + w_m} \\
\mathcal{E}_{denom,w_f} &= \frac{\partial\left(p_n - \eta p_e\right)}{\partial w_f}\frac{w_f}{p_n - \eta p_e} = \mathcal{E}_{p_n,w_f}\frac{p_n}{p_n - \eta p_e}
\end{aligned}$$

and

$$\mathcal{E}_{p_n,w_f} \equiv \frac{\partial p_n}{\partial w_f}\frac{w_f}{p_n} = \frac{\phi^{\frac{1}{1-\rho}} w_f^{\frac{\rho}{\rho-1}}}{\phi^{\frac{1}{1-\rho}} w_f^{\frac{\rho}{\rho-1}} + (1-\phi)^{\frac{1}{1-\rho}} p_m^{\frac{\rho}{\rho-1}}} \in (0,1)\,.$$

The question is how optimal fertility varies across couples that represent different income deciles for a given year, or the same decile across years. These couples differ on w_m and w_f. From (C.5) we see that for n^* to decline across income deciles in 1980, as was observed in the data, the price effect of Δw_f must dominate the income effect of both Δw_f and Δw_m, where the Δ's are taken across consecutive income deciles. Moreover, for n^* to increase between 1980 and 2010 for couples representing high income deciles, the price effect due to Δw_f must yield to the income effect due to both Δw_f and Δw_m. In this case, the Δ's refer to changes over time for a fixed decile. Because the effect of w_m on n is always positive ($\mathcal{E}_{num,w_m} > 0$), we focus on investigating the effect due to w_f alone

$(\mathcal{E}_{num,wf} - \mathcal{E}_{denom,w_f})$. This is done in the two propositions to follow. However, bear in mind that to understand the profile of optimal fertility across income deciles or over time, we need to consider the combined effects of both w_m and w_f.

Proposition 2 . *(Monotonicity and limit of $\partial n^*/\partial w_f$). If $\rho \in (0,1)$, i.e. inputs in the home production are substitutes, then (a) $\partial n^*/\partial w_f$ is monotonically increasing in w_f and (b) strictly positive for a large enough w_f, i.e. $\lim_{w_f \to \infty} \partial n^*/\partial w_f > 0$.*

Proof. Proof of (a). Differentiating (C.3) with respect to w_f and omitting the positive constant term gives

$$\frac{\partial n^*}{\partial w_f} \propto \frac{(p_n - \eta p_e) - (w_f + w_m)\frac{\partial p_n}{\partial w_f}}{(p_n - \eta p_e)^2} = \frac{1 - \left(\frac{w_f + w_m}{w_f}\right)\mathcal{E}_{p_n,w_f}\frac{p_n}{p_n - \eta p_e}}{p_n - \eta p_e}.$$

The denominator is positive. To show that the ratio is monotonically increasing, it suffices to show that the negative term in the numerator is made up of positive and monotone decreasing functions of w_f. This is seen from obtaining a negative derivative for each of the product terms:

$$\frac{\partial}{\partial w_f}\left(\frac{w_f + w_m}{w_f}\right) = -\frac{w_m}{w_f^2} < 0,$$

$$\frac{\partial}{\partial w_f}\mathcal{E}_{p_n,w_f} = \frac{\left(\frac{\rho}{\rho-1}\right)\phi^{\frac{1}{1-\rho}} w_f^{\frac{\rho}{\rho-1}-1}}{\left(\phi^{\frac{1}{1-\rho}} w_f^{\frac{\rho}{\rho-1}} + (1-\phi)^{\frac{1}{1-\rho}}\, p_m^{\frac{\rho}{\rho-1}}\right)^2} < 0, \text{ when } \rho \in (0,1)\,,$$

and

$$\frac{\partial}{\partial w_f}\left(\frac{p_n}{p_n - \eta p_e}\right) = \frac{-\eta p_e \frac{\partial}{\partial w_f} p_n}{(p_n - \eta p_e)} < 0,$$

where the last inequality follows from showing that $\frac{\partial}{\partial w_f} p_n > 0$:

$$\frac{\partial p_n}{\partial w_f} = \frac{1}{A}\left[\alpha_1 w_f^{\sigma} + \alpha_2 p_m^{\sigma}\right]^{\frac{1}{\sigma}-1} \alpha_1 w_f^{\sigma-1} = \frac{\alpha_1}{A}\left[\alpha_1 + \alpha_2\left(\frac{p_m}{w_f}\right)^{\sigma}\right]^{\frac{1-\sigma}{\sigma}} > 0.$$

(b) Proof of (b). Because the limit of a product of functions is equal to the product of limits, we obtain

$$\lim_{w_f\to\infty}\left(\frac{w_f+w_m}{w_f}\right)\mathcal{E}_{p_n,w_f}\frac{p_n}{p_n-\eta p_e}=\lim_{w_f\to\infty}\left(\frac{w_f+w_m}{w_f}\right)\cdot\lim_{w_f\to\infty}\mathcal{E}_{p_n,w_f}\cdot\lim_{w_f\to\infty}\frac{p_n}{p_n-\eta p_e}.$$

Each limit can then be obtained. First,

$$\lim_{w_f\to\infty}\left(\frac{w_f+w_m}{w_f}\right)=1.$$

Second,

$$\lim_{w_f\to\infty}\mathcal{E}_{p_n,w_f}=\lim_{w_f\to\infty}\frac{\phi^{\frac{1}{1-\rho}}}{\phi^{\frac{1}{1-\rho}}+(1-\phi)^{\frac{1}{1-\rho}}\left(\frac{p_m}{w_f}\right)^{\frac{\rho}{\rho-1}}}=0,$$

which follows from $\lim_{w_f\to\infty}\left(\frac{p_m}{w_f}\right)^{\frac{\rho}{\rho-1}}=\infty$ implied by the assumption that $\rho\in(0,1)$. To derive the final limit, we first note that

$$\lim_{w_f\to\infty}p_n=\lim_{w_f\to\infty}\left[\phi^{\frac{1}{1-\rho}}w_f^{\frac{\rho}{\rho-1}}+(1-\phi)^{\frac{1}{1-\rho}}p_m^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}}=(1-\phi)^{-\frac{1}{\rho}}p_m>0$$

whenever $\rho\in(0,1)$. It follows that the third limit is

$$\lim_{w_f\to\infty}\frac{p_n}{p_n-\eta p_e}=\frac{(1-\phi)^{-\frac{1}{\rho}}p_m}{(1-\phi)^{-\frac{1}{\rho}}p_m-\eta p_e}\in(1,\infty).$$

The product of these three limits is zero. It follows that

$$\lim_{w_f\to\infty}\frac{\partial n^*}{\partial w_f}\propto\lim_{w_f\to\infty}\frac{1-\left(\frac{w_f+w_m}{w_f}\right)\mathcal{E}_{p_n,w_f}\frac{p_n}{p_n-\eta p_e}}{p_n-\eta p_e}=\frac{1}{(1-\phi)^{-\frac{1}{\rho}}p_m-\eta p_e}>0.$$

■

Corollary 1 *In the region of w_f where the solution is interior, n^* is either U-shaped or monotonically increasing in w_f.*

Proof. Observe from (C.2) that e^* is monotone increasing in w_f whenever e^* is interior. Thus, there exists a well-defined lowest wage marking the interior solution $\underline{w_f} \equiv \inf \{w_f | e^* (w_f) > 0\}$. If $\partial n^* / \partial w_f \left(\underline{w_f}\right) \geqslant 0$, i.e. n^* is non-decreasing near $\underline{w_f}$, then we know by proposition 2 that $\partial n^* / \partial w_f > 0$ for larger w_f. In this case, n^* is strictly increasing in the region of $w_f \geqslant \underline{w_f}$. If, however, $\partial n^* / \partial w_f \left(\underline{w_f}\right) < 0$, i.e. n^* is decreasing in w_f near $\underline{w_f}$, then we know by proposition 2 that $\partial n^* / \partial w_f$ will monotonically increase with w_f becoming strictly positive for a large enough w_f. In this case, n^* is a U-shape function of w_f in the region of $w_f \geqslant \underline{w_f}$. ■

D Education Robustness

There has been increasing interest in rising returns to education and rising education costs in the literature. We have so far abstracted from these issues, using the empirical relationship between income and college attainment in 1980 in order to control for changing education rates over time, instead focusing on differential fertility. Is it possible, however, that changes in college returns and costs could be driving changes in differential fertility? In principle, rising education costs could lead to more fertility through a quantity-quality tradeoff, potentially yielding changing patterns fertility by income. This effect might be mitigated by rising returns to education.

We now allow both the college premium, as described in (1), and education costs (p_e) to change over time. The time dependent parameters $p_{e,t}$ and ω_t capture the increase in the price of education and the rise in the college premium. The only other change we make to the setup of the model is that we replace (2) with:

$$u = \ln (c + \bar{c}) + \alpha \ln (n) + \tilde{\beta} \ln (w) . \tag{D.6}$$

That is, we introduce a constant $\bar{c}$ into the consumption function. This allows for non-homotheticity.[46]

[46] The calibration sets $\bar{c}$ close to 0 in the benchmark model, so we do not include it in the analysis there.

Relative to the calibration strategy described in Section 4, we only need to describe three things: how p_e changes over time; how ω is calibrated; how $\bar{c}$ is identified.

Beginning with p_e, we normalize $p_{e,1980} = 1$ as before. Although education expenditures map into all possible education-related expenditures per child, we take the stand that college education cost changes accurately describe general changes over time. We therefore choose to proxy the increase in the price of education by the increase in the effective price of college. Using institutional survey data available through the National Center for Education Statistics, we obtain that an annual cost of a public 4-year college is approximately \$6,400. This includes tuition and room & board, net of grants and scholarships. This quantity for the most recent year available is \$7,887, an increase of a 22%. We thus set $p_{e,2010} = 1.22$. ω in our model captures the lifetime return to college. This is different from the lifetime college premium which simply refers to the observed difference between the earnings of college graduates and other workers. Hendricks & Leukhina (2017) measure the role of ability selection in lifetime earnings premium (for the 1980 high school graduates) to be approximately a half of the observed college premium. The remaining half is the average return to college. Hence, we calibrate the return to college in 1980 and 2010 to the half of the observed cross-sectional lifetime premium (measured from the 1980 Census and 2010 ACS). Thus, we set $\omega^{1980} = 1.25$ and $\omega^{2010} = 1.40$). Finally, as the model is already greatly overidentified, $\bar{c}$ does not need extra moments for identification. We recalibrate the rest of the parameters as before. The calibrated parameters are as follows:

Notice that the change in the price of is somewhat lower than in the benchmark exercise while the pace of technological advancement (A) is somewhat faster. The other parameters are quite similar.

The results are quite similar. In the model, college attainment due to differential fertility rises modestly, by 0.4 percentage points, but when recalculating holding the cost of marketization constant, this statistic falls by 1.4 percentage points, leading to a total bias from ignoring marketization of 1.8 percentage points. While this result is somewhat weaker than the benchmark results, it is still quantitatively meaningful.

Parameter	Interpretation	Value
α	Weight on # children	0.15
β	Weight on quality of children	0.22
θ	Exponent π	0.60
b	Scaling	1.82
η	Basic edu.	0.47
ϕ	Share of mother's time	0.92
ρ	Elasticity wife/m	0.63
$\bar{c}$	Consumption constant	25.81
A_{1980}	TFP child production, 1980	4.30
A_{2010}	TFP child production, 2010	0.7 % annual growth
$p_{m,1980}$	Price of market substitutes 1980	1
$p_{m,2010}$	Price of market substitutes 2010	1.6% Annual decrease
$p_{e,1980,2010}$	Cost of education	1, 1.22
$\omega_{1980,2010}$	Returns to college degree	1.25, 1.40

Table D.2: Parameters- Education Robustness

E Normalization of Parameters

E.1 Normalizing p_e

Notice that in our model we can normalize $p_e = 1$ (or any other value), without affecting other meaningful quantities which are mapped to the data. At the interior solution we have

$$
\begin{aligned}
e^* &= \frac{p_n}{p_e}\frac{\beta\theta}{\alpha}\ln(\omega) - \eta \\
p_e e^* &= p_n\frac{\beta\theta}{\alpha}\ln(\omega) - p_e\eta
\end{aligned}
$$

The last equation shows that scaling up p_e by any factor, requires reducing e^* and η by the same factor to keep the product $p_e e^*$ unchanged, e.g. $\forall\varepsilon > 0$

$$
\begin{aligned}
p_e e^* &= p_e\varepsilon\frac{e^*}{\varepsilon} \\
p_e\eta &= p_e\varepsilon\frac{\eta}{\varepsilon}
\end{aligned}
$$

Only the product $p_e e^*$ enters the solution for n, so the solution to n will not change due to the scaling above. Finally, although e itself is meaningless, the quantity $\pi(e)$ is used to target college attainment rates in the data. However, the parameters inside $\pi(\cdot)$ can be scaled as follows, to keep it unchanged:

$$\pi\left(\frac{e}{\varepsilon}\right) = \ln\left(b\varepsilon^{\theta}\left(\frac{e}{\varepsilon} + \frac{\eta}{\varepsilon}\right)^{\theta}\right) = \ln\left(b\left(e+\eta\right)^{\theta}\right) = \pi(e)$$

Thus, the solution to the model, in terms of n and $\pi(e)$, is invariant to the following transformation of parameters:

$$\tilde{p}_e = p_e\varepsilon, \tilde{\eta} = \frac{\eta}{\varepsilon}, \tilde{b} = b\varepsilon^{\theta}, \tilde{e} = \frac{e}{\varepsilon} \ \forall\varepsilon > 0$$

E.2 Normalizing p_m

In this section we show that we can normalize p_m to any value, without affecting the key variables: p_n, t_f and mp_m. The solution to p_n from the cost minimization problem can be rewritten as follows:

$$\begin{aligned} p_n &= \frac{1}{A}\left[\phi^{\frac{1}{1-\rho}} w_f^{\frac{\rho}{\rho-1}} + (1-\phi)^{\frac{1}{1-\rho}} p_m^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}} \\ &= \left[A^{\frac{\rho}{1-\rho}}\phi^{\frac{1}{1-\rho}} w_f^{\frac{\rho}{\rho-1}} + A^{\frac{\rho}{1-\rho}}(1-\phi)^{\frac{1}{1-\rho}} p_m^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}} \end{aligned}$$

First we show that when scaling p_m by $\varepsilon > 0$, we can find adjustments to A and ϕ to keep p_n unchanged:

$$\begin{aligned} \tilde{A}^{\frac{\rho}{1-\rho}}\tilde{\phi}^{\frac{1}{1-\rho}} w_f^{\frac{\rho}{\rho-1}} + \tilde{A}^{\frac{\rho}{1-\rho}}\left(1-\tilde{\phi}\right)^{\frac{1}{1-\rho}}(p_m\varepsilon)^{\frac{\rho}{\rho-1}} &= A^{\frac{\rho}{1-\rho}}\phi^{\frac{1}{1-\rho}} w_f^{\frac{\rho}{\rho-1}} + A^{\frac{\rho}{1-\rho}}(1-\phi)^{\frac{1}{1-\rho}} p_m^{\frac{\rho}{\rho-1}} \\ \left[\tilde{A}^{\frac{\rho}{1-\rho}}\tilde{\phi}^{\frac{1}{1-\rho}} - A^{\frac{\rho}{1-\rho}}\phi^{\frac{1}{1-\rho}}\right] w_f^{\frac{\rho}{\rho-1}} &= \left[A^{\frac{\rho}{1-\rho}}(1-\phi)^{\frac{1}{1-\rho}} - \tilde{A}^{\frac{\rho}{1-\rho}}\left(1-\tilde{\phi}\right)^{\frac{1}{1-\rho}}\varepsilon^{\frac{\rho}{\rho-1}}\right] p_m^{\frac{\rho}{\rho-1}} \end{aligned}$$

Since w_f and p_m are fixed at arbitrary values, we have the following system with $\tilde{A}$ and $\tilde{\phi}$:

$$\begin{aligned}\tilde{A}^{\frac{\rho}{1-\rho}}\tilde{\phi}^{\frac{1}{1-\rho}} &= A^{\frac{\rho}{1-\rho}}\phi^{\frac{1}{1-\rho}} \\ \tilde{A}^{\frac{\rho}{1-\rho}}\left(1-\tilde{\phi}\right)^{\frac{1}{1-\rho}}\varepsilon^{\frac{\rho}{\rho-1}} &= A^{\frac{\rho}{1-\rho}}\left(1-\phi\right)^{\frac{1}{1-\rho}}\end{aligned}$$

Dividing through, and solving for $\tilde{\phi}$:

$$\begin{aligned}\frac{\tilde{A}^{\frac{\rho}{1-\rho}}\left(1-\tilde{\phi}\right)^{\frac{1}{1-\rho}}\varepsilon^{\frac{\rho}{\rho-1}}}{\tilde{A}^{\frac{\rho}{1-\rho}}\tilde{\phi}^{\frac{1}{1-\rho}}} &= \frac{A^{\frac{\rho}{1-\rho}}\left(1-\phi\right)^{\frac{1}{1-\rho}}}{A^{\frac{\rho}{1-\rho}}\phi^{\frac{1}{1-\rho}}} \\ \left(\frac{1-\tilde{\phi}}{\tilde{\phi}}\right)^{\frac{1}{1-\rho}} &= \left(\frac{1-\phi}{\phi}\right)^{\frac{1}{1-\rho}}\varepsilon^{\frac{\rho}{1-\rho}} \\ \frac{1-\tilde{\phi}}{\tilde{\phi}} &= \left(\frac{1-\phi}{\phi}\right)\varepsilon^{\rho} \\ \tilde{\phi} &= \frac{1}{1+\left(\frac{1-\phi}{\phi}\right)\varepsilon^{\rho}} = \frac{\phi}{\phi+\left(1-\phi\right)\varepsilon^{\rho}} \in [0,1]\end{aligned}$$

Notice that if $\varepsilon = 1$, then $\tilde{\phi} = \phi$. If $\varepsilon > 1$, then $\tilde{\phi} < \phi$, which does not make sense. Finally, solving for $\tilde{A}$ gives

$$\begin{aligned}\tilde{A}^{\frac{\rho}{1-\rho}} &= A^{\frac{\rho}{1-\rho}}\left(\frac{\phi}{\tilde{\phi}}\right)^{\frac{1}{1-\rho}} = A^{\frac{\rho}{1-\rho}}\left[\phi+\left(1-\phi\right)\varepsilon^{\rho}\right]^{\frac{1}{1-\rho}} \\ \tilde{A} &= A\left[\phi+\left(1-\phi\right)\varepsilon^{\rho}\right]^{\frac{1}{\rho}}\end{aligned}$$

Thus, scaling p_m by a factor $\varepsilon > 0$, and adjusting the share parameter and productivity as above, keeps p_n fixed.

Now, we express t_f and m in terms of p_n

$$\left(Ap_n\right)^{\frac{1}{\rho-1}} = \left[\phi^{\frac{1}{1-\rho}}w_f^{\frac{\rho}{\rho-1}} + \left(1-\phi\right)^{\frac{1}{1-\rho}}p_m^{\frac{\rho}{\rho-1}}\right]^{\frac{1}{\rho}}$$

Plug the bracketed term into t_f and m

$$
\begin{aligned}
t_f^n &= \frac{\left(\frac{\phi}{w_f}\right)^{\frac{1}{1-\rho}}}{A\left[\phi^{\frac{1}{1-\rho}} w_f^{\frac{\rho}{\rho-1}} + (1-\phi)^{\frac{1}{1-\rho}} p_m^{\frac{\rho}{\rho-1}}\right]^{\frac{1}{\rho}}} n = \frac{\left(\frac{\phi}{w_f}\right)^{\frac{1}{1-\rho}}}{A\left(Ap_n\right)^{\frac{1}{\rho-1}}} n = \frac{\left(\frac{\phi}{w_f}\right)^{\frac{1}{1-\rho}} \phi^{\frac{1}{\rho-1}}}{A^{\frac{\rho}{\rho-1}} \phi^{\frac{1}{\rho-1}} p_n^{\frac{1}{\rho-1}}} \\
m &= \frac{\left(\frac{1-\phi}{p_m}\right)^{\frac{1}{1-\rho}}}{A\left[\phi^{\frac{1}{1-\rho}} w_f^{\frac{\rho}{\rho-1}} + (1-\phi)^{\frac{1}{1-\rho}} p_m^{\frac{\rho}{\rho-1}}\right]^{\frac{1}{\rho}}} n = \frac{\left(\frac{1-\phi}{p_m}\right)^{\frac{1}{1-\rho}}}{A\left(Ap_n\right)^{\frac{1}{\rho-1}}} n = \frac{\left(\frac{1}{p_m}\right)^{\frac{1}{1-\rho}}}{A^{\frac{\rho}{\rho-1}} \left(1-\phi\right)^{\frac{1}{\rho-1}} p_n^{\frac{1}{\rho-1}}}
\end{aligned}
$$

We showed that the term $A^{\frac{\rho}{\rho-1}} \phi^{\frac{1}{\rho-1}}$ is unchanged due to scaling of p_m, which means that t_f is unchanged. However, the term $A^{\frac{\rho}{1-\rho}} \left(1-\phi\right)^{\frac{1}{1-\rho}}$ increases by a factor of $\varepsilon^{\frac{\rho}{1-\rho}}$. Thus, the effect of scaling p_m by a factor of $\varepsilon > 0$, and adjusting A and ϕ to keep p_n constant, gives:

$$
mp_m = \frac{\left(\frac{1}{p_m \varepsilon}\right)^{\frac{1}{1-\rho}} \left(p_m \varepsilon\right)}{A^{\frac{\rho}{\rho-1}} \left(1-\phi\right)^{\frac{1}{\rho-1}} \varepsilon^{\frac{\rho}{1-\rho}} p_n^{\frac{1}{\rho-1}}} = \frac{p_m^{\frac{\rho}{1-\rho}} \varepsilon^{\frac{\rho}{1-\rho}}}{A^{\frac{\rho}{\rho-1}} \left(1-\phi\right)^{\frac{1}{\rho-1}} \varepsilon^{\frac{\rho}{1-\rho}} p_n^{\frac{1}{\rho-1}}} = \frac{p_m^{\frac{\rho}{1-\rho}}}{A^{\frac{\rho}{\rho-1}} \left(1-\phi\right)^{\frac{1}{\rho-1}} p_n^{\frac{1}{\rho-1}}}
$$

Notice that ε cancels out, and therefore does not affect mp_m.