Assessing Default Risk of Israeli Companies Using a Structural Model[1]

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הערכת סיכון חדלות הפירעון של חברות בישראל באמצעות מודל מבני

אנה ססי - ברודסקי

תקציר

מטרתו של מחקר זה הינה לאמוד את ההסתברויות לחדלות פירעון של חברות ולבחון באיזו מידה הסיכון לחדלות פירעון מסביר את מחירי השוק של אג״ח הנסחרות בבורסה. המאמר בוחן שאלות אלו במסגרת המודל שהציע Merton לתמחור חוב סחיר והוא מיושם על מספר רב של חברות שמניותיהן נסחרו בבורסה בת״א בשנים 2004-2010. חברות אלה מוצלבות עם מאגר הכולל את כל החברות אשר נכנסו במשבר הפיננסי שהחל בשנת 2008 לתהליך של הסדר חוב. מהתוצאות עולה כי המודל אכן מספק יכולת לזהות חברות בעייתיות תקופה מראש, אולם תוך טעויות לא מבוטלות בזיהוי. כמו כן עולה מהמחקר כי סיכון חדלות הפירעון מסביר רק חלק קטן מאד מהמרווח הנצפה בשוק על מחיר ה-אג״ח הקונצרניות, בעוד שסיכוני שוק אחרים לצד הנחות ביחס לעלויות פשיטת רגל יכולים לתרום ליכולת ההסבר.

Assessing Default Risk of Israeli Companies Using a Structural Model

Ana Sasi-Brodesky

Abstract

The aim of this study is to assess the default probabilities of public companies and to explore the extent to which these probabilities explain bond prices in the market. These questions are examined under the framework that was proposed by Merton to price corporate debt, applied on a large number of public companies listed on the Tel Aviv Stock Exchange in the period 2005-2010. The data is cross-checked with a list of companies that underwent debt repayment difficulties during that period, which includes the 2008 financial crisis. The results show that the model is able to identify problematic firms some time in advance, though with significant errors in identification. The results also show that default risk can explain only a small part of the corporate bond spreads, while other market risks along with the introduction of bankruptcy cost assumptions can contribute considerably to the explanatory power of the model.

1. INTRODUCTION

This study focuses on the model proposed by Merton (1974) to price corporate debt. It tests the model's ability to estimate the default probability of public companies, and evaluates to what extent this probability explains the market price of corporate bonds. In the first stage, the model is applied to a database of companies whose equity was listed for trading on the Tel Aviv Stock Exchange (TASE) in 2005-2010, and cross-checks the data with cases of defaults/debt reorganizations of companies during this period.

The results indicate that with the help of default probabilities derived from the model for Israeli companies, companies that encountered difficulties as a result of the 2008 crisis can be singled out as early as 2006 (2-3 years before the event itself), but with notable errors in identification.

However, the Merton Model does not just estimate the default probability of a company; it also proposes a theoretical framework for calculating the debt price that reflects this risk. Therefore, in the second stage, the study also examines the relationship between the spreads derived from the model and actual bond prices observed in the market. As in previous studies, the model spreads obtained are significantly smaller than those observed in the market, even when the existence of bankruptcy costs is assumed. For this purpose, the effect of adding other risk factors on bond spreads in Israel was tested.

Based on the two uses of the model mentioned above, the study is divided into two parts: Part 1 describes in detail the Merton model, how it was applied in the study, and its success in forecasting defaul for Israeli companies. Part 2 presents the theoretical debt price derived from the model in Part 1, and how it compares with the bond prices observed in the market. A Summary of the study is presented at the end of Part 2.

2. ASSESSING DEFAULT PROBABILITY

a. Introduction to Part 1

Part 1 tests the ability of the Merton Model (1974) to assess default probability for public companies. This part is organized as follows: Section 2.2 reviews the relevant literature. Section 2.3 describes the Merton Model in detail and discusses its implementation in the study. Section 2.4 describes the database used for the study. Section 2.5 presents the risk indices obtained from the model and their ability to predict default events for Israeli companies.

b. A Review of the Literature

Assessment of a company's default risk and the effect of that risk on the price of its debt have been discussed extensively in the research literature. One of the generally accepted models for predicting default is Altman's Z-Score (1968), which is based on balance sheet risk indices and historical probabilities of default. Zilberman, Hachmon, Kahn, and Gur Gershgoren (2012) found that Altman's original index was not successful in predicting default for Israeli companies in 2007-2010. Based on Altman's methodology, they built a new index for predicting default for non-manufacturing companies in the Israeli bond market, and obtained better results in discriminating between companies that entered into a debt reorganization and other companies a year or two before a default event, while the results of the model one quarter before the event were not as good.

Other models rely on bond market prices to predict default; in these models, the spread between the return on a corporate bond and the return on a government bond is taken to represent the expected loss to the bondholder from default. Empirical studies, however, have shown that the risk of default, whether theoretical or historical, explains only a small portion of the actual observed spreads (Collin-Dufresne, Goldstein, & Martin (2001), Elton et al. (2001)). Therefore, attributing the entire spread to default risk exaggerates the scope of this risk.

Merton (1974) proposed a structural framework (in contrast to the Altman model, for example, which does not rely on corporate finance theory, but proposes variables that an empirical test shows to be correlated with the ability to predict default) for assessing and pricing a company's risk of default and evaluating the expected loss to the debt holders, which relies on the theoretical basis of options theory. Black and Scholes (1973) and Merton (1973) developed a relatively simple formula for pricing options. As will be demonstrated later in the paper, they pointed out the similarity between a company's payoffs to its creditors and equity holders and the payoffs from holding options on the company's assets, thereby laying the foundations for the Merton Model (1974) discussed in this article. This model continues to serve as the most well-known and basic structural model for evaluating company debt and, to the best of my knowledge, no attempt has yet been published to apply it to market data in Israel.

Tudela and Young (2003) applied an expanded version of the Merton model—in which default can occur at any stage during the lifespan of the debt, not only at maturity as assumed in the original model—to non-financial companies in the UK in 1990-2001. They concluded that the model discriminates rather well between companies that defaulted and those that did not a year before the event, with relatively small errors of classifying companies in the wrong group.

Geske (1977) also developed an expanded model based on Merton's Model, which makes it possible to price coupon-paying bonds. Delianedis & Geske (2003) applied this model to a sample of 600-1,000 American companies in 1988-1999, and showed that the risk indices obtained from the model can predict rating migrations of the issuing company. Moody’s KMV (hereafter – “MKMV”), one of the world’s largest rating companies, has adopted the model’s theoretical framework with its own additions (Crosbie and Bohn (2003)) and uses it for daily tracking of default risks of companies around the world.

c. The Merton Structural Model for Assessing Credit Risk of Public Companies

1. *Background*

The model proposed by Merton (1974) describes a simple company structure, under which the market value of the company's assets (V) equals the market value of its equity (S) together with the market value of its debt (D), under the assumption that all the debt is paid off at a single date in T years, and the debt does not pay coupons:

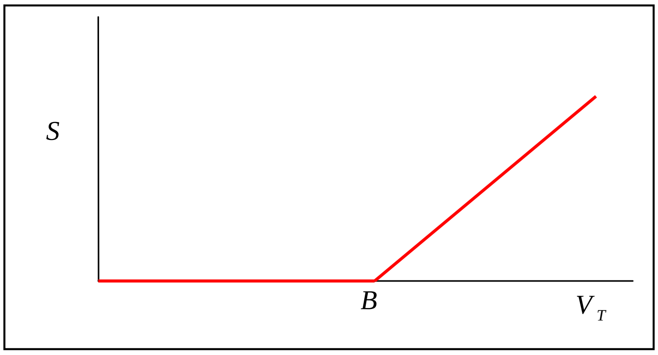
(1) $V = S + D$

The risk of default is due to the fact that the company’s assets, V, are subject to a stochastic process. This process is described in the model by a random walk, similar to the path of the return on equity in the Black and Scholes option pricing model:

(2) $$\frac{dV}{V} = \mu dt + \sigma_V \varepsilon \sqrt{dt} \qquad \varepsilon \sim N(0,1)$$

Since the future value of the company’s assets is subject to uncertainty, the company’s ability to repay its debt at maturity is not certain. As we can see in Figure 1, the proceeds from holding the company’s equity, S, are compared to holding a call option on the company’s assets ,V, with a strike price equal to the face value of its debt, B.

Figure 1: Payment to the Equity holders



If the value of the company's assets at maturity is less than its debt liabilities ($V_T < B$), then the equity holders receive nothing, and all the company's assets go to the creditors. On the other hand, if the company has sufficient assets to meet its obligations ($V_T > B$), then the equity holders receive all the assets that are left after the debt is paid, and the company is liquidated. It therefore follows that the payment to the equity holders at time T is $\max(V_T - B, 0)$.

In a world in which:

a. The capital markets are perfect – there are no transaction costs or taxes, and all individuals have free access to all existing information;
b. A default event incurs no accompanying costs other than a decline in the value of the company's assets;
c. All individuals can borrow and short sell with no restrictions;

The Black and Scholes option pricing formula can be used to describe at any given moment the relationship between the company's equity value, the market value of its assets, and the standard deviation of those assets, as follows:

(3) $$S = VN(d_1) - Be^{-r_{f,T}T}N(d_2)$$ [1]

$$d_1 = \frac{\ln(V/B) + (r_{f,T} + \frac{1}{2}\sigma_V^2)T}{\sigma_V\sqrt{T}}$$

$$d_2 = d_1 - \sigma_V\sqrt{T}$$

The company's assets have an average return of μ (in a risk-averse world, $\mu > r_f$), and this return has a normal probability distribution, meaning that V, the company's assets, has a

[1] N() denotes the cumulative probability function of the standard normal probability distribution.

lognormal probability distribution. The following chart depicts the probability distribution of the company's assets at time T:

Figure 2: The Probability Distribution of the Company's Assets at Time T

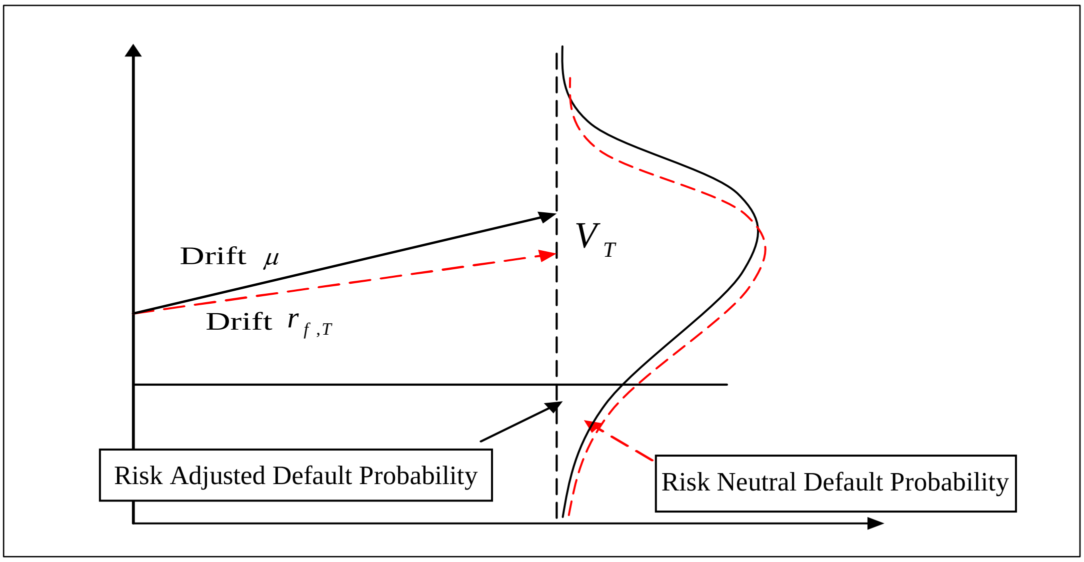


Figure 2 presents two probability distributions for V. One is the distribution assuming that the return on the assets is equal to μ, and is shown by a solid line. The second is the distribution assuming that the expected return on assets is $r_{f,T}$, and is shown by a dotted line. This probability, called the risk neutral default probability (RNDP), equals $1-N(d_2)=N(-d_2)$, where d_2 is taken from Equation (3) above. In order to obtain the "true" probability of default, the assumption of a return equal to the risk-free interest rate should be replaced by the true return on the assets, meaning that the risk-adjusted probability of default is:

(4) $$N(-d_2^{\mu}) = N\left(-\frac{\ln(V/B)+(\mu-\frac{1}{2}\sigma_V^2)T}{\sigma_V\sqrt{T}}\right)$$

2. *Application of the Model*

Before applying the model to the data, we must examine a number of assumptions upon which the model is based that may turn out to be problematic. I will also discuss the limitations concerning the availability of the data.

One of the main parameters that affect the result of the calculation of equation 3 and influence the derived value of V, is the selection of B—the face value of the company's debt. In the Merton Model, this value is simply equal to the total company debt which, based on the model's assumptions, does not pay coupons and is paid off at a single date, T. However, in reality, companies generally issue debt that pays periodic coupons and that is issued for various maturity ranges, and that debt is distributed among creditors with a certain order of preference.

In addition, since the company needs to meet coupon payments throughout the debt's lifespan, a default may happen before T if the company is unable to meet one of the payments. Alternatively, and as opposed to what is described in the model, in certain cases, creditors initiate a takeover of the company when they fear that the company's assets are insufficient for repaying the debt and that the company has low chances of increasing its assets to a sufficient level by the maturity date.

So how can we deal with this problematic nature? MKMV claims that in order to obtain the best performance of the model in actuality, the most appropriate value for B is the company's total current debt and half of its long-term debt (including expected interest payments). In addition, MKMV does not look at the debt maturity date, but calculates the probability derived from the model for a series of ranges – one year, two years, etc. In other studies based on the structural model, the value of B is set endogenously as part of the model's solution. In order to deal with the problem of multiple maturity dates, Geske (1977) proposed specifying the debt coupon payments as compound options. However, Geske and Delianedis (2003) found that the default probability generated by the original simple model that has only non-coupon debt with a single maturity date is not materially different from the probability generated by the Geske (1977) model that allows for a number of maturity dates.

In order to stick to the theoretical lines of the original model, and in light of the findings of Geske and Delianedis, I have chosen to specify B as the face value of total company debt, without coupon payments, redeemed at a single date, T, which is set to be equal to the duration of the debt.

Another problem in applying the model is the fact that the return on the assets, μ, is unknown. There are methods in the literature that attempt to calculate this return. For instance, MKMV simultaneously assesses the series of company assets, V, their return, μ, and the standard deviation of the assets, σ_V.

In this study, I have chosen not to attempt to measure μ, and instead to look at the risk-neutral probability distribution of V—RNDP—assuming that the return on the assets is the risk-free return. In the "real", risk adjusted world, RNDP will always constitute an upper bound for the real probability of default. Even though I calculate the risk-neutral default probability, there is a basis to assume that RNDP contains information of relevance for default risk adjusted to the company's credit risk. In particular, when comparing companies with similar characteristics from the same industrial sectors, in which it is reasonable to assume that the return on their assets is similar, a higher RNDP will correspond to a company with a higher probability of default. In addition, RNDP and the "true" probability of default respond similarly to changes in the company's characteristics: both rise with an increase in the standard deviation of the assets or a rise in liabilities, and fall with an increase in the company's assets or a rise in the risk-free interest rate. In addition, Delianedis and Geske (2003) showed that RNDP enables the prediction of future changes in the company's credit risk (a change in rating or default).

In the event that all of the company's debt is traded in the market and D is known, V can then be directly derived from Equation (1), and σ_V can be derived from Equation (3), leading to RNDP. However, when not all of the debt is traded, both V and σ_V are unknown, and Equation (3) itself is not sufficient to extract the RNDP. I have therefore used the company's stock data in order to derive σ_V. Since the equity value in the model is a function of the assets, which are distributed according to the outline in Equation (2), the use of Ito's Lemma, which makes it possible to find the differential of a function of a random variable, generates the following connection:

(5) $$\sigma_S = \frac{\partial S}{\partial V}\frac{V}{S}\sigma_V = N(d_1)\frac{V}{S}\sigma_V$$

Equation (5) shows the instantaneous connection between the standard deviation of the equity and the standard deviation of the assets, as described by Jones et al (1984).

The d_2^{μ} value from Equation (4) is also called Distance to Default (DD), since it describes the "quantity" of standard deviations between the company's current situation and a default situation. According to the model's assumption, this value is normally distributed. MKMV claims that the assumption of normal distribution does not properly describe the actual default distribution. In particular, they claim that in historical data, we can see that the default probability distribution as a function of the DD value has a longer tail than that of the normal distribution. For this reason, MKMV uses historical default data to find the default probability for each DD value (Crosbie and Bohn (2003)). MKMV asserts that this mapping is a one-to-one correspondence, i.e. the default risk factor derived from the model contains full information about the company's default risk, with no adjustment being necessary for the sector, company size, or geographic location. Since I have no historical data on the connection between DD and default, I am continuing in the framework of the original model, assuming normal distribution of DD. Since MKMV found that the mapping between the theoretical probability and the historical distribution is a one-to-one correspondence, the theoretical probability should provide ordinal information regarding the default risk ("which company is more at risk") even if it is not cardinal (it is impossible to relate to theoretical probability level as equal to actual probability of default).

3. *The Database*

The database used to test the model in this study includes 277 public companies whose stocks were traded on the TASE in January 2004-July 2010. The final sample includes companies that meet the minimal liquidity condition—an average of at least six trading days per month, a limitation which shrinks the database by 30 percent. The sample includes only companies with public debt—even though bond data is not necessary in order to assess the model—because the default / reorganization pool includes only companies where at least part of their debt is held by the public. 2004 was used only for estimation of the equity's volatility, not for the subsequent tests and regressions, which were conducted for the January 2005-July 2010 period. There were no companies in the sample that entered debt reorganization before 2008.

The companies' quarterly and yearly balance sheet data up until 2008 appear in the Duchas system, while after that, the data appear in the Magna system, excluding companies from the banking and insurance sector.[2] Total debt, B, equals the company's total current and long-term liabilities, according to the discussion in the previous section. Time to maturity, T, equals the MacCauley duration calculated for current and long-term liabilities under the assumption that the current liabilities will be repaid within six months (the middle of the period to which the current liabilities refer), and the long-term liabilities will be paid in four years. This calculation of the duration makes it possible to distinguish between a company with large current liabilities and few long-term liabilities, which is more risky in

[2] In addition, the accounting data do not include dual-listed companies since the switch to international accounting standards, i.e. in the fourth quarter of 2007 or the beginning of 2008.

the short term, and a company with the same total liabilities, but a larger proportion of long-term liabilities. The selection of four years as the maturity of the long-term liabilities is to a large extent arbitrary, because there is no available data for the maturity of the non-traded part of the debt. At the same time, a test was conducted to see how much effect this decision has on the results, and alternative terms to maturity for long-term liabilities were tried. While the decision regarding the term to maturity of the long-term liabilities affects the probabilities of default obtained from the model, it does not affect the degree of success in discriminating between problematic companies and stable ones (see Figure 4b below).

The market value of the company's equity is taken from TASE trading figures, and the annual standard deviation of equity, σ_S, is estimated using an Exponentially Weighted Moving Average (EWMA), with a decay factor (also called the smoothing constant) of 0.88. The estimation of the standard deviation is based on the weekly returns on each company's equity. This calculation tacitly assumes that the expected return is zero. The decay factor was estimated using maximum likelihood method on the weekly return on the Tel Aviv 25 Index (the 25 stocks with the highest market value on the Tel Aviv Stock Exchange) from January 2004- July 2010,[3] in accordance with the sample period, and is used to estimate the standard deviation of all the companies in the sample:

(6) $$\sigma_{S,t}^2 = \lambda\sigma_{S,t-1}^2 + (1-\lambda)u_{S,t-1}^2$$
$$\lambda = 0.88$$

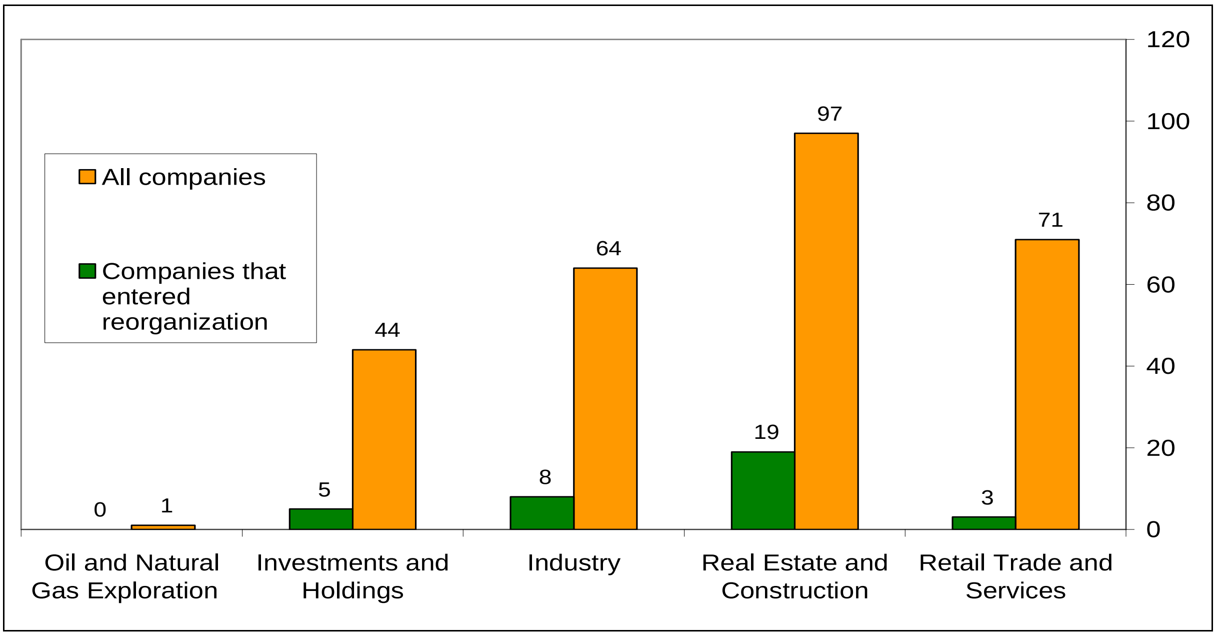
Where σ_S is the standard deviation of equity and u_S is the return.

In contrast, MKMV gives greater weight to the historic returns on equity in determining the volatility: the standard deviation of equity for US and Europe based companies is estimated using weekly returns of the past three years, and for other geographic regions using five years of monthly returns. Calculating the standard deviation using the EWMA model makes it possible to give greater weight to recent developments in the stock market and to derive the investors' most up-to-date expectations, while the historical calculation used at MKMV gives greater weight to the company's long-term performance. Later, we will see from the results that the market bond price is indeed affected to a great extent by developments in the stock market, and responds to changes in risk in this market. The risk-free interest rate is taken from the zero yield curve calculated at the Bank of Israel.

[3] For further explanation of the methods of estimating standard deviation, see J.C. Hull, "Options, Futures, and Other Derivatives," Chapter 15.

Up until July 2010, 35 companies in the database had announced difficulties in meeting their debt obligations.[4] The date of default was selected as the earliest of (1) a delay in the payment of interest; (2) a petition by the company to issue a stay of proceedings; (3) a demand by the bondholders for immediate repayment of the debt; (4) an announcement by the company that it wishes to negotiate with the bondholders on debt reorganization; or (5) the appointment of a bondholders delegation for the purpose of negotiation. All of the default events in the sample took place from 2008 onwards. As of July 2012, five of the 35 companies had reached liquidation/deletion, while the other thirty had reached a reorganization of debt agreement with the creditors. Negotiations on debt reorganization lasted an average of a year and a half. Most of the companies continued to publish statements, and trading in their equity continued even after notice of their difficulties. Even so, I have excluded these companies from the sample at this stage, since trading was not always continuous, and since the stock price reflected, inter alia, various expectations concerning the results of the negotiations, and not just the state of the company's assets.

Figure 3: The Probability Distribution of the Companies in the Sample



[4] One of the companies is omitted because it did not have enough observations. A list of the companies that encountered difficulties is found in Appendix 2.

4. *The Model's Ability to Identify Companies that Later Encountered Difficulty Repaying Their Debt*

In this section, I tested whether the risk indices obtained from the model, primarily the RNDP, indicate rising credit risk, and how long in advance the index changes enough to indicate a problem. Table 1 displays the basic statistics with respect to the results of the model, while Figures 4A and 4B below present the median and the average of RNDP over the sample period.

Table 1: Basic Statistics of the Sample Observations

	Average	Median	Standard Deviation
Assets (Millions of NIS)	3,347	467	9,859
Leverage(Be^{-rT}/V)	0.70	0.68	1.04
Standard Deviation in Assets	0.24	0.16	0.38
RNDP	12.96%	2.87%	21.36%

This table presents the basic statistics with respect to the results of the RNDP estimate. The assets, standard deviation in assets and RNDP are estimated within the model.

Figure 4A: Development of RNDP Median and Average Over Time

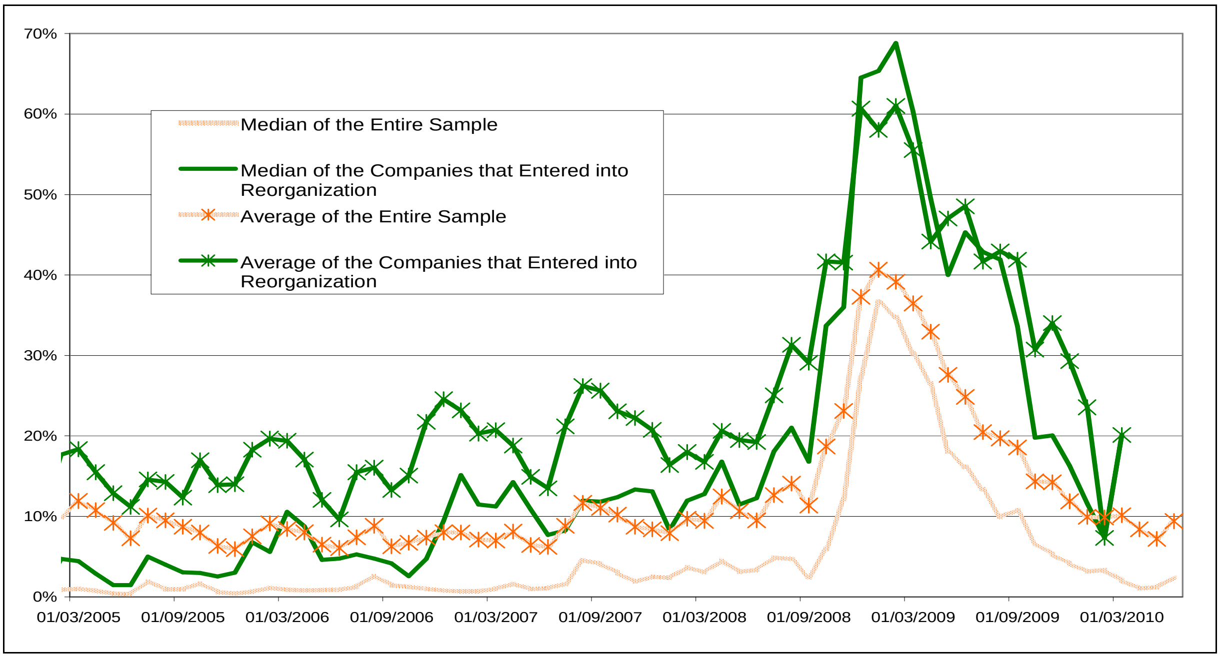
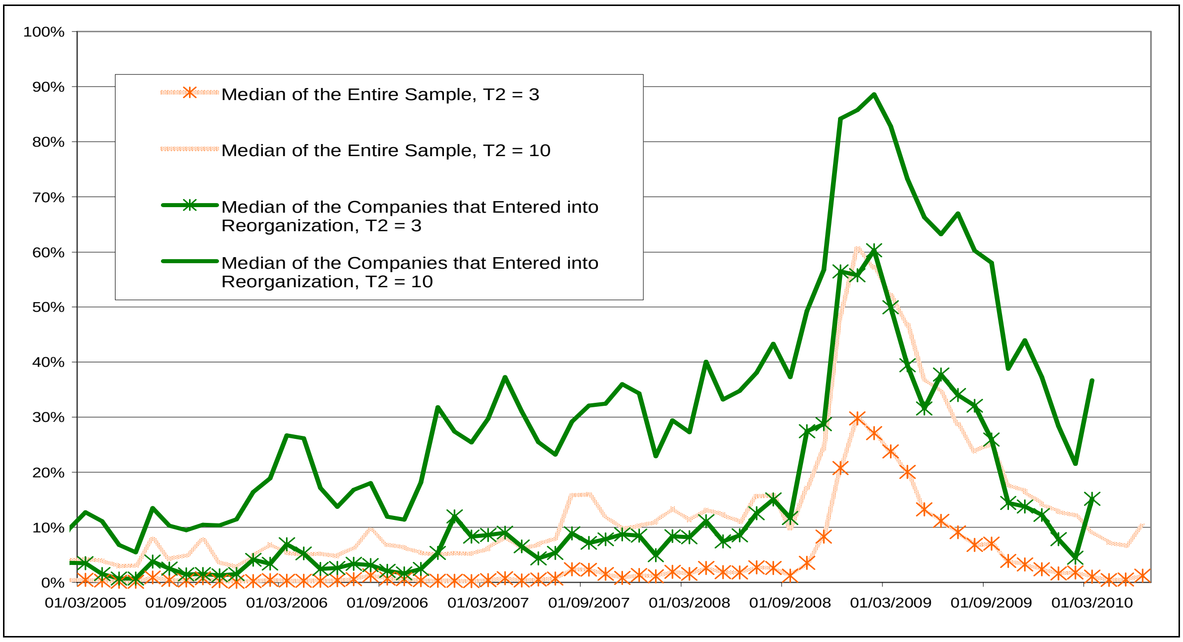


Figure 4B: Sensitivity of the RNDP Median to the Term to Maturity of Long-Term Liabilities



The number of companies in the group of companies that entered into reorganization declined consistently after October 2008 (once a company entered into reorganization, it was removed from the sample), so that the last observation in the chart, in March 2010, actually includes only two companies. The charts show that the risk indices did rise markedly around the peak crisis period. The indices rose for the group of companies that entered into reorganization and for the other companies as well. In addition, the gaps in levels between the two groups widened during the crisis.

Figure 4B displays the sensitivity of the RNDP calculation to the term to maturity of the long-term liabilities (as discussed in Section 4.2 of this article). We can see that extending the term to maturity increases the probability for both groups, and shortening it decreases the probability. At the same time, the chart clearly shows that the model's ability to discriminate between the two groups is not affected by the selection of the term to maturity of long-term liabilities, as reflected in the statistical tests presented later on.

In order to check whether the model is successful in discriminating between companies that entered into difficulties and those that did not, the RNDP levels of all companies were checked for December 2006, December 2007, and December 2008, while a majority of the companies in the sample announced that they were having difficulties during 2009.

A non-parametric test was conducted to check the hypothesis that the RNDP values of the companies that entered into difficulties were higher than the corresponding values of the companies that did not enter into difficulties (the Mann-Whitney U test).[5] For the entire sample, the results of this test were significant for all three periods: at the end of 2006, at the end of 2007, and at the end of 2008. When conducted only for the real estate sector, the results of this test were significant only for December 2007.

Table 2: Mann-Whitney U Test

	The entire sample		Real Estate	
	Observations (entered / did not enter into difficulties)	p-value	Observations	p-value
December 2006	29/189	0.00	17/59	0.18
December 2007	31/214	0.00	18/69	0.09
December 2008	28/230	0.00	15/75	0.18

This table presents the results of the non-parametric test (i.e. there is no assumption regarding the distribution of probability of the observations in the sample). This test was conducted to check the hypothesis that the RNDP values of the companies that entered into difficulties where higher than the corresponding values of the companies that did not enter into difficulties (Mann-Whitney U test). The test was conducted at three different points in time – two years before the crisis, one year before it, and during it. The two left-hand columns reconstruct the test for companies in the real-estate sector only.

At this stage, a logit regression was run for the binary variable of entering into debt repayment difficulties. The regression was run three times, each time with a single explanatory variable – the company's RNDP in December 2006, December 2007, and December 2008.

Table 3: Results of the Logit Regression

	Interceptor	RNDP coefficient	What does the RNDP need to be so that the chance of encountering difficulties will be 10 percent?
December 2006	-2.32	3.81	3.29%
P-value	0.000	0.000	
Pseudo R^2	0.11		
December 2007	-2.45	4.39	5.67%
P-value	0.000	0.000	
Pseudo R^2	0.10		
December 2008	-3.24	2.39	43.79%
P-value	0.000	0.000	
Pseudo R^2	0.09		

This table presents the results of the logit regression of RNDP values on the binary variable of encountering or not encountering debt repayment difficulties. As we can see, in 2008, the same chance of entering financing difficulties can be applied to a much higher value of RNDP than its value a year or two earlier, against the background of an increase in the RNDP values for all types of companies during this period.

As can be seen in the table, in each of the three regressions, the effect of RNDP on the probability of entering into difficulties is significant.

[5] The RNDP values do not have a normal probability distribution; they have a positive skewness.

Next, I assessed the model's ability to discriminate between companies that entered into difficulties and those that did not. For this purpose, I examined Type I errors (the number of companies that entered into reorganization but that the model classified as safe) and Type II errors (the number of companies that did not enter into reorganization, but that the model classified as problematic companies) made by the model at different levels of "decision rules," and in different periods. Table 4 displays the results of the test, in which a different threshold of the RNDP distribution was selected each time as the "decision rule." When a smaller proportion of the distribution (higher threshold) was considered to include problematic companies, there were more Type I errors and fewer Type II errors.

Table 4A: Results of the Test of the Model's Discriminatory Ability (percent)

Sample	Type of Error	Threshold		
		50%	40%	30%
	I	20.7	27.6	41.4
December 2006	II	45.5	34.9	25.4
	I	20.0	22.6	41.9
December 2007	II	42.9	34.6	25.7
	I	19.4	25.0	39.3
December 2008	II	43.6	35.7	26.1

This table presents the model's ability to discriminate between safe companies and problematic companies during various periods before the default event. For each period, the RNDP observations are divided into two groups: The observations in the part of the distribution above the "decision threshold" are classified as problematic companies, and all of the observations below the threshold are classified as safe. Later on, I examined how many companies in each group were mistakenly classified. A Type I error notes companies that the model classified as safe and which entered into default. A Type II error notes companies that the model classified as problematic and which did not later encounter financing difficulties.

The table shows that the best threshold for discriminating between safe and problematic companies was around the upper 40 percent of the probability distribution. The model's discriminatory ability was best at the end of 2007, but it still incorrectly classified about one-third of the companies in each group. It is worth noting that the model's ability to discriminate is similar to the performance of the Altman index. In particular, Zilberman et al. (2012) divided the results of the Altman index into three groups: safe companies, a gray area, and companies in distress. Assuming that the gray area of their index is included in the group of problematic companies, the proportion of Type I errors in the sample was 26.9 percent, and the proportion of Type II errors was 39.4 percent a year before the crisis. In out-of-sample forecasting, the proportion of Type I errors was zero, and the proportion of Type II errors was 41.3 percent.

The results in the above table show that we cannot select a fixed discrimination threshold in order to distinguish between safe and problematic companies, since during the crisis period (end of 2008), the RNDP values increased significantly for all companies in the sample against the background of uncertainty in the capital markets (see Figures 4A and 4B). This phenomenon makes it difficult to analyze the chance in a certain company's risk over time and also makes it difficult to compare different companies at various points in time.

In the third stage, I made a comparison that was intended to check whether the Merton Model predicts default events better than the naïve alternative—observing the bond yield spreads during various periods before the default. This examination was done on a smaller sample than the previous test, a sample of 214 companies (the database is detailed in Part B of the article) and is outlined in Table 4B.

Table 4B: Results of the Test of Discriminatory Ability between Companies By Bond Yields (percent)

Sample	Type of Error	Threshold		
		50%	40%	30%
December 2006	I	23.1	30.8	53.9
	II	45.7	35.8	27.2
December 2007	I	28.6	42.9	61.9
	II	46.7	37.0	28.9
December 2008	I	33.3	38.9	50.0
	II	47.8	37.7	27.7

This table presents the results of the attempt to forecast default events during different periods by bond spreads. The bond spreads were taken from TASE releases. During each period, the companies were divided into two groups by their bond yields during the referenced periods: All of the observations in the distribution above the threshold noted in the heading of the column are classified as problematic companies, and all observations below the threshold are classified as safe companies. We then examined how many companies in each group were mistakenly classified. A Type I error notes companies that encountered difficulties but which the model classified as safe. A Type II error notes companies that did not encounter financing difficulties, but which the model classified as problematic. Sorting the companies by the size of the spread does not take into account the time range to the bond's maturity.

Table 4B shows that when we discriminate among companies by bond spreads, the identification errors are larger than the model's errors, and at best can reach a minimum of about 40 percent of the observations in each of the groups. In addition, it is easier to compare between companies in the model than in a framework based on yield spreads, since the spreads are not derived only from default risk, but also from differences in the characteristics of the bonds and in their range to maturity, while comparison based on RNDP is not dependent on these variables.

In summary, it seems that the model is successful in identifying companies liable to enter into debt repayment difficulties, even two to three years in advance. At the same time, the model’s accuracy is limited, and errors in identification amount to about 30 percent.

Furthermore, no fixed threshold of RNDP can be set for the purpose of discriminating between problematic companies and stable ones.

3. TO WHAT EXTENT DOES THE DEFAULT PROBABILITY EXPLAIN THE PRICES OF COMPANIES' TRADABLE DEBT?

a. Introduction

As stated, the Merton Model provides both a forecast of the expected value of assets as well as their distribution at maturity. In other words, the model estimates the value of the expected loss to bond holders. Since the distribution of the expected loss is known, it can be reflected in the current price of debt. Many studies have focused on the question of whether the price of debt as derived from the model is similar to bond prices on the market. Part 2 of the article attempts to deal with this question as well.

At this point, it is worth noting that the theoretical price does not include a risk premium, since the model assumes that the investor can create a portfolio that is completely risk-free by combining the company's equity and debt in the asset portfolio and the ability to purchase and short sell from both of these assets.

This part of the article is structured as follows: Section 3.2 reviews the relevant literature. Section 3.3 outlines the theoretical price of debt derived from the model. Section 3.4 shows how the addition of bankruptcy costs influences the projected price. Section 3.5 tests the connection between the model spread and the market spread on corporate bonds. Section 3.6 focuses on companies for which the debt value derived from the model more closely follows actual bond prices. A summary of the article appears at the end of Part 2.

b. A Review of the Literature

There is broad agreement in the literature that in the simple framework of the Merton model, its forecasts of the corporate bond spreads (the difference in yield vs the risk-free interest rate) are markedly lower than what is observed in practice (Jones, Mason and Rosenfield (1984); Kim, Ramaswamy and Sundaresan (1993)). This result is not unique to the structural model: the use of historical default distributions and losses due to default leads to the conclusion that default risk alone explains less than one-fifth of the spreads observed on bonds in the market (Elton et al. (2001)).

The attempts to deal with this result can be divided into two groups: expanding the Merton Model such that it evaluates larger losses to bondholders, for instance by accounting for the complex structure of the debt (Geske (1977)), by adding interest rate risk (Kim, Ramaswamy and Sundaresan (1993)), or by adding other endogenous variables that influence the default probability—such as making the debt structure and timing of default internal decisions made by the company (Leland and Toft (1996)). Alternatively, there are studies that do not attempt to increase the spreads derived from the model. As such, they agree that the default risk cannot fully explain the spreads in practice, and they claim that some of the spread can be explained by other risks that bondholders have—bankruptcy costs, taxes or lack of liquidity (Delianedis and Geske (2001)). This study assesses the two types of attempts at dealing with the problem of model spreads being too small.

c. Deriving the Theoretical Price in the Merton Model

Let us again observe Equation (1):

(1) $$V = S + D$$

If we organize the elements of the equation differently, we find that D, the market price of the debt, equals V, the market price of the assets, minus S, the market price of the equity. Another way of looking at the price of the debt is the debt excluding the risk of default, minus the insurance premium that the debt holder would have to pay in order to insure herself against that risk.

(7) $$\begin{aligned} D &= V - S = VN(-d_1) + Be^{-rT}N(d_2) = Be^{-rT} - P \\ P &= -VN(-d_1) + Be^{-rT}N(-d_2) \end{aligned}$$

Consequently, the model spread on the company's debt liabilities derived from the model equals:

(8) $$y_T - r = -\frac{1}{T}\ln\left(\frac{D}{B}\right) - r = -\frac{1}{T}\ln\left(\frac{D}{Be^{-rT}}\right)$$

Where y_T is the yield to maturity on debt maturing in T years.

For the sake of comparison between the model spreads and bond prices in the market, I used a database that included the market data for all traded bonds in Israel for the period January 2005-July 2010. The sample was restricted to include only simple CPI-linked or nominal corporate bonds (excluding convertible bonds, multi-maturity bonds, etc.). The market spread is equal to the difference between the yield to maturity of the bond and the corresponding yield according to the bond duration from the zero curve (nominal or real, depending on the bond's linkage features). If the company had issued more than one series of bonds that had not yet been paid up, the market spread was taken as the weighted average (according to the size of the issue) of the spreads of the company's various bond issues. 214 companies remained after cross-checking with the sample used for the calculation of default probabilities. The rating assigned to the company was the highest rating between the company's different issues.

The model spread of the debt was derived from Equation (8) above, where the risk-free interest rate was obtained, as before, from the nominal zero curve calculated at the Bank of Israel. Note that the model spread refers simultaneously to all the company's liabilities. The model spread is, therefore, an average of the spreads on the various series of the company's bonds and of the outstanding non-traded debt. The display of the model spreads was not grouped by duration because this duration does not equal the real duration of the debt, regarding which there is no data. In contrast, the real spread is presented by duration, since the duration of the part of the debt that is traded is known. The distribution of the market spread by duration is presented in four categories, with the duration rounded to the nearest whole number, where bonds with duration of more than four years are included in the four-year group.

The following table displays a comparison of the median of the spreads, both market and model, according to rating groups. The standard deviations of the spreads for each rating group are also displayed.

Table 5A: The Median Market and Model Spreads, First Panel (January 2005-December 2007) (percent)

	Rating		
	AAA – AA-	Below A+	Not rated
Market median 1 year duration	No observations	2.42	4.72
Market median 2 years duration	0.69	2.07	3.19
Market median 3 years duration	0.82	1.62	3.39
Market median 4 years duration	1.08	1.66	3.27
Total median	1.05	1.69	3.33
Standard deviation	0.30	0.98	5.39
Model median, $T_2 = 4$	0.01	0.07	0.12
Model standard deviation	0.78	1.84	12.90
Model's explanatory ability	1.12	4.09	3.51
Model median, $T_2 = 3$	0.00	0.03	0.07
Model median, $T_2 = 5$	0.02	0.11	0.18
Model median, $T_2 = 10$	0.08	0.31	0.46

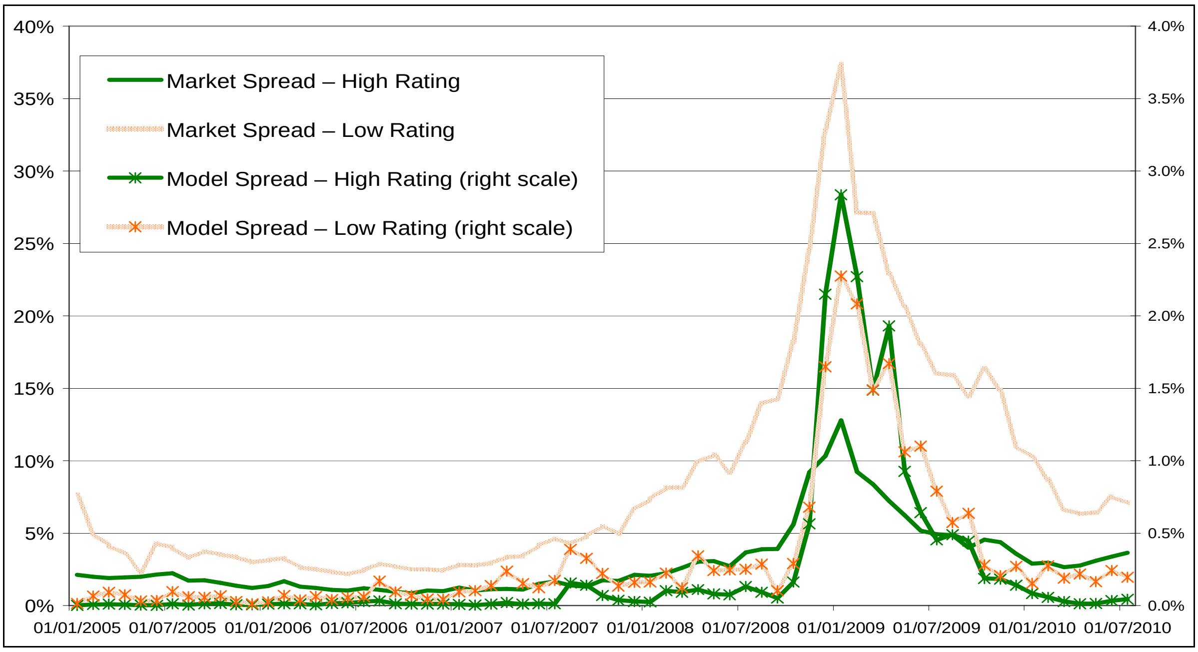
Table 5B: The Median of the Market and Model Spreads, Second Panel (January 2008-June 2010) (percent)

	Rating		
	AAA – AA-	Below A+	Not rated
Market median 1 year duration	1.70	11.27	13.82
Market median 2 years duration	1.38	5.62	15.83
Market median 3 years duration	1.68	6.26	13.89
Market media 4 years duration	2.10	3.99	8.28
Total median	2.05	5.14	13.69
Standard deviation	2.23	13.66	26.80
Model median, $T_2 = 4$	0.10	0.25	0.39
Model standard deviation	5.75	10.95	20.94
Model's explanatory ability	4.96	4.92	2.82
Model median, $T_2 = 3$	0.06	0.16	0.25
Model median, $T_2 = 5$	0.14	0.34	0.52
Model median, $T_2 = 10$	0.26	0.74	1.08

Tables 5A and 5B: These two tables present the median values of the model spreads and of the actual spreads by rating groups and by duration. The table divides the medians into two periods, from January 2005 to December 2007, and from January 2008 to June 2010 (the period that included the crisis). The table rounds off the duration of the bonds to the nearest whole number, where all bonds with duration of longer than four years are included in the four-year group. The three bottom rows of each table present the results of the model spreads under various assumptions regarding the duration of the debt—by changing the time to maturity of the company's long-term liabilities.

The following figure displays the development of the median spreads over time, both the market spreads and the model spreads derived from the model, according to rating categories and not according to duration. The high rating category includes companies that were rated higher than BB+ during most of the sample period, which in effect includes all the rated companies (there were specific observations in which the rating was lower). The lower rating category includes all the other companies, i.e. companies that actually had no rating for most of the sample period.

Figure 5: Development of the Model Spreads and Market Spreads Over Time



From the tables and the chart, we can see that the spreads derived from the model are considerably smaller than the spreads observed in the market. Furthermore, the difference in percentages between the model spreads and the actual spreads is greater when the quality of the debt is lower. The standard deviation is much larger, relatively, among the model observations, and is of the same level as the market spreads, despite the difference in the values themselves. When we examine what percentage of the actual spread is explained by the model spread, we see that this explanation does not exceed 5 percent in most of the columns, while the explanation is slightly better in the groups of rated companies in the second panel, and is especially poor for the highest rating group in the first panel. The chart indicates a resemblance in the development of the model spreads and market spreads over time: the correlation in the two groups between the median market spread and the median model spread stands at 0.9. During the crisis, we can see that, in contrast to what happened to the market spreads, the model spread in the group of rated companies reached a higher peak than in the group of unrated companies.

It is worth noting that the results obtained from the Merton Model do not differ in magnitude from the spreads derived from the historical default distribution. In particular, Elton et al. (2001), relying on the historical default events in the US, found that the average spread of A-rated companies for a one-year maturity, derived solely from the risk of default, should be 0.04 percent, while the spread on BBB-rated companies (equivalent to a AAA-AA rating in Israel, according to S&P) should equal 0.11 percent.

So what are the inputs required so that the model generates spreads similar in magnitude to the actual spreads? I conducted a simulation that calculated the standard deviations required for assets and equity in order to obtain model spreads equal to the actual spreads. I assumed that the market spread of the company's debt is the correct spread for all of its liabilities, and obtained a new theoretical market value for the entire debt. I then obtained the new market value of the assets from a schema of this theoretical market value of the debt and the market value of the equity (according to Equation (1)). Using Equation (3), I obtained the implied standard deviation of the assets, σ'_V. I then calculated the required standard deviation of the equity, σ'_S, from Equation (5). The average standard deviation for equity, σ_S, equals 57 percent for all the companies in the sample, with a standard deviation of 42 percent. On the other hand, the average required standard deviation of equity, σ'_S, equals 168 percent, with a standard deviation of 107 percent, i.e. the standard deviations obtained in the simulation are much larger and not very reasonable.

It therefore seems that the simple framework of the model cannot generate spreads that are similar to those seen in the market, and that to do so, we need to expand the model or take other risk factors into account.

d. Partial Repayment of the Debt to the Bondholders in Case of Bankruptcy

1. *Background*

Calculation of the model spreads relies on the assumption that when a company reaches a state of default, the debt holders take over the company and receive the full value of its remaining assets, or the value of the assets corresponding to their relative claim on the debt. In practice, when a company goes into liquidation, or in a situation in which negotiations are conducted for repayment of the debt, the debt holders lose a certain proportion of the company’s assets as a “deadweight loss” – payments to trustees and lawyers, and the decrease in the company’s value at time of liquidation versus its value when the company was active. In addition, sometimes bondholders do not receive the full relative value of the company’s assets because of obligations to other creditors that get higher priority than the bondholders.

It seems that in the eyes of bondholders, the value of an active company is higher than the value of an insolvent company, i.e. it is higher than the value of all of its tangible assets. We can find evidence of this in the fact that the debt holders generally prefer a debt arrangement with the company's ownership over liquidation. As of July 2012, most of the companies in the sample that had announced difficulties had reached debt arrangements with bondholders. (These arrangements are known as "haircuts" since they include some deletion of the company's liabilities to its creditors). Just five companies in the sample had reached the state of liquidation, 15 percent of all companies. It seems, therefore, that in most of the cases, the bondholders would rather agree to shrink the company's liabilities and enable it to continue operating instead of liquidating. This phenomenon of preferring a "haircut" over liquidation is not unique to the Israeli market. According to evidence from the US market, in a situation of negotiation on the debt terms, the debt holders compromise on a value of 50-80% of the original obligations (Franks and Torous (1994)). A study on a number of countries in eastern Asia reached similar conclusions. The study looked at the phenomenon of bankruptcy and debt arrangements during the crisis that took place there in 1997 and 1998, and found that 83 companies in the sample filed a request to liquidate, while 561 companies reached an agreement with their creditors (Claessens et al. 2003). The ratio obtained is slightly lower than the current sample: 13 percent. This phenomenon justifies the claim that some of the company's assets "get lost" when the company liquidates.

I tested how the debt price was affected when bankruptcy costs were added to the model in two ways: relative costs and fixed costs. These tests are described in the next two sections accordingly.

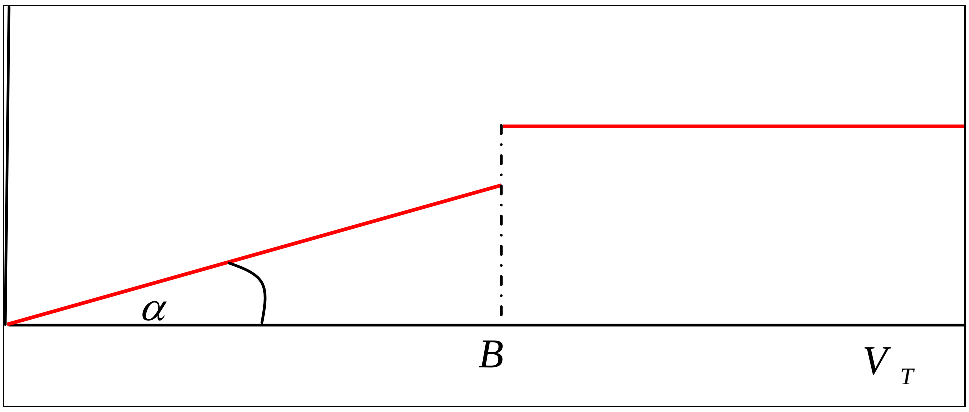
2. *Bankruptcy costs*

Following the study of Delianedis and Geske (2001), I add to the model bankruptcy costs, C, with the following specification:

$$(9) \quad \begin{matrix} V_T > B \\ V_T \leq B \end{matrix} \qquad C = \begin{cases} 0 \\ (1-\alpha)V \end{cases}$$

Meaning that the payments to the bondholders, taking bankruptcy costs into account, appear as follows:

Figure 6: Payment to Bondholders Taking Bankruptcy Costs Into Account



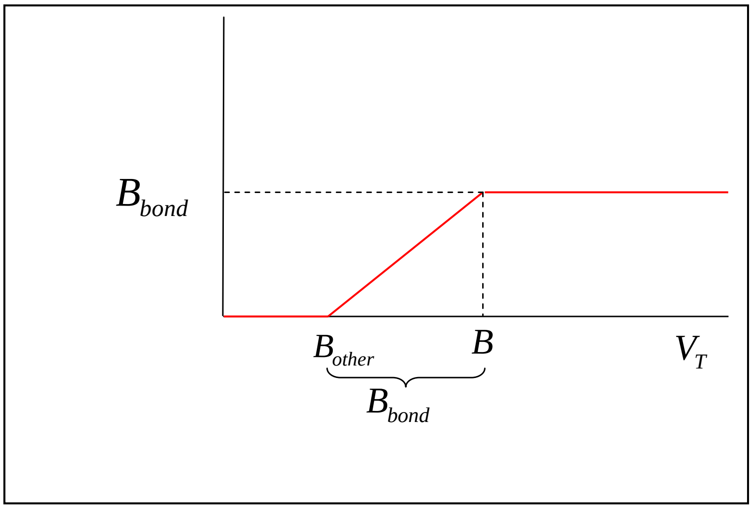
The burden of bankruptcy costs falls entirely on the debt holders, and does not affect the payments to the equity holders. It therefore does not affect the calculation of the company value, V, or the calculation of the probability of default in the preceding section of the study. However, it does have an effect on the value of the debt:

(10)
$$D = V - S - C = Be^{-rT} - P_\alpha$$
$$P_\alpha = -\alpha VN(-d_1) + Be^{-rT} N(-d_2)$$

3. *Order of Priorities of the Bondholders*

Alternatively, in the absence of available data concerning the company's debt structure, including the priority order of its various creditors, I will assume the extreme case: the bondholders have the lowest priority among all creditors of the company, meaning that even when the company, after declaring bankruptcy, has a positive value, the bondholders can be left with nothing. The following chart describes the payment to the bondholders in this situation:

Figure 7: The Payment to the Bondholders under the Assumption that the Public Debt Has Lower Priority than All the Other Creditors



$B = B_{bond} + B_{other}$ - The face value of the company's liabilities equals the sum of its obligations to the bondholders and other liabilities.

B_{bond} - The obligations to the bondholders

B_{other} - The obligations to other creditors

As previously, the change does not affect the company value, V. The effect on the value of the bondholders' debt is calculated as follows:

$$
\begin{aligned}
&D_{bond} = V - D_{other} - S = C(V, B_{other}) - S = \\
(11)\quad &= VN(k_1) - B_{other} e^{-r_{f,T}T} N(k_2) - VN(d_1) + Be^{-r_{f,T}T} N(d_2)
\end{aligned}
$$

$$
k_1 = \frac{\ln(V / B_{other}) + (r_{f,T} + \frac{1}{2}\sigma_V^2)T}{\sigma_V \sqrt{T}}
$$

$$
k_2 = k_1 - \sigma_V \sqrt{T}
$$

where the symbol $C(V, B_{other})$ refers to the value of a call option on the underlying asset, V, with a strike price of B_{other} .

4. *Testing the Effect of Bankruptcy Costs/Order of Priorities on the Model Spreads*

The following charts display a comparison of the medians of the spreads in the market and the medians of the model spreads obtained as a result of adding bankruptcy costs or payment to other debt holders. Adding bankruptcy costs is displayed for a number of possible levels of α :

Figure 8A: Development of the Theoretical and Market Spreads Depending on α and on Creditors Priority Rules, High Rating

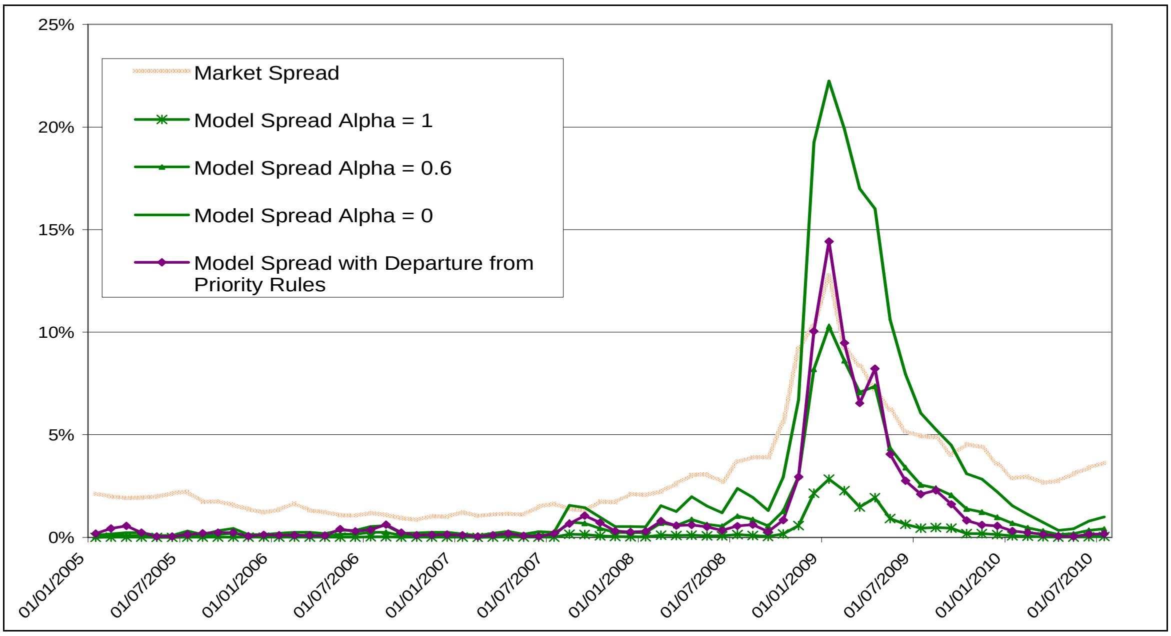
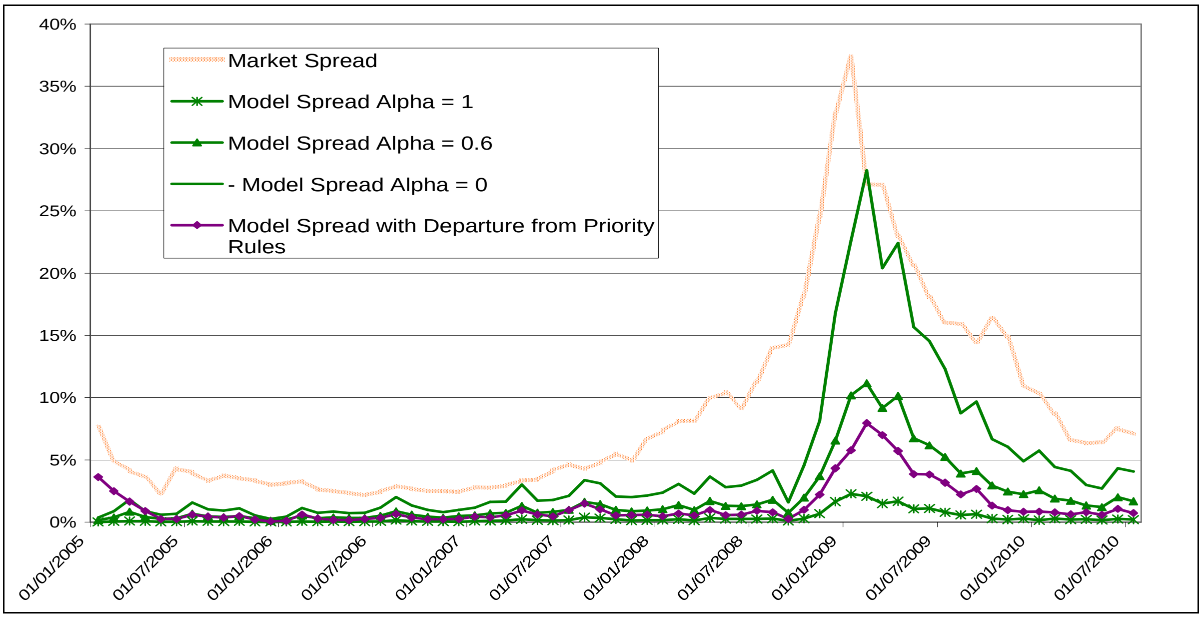


Figure 8B: Development of the Theoretical and Market Spreads Depending on α and on Creditors Priority Rules, Low Rating



Figures 8A and 8B display the spreads derived from the model for different values of α, and also for the assumption that the bondholders' debt has the lowest priority, while in the extreme situation, the results are displayed under the assumption that in a liquidation, the debt holders receive nothing, i.e. α = 0, and the bankruptcy costs amount to 100% of the value of the remaining assets. Since the default risk is the only risk to the bondholders in the model, a situation of maximal bankruptcy costs constitutes an upper bound for the derived market spread, meaning that even if we have more precise information about the priorities of the company's creditors, the spread on the various debt issues cannot exceed the model spread in a case of $\alpha = 0$.

We can see that adding an assumption about additional costs to bondholders in the event of liquidation leads to different outcomes for the two rating groups: for the group of unrated companies, even under an assumption of maximal bankruptcy costs, the spreads derived from the model remain lower than the market spreads, even at the peak of the recent crisis. On the other hand, in the high rating group, under an assumption of maximal bankruptcy costs, the spreads derived from the model reach a level of 23 percentage points, 10 percentage points higher than the market spreads. A similar result is obtained for a case where the bondholders' debt has the lowest priority.

In their study, Delianedis and Geske (2001) examine what model spreads are obtained when selecting a value of 60 percent for α. This value represents a 40 percent loss for the bondholders as a result of bankruptcy and negotiation expenses and the sharing of assets with other creditors. They found that the model spreads calculated with such bankruptcy costs explain only 5 percent of the market spreads of highly rated bonds, and close to 20 percent for medium rated bonds. According to their calculations, the spread for a BBB rating with bankruptcy costs should be between 0.14 percentage points and 0.26 percentage points.

The current sample shows that bankruptcy costs of 40 percent of the company's value lower the theoretical debt price (the difference between P and $P_{60\%}$) by an amount equal to 2.35 percent of the company's value for companies rated above A+ and 4.26 percent for unrated companies. By comparison, Almeida and Philippon (2007) examined the pricing of bankruptcy costs in the market and found that bankruptcy costs measured according to historical data lower bond prices by 3.83 percent of company value for A-rated companies and 9.54 percent for B-rated companies.

5. *Examining the Connection between the Model Spread and the Corporate Bond Spread*

It is therefore clear that the magnitude of the spreads derived from the model are significantly smaller than the actual spreads for most of the sample period, even if bankruptcy costs are added. Nevertheless, it is worth assessing whether the model spreads can offer an explanation of the changes occurring in the market spreads over time. For this purpose, regressions were run on the connection between the median model spread and the median market spread. The model spreads used for the regressions were calculated under the assumption of 40 percent bankruptcy costs, as discussed earlier. Figures 8A and 8B show that from the standpoint of magnitude, a loss of 40 percent leads to spreads similar to the spreads obtained on the assumption that bondholders' debt has the lowest priority.

Both the actual spread used as the dependent variable and all explanatory variables marked with "%" were multiplied by 100 in all the regressions, i.e. they are given in percentage points. The slope of the term premium is defined in the regressions as the difference between the four-year yield and the one-year yield from the nominal zero curve.

Table 6: Regression of Medians According to Rating Groups[6]

Explanatory Variable	AAA-AA-		Below A+		Unrated	
Model spread (%)	0.43***	0.18***	1.19***	0.86***	2.77***	1.76***
Yearly risk-free nominal interest (%)		-0.32***		-0.21		1.67
Term Premium slope (%)		-0.01		0.52		0.01
Average monthly standard deviation on the TA25 (%)		0.03***		0.07***		0.29***
Cumulative monthly yield on the TA25 (%)		-0.02***		-0.08***		-0.12*
Interceptor	1.41***	1.97***	2.06***	1.2	3.37***	-3.62
Observations	67	67	67	67	67	67
R^2	0.68	0.90	0.85	0.93	0.79	0.87

This table shows six regressions of various variables on the median market spread by rating groups. For the purpose of the regressions, both the dependent variable and all of the explanatory variables marked with % were multiplied by 100, i.e. they were given in percentage points.

The explanatory power of all the regressions is quite high; even when no other explanatory variable is included other than the model spread, the explanatory ability is between 70 percent and 80 percent. When other macro variables are added, the R^2 rises to 90 percent. In all three rating categories, the correlation between the median market spread and the median model spread is around 0.85 for the entire period. This correlation is much lower when the sample is restricted to the January 2005-January 2008 period (without the crisis), and falls to 0.3 for the highest rating group and the unrated group, and to a negligible level for the medium-rated group. This result is similar to that of Delianedis and Geske (2001), who tested the correlation between market spreads and model spreads for the US in 1991-1998. The correlations obtained in their study are high when the entire period is used and much lower when the LTCM crisis in the fall of 1998 is excluded from the sample.

We can see that the slope of the zero interest rate curve is not significant in any of the regressions. The return on the Tel Aviv 25 Index for the most recent month has no

[6] In this table and in all the following tables displaying regression results, ***,**, and * mean that the coefficient is significant at a 1 percent, 5 percent, and 10 percent level of significance, respectively.

significant effect on the market spread, but the risk in the stock market, as reflected in the average standard deviation in the Tel Aviv 25 Index in that month, has a significant effect on the spread in the expected direction. This effect increases with the decrease in the quality of the debt. In addition, in the groups of low-rated and unrated companies, after the market variables are added, the regression interceptors lose their significance. In contrast, there remains a constant element in the high rated companies that is not explained by the variables used in the regression.

The explanatory power of the model for market spreads declines markedly when we move to a regression for individual observations instead of medians. 125 (58 percent) of the 214 companies in the comparison have a significant positive correlation between the series of model spreads and the series of market spreads. In addition, these percentages are higher for the rated companies (as stated, all higher than BB+)—71 percent, compared with 44 percent for unrated companies. Appendix 1 shows the relationship between the market spread and the spread derived from the model for a number of randomly-selected companies representing a variety of industrial sectors, company sizes, and leverage ratios. The following tables show the results of regressions on individual observations of companies, where each table expands the group of explanatory variables over its predecessor. For the 214 companies in the sample, there are 8,715 observations in which there are data for both market spread and model spread.

Table 7A: Regression of Model spread on Market Spread – Effect of the Equity's Liquidity and the Company Rating

Explanatory Variable	The entire sample	Low liquidity	Intermediate liquidity	High liquidity	High rating	Low rating or unrated
Model spread (%)	0.23***	0.09***	0.31***	0.43***	0.31***	0.22***
Interceptor	7.73***	10.63***	9.22***	2.66***	3.91***	11.68***
Observations	8,493	1,418	4,585	2,490	4,297	4,196
R^2	0.08	0.02	0.09	0.29	0.20	0.06
Average daily trading volume in bonds (NIS million)	1.62	0.21	0.39	4.68	2.9	0.38

Table 7A shows the regression of the model spread on the market spread, which examines how the equity's liquidity and the company's debt rating influence the connection between the two spreads. A rather low R^2 of 8 percent is obtained for the sample as a whole. One of the main problems in applying the model lies in the fact that calculation of the model is greatly influenced by the trading data of the equity. This raises concern that equity with insufficient trading will cause misevaluation of the standard deviation of the stock, thereby leading to misevaluation of the standard deviation of the assets. In addition to a possible error in calculating the standard deviation, low trading in equity undermines one of the model's central theoretical assumptions – the ability to hedge the risk of holding bonds with equity. In order to make an effective hedge, it is necessary to continuously adjust the quantity of bonds and equity (or options on equity) in the portfolio. Low trading in equity loosens the connection described in the model between the share price and the bond price. I therefore tested whether the share's degree of liquidity affects the relationship between the theoretical and market spreads. For this purpose, I divided the sample into three groups: companies with low liquidity in their equity (an average of less than 10 days per month in which trading in the stock took place), companies with average liquidity in their equity (10-20 days per month of trading), and companies with high liquidity (over 20 days per month of trading in their stocks, meaning that trading took place every day). Following the initial filtering of the data, the sample doesn't include companies with an average of less than six trading days per month in the stock. As expected, R^2 rises significantly with higher liquidity in the stock. Furthermore, as we can see in the bottom row of Table 7A, there is a strong correlation between trading frequency in equity and in bonds; companies in the groups with higher frequency of trading in equity also have higher average turnovers in their bonds. Therefore, other than a more accurate estimation of the standard deviation, other effects are also possible here—for example, a lower liquidity premium in the spread on the bonds, or an increase in the quality of the bond price, reflecting more up-to-date information.

The connection between the model spread and the market spread is stronger for high rated companies (above BB+) than for low rated and unrated companies. This result is not surprising, given the fact that on the average, bonds in the high-rated group also have higher turnovers and a higher frequency of trade in their company's equity.

Tables 7B and 7C show the attempts to explain the gap between the model and actual spread by adding market variables and characteristics of the company to the regression.

Table 7B: Adding Variables Specific to the Company

Explanatory Variable	1	2	3	4	5	6
Model spread (%)	0.24***	0.17***	0.23***	0.16***	0.16***	0.79***
Leverage		2.68***		2.71***	2.48***	0.12
Log of the value of company assets			-1.04***	-1.06***	0.15	-0.82***
Duration					-3.07***	-2.36***
Standard deviation of assets (%)						-0.41***
Dummy variable for low rated debt	7.36***	7.34***	5.38***	5.33***	4.52***	3.57***
Interceptor	4.18***	2.50***	12.38***	10.83***	13.49***	24.43***
Observations	8,493	8,493	8,493	8,493	8,493	8,493
R^2	0.15	0.17	0.16	0.18	0.24	0.34

After adding variables specific to the company, Table 7B shows that adding the duration contributes significantly to the explanatory power of the regression. The duration has a negative impact on the market spread. This makes sense, given the fact that in most rating categories, the spread curve decreases with the term to maturity (see Tables 5A and 5B). The strong explanatory power of the duration could be due to the fact that the debt's term until maturity in the calculation of the model spread was selected arbitrarily, and given more exact information about the maturity of the company's debt, it would be possible to include this effect in the calculation of the model spread. The size of the company (logarithm) usually has a negative impact on the market spread. The company's leverage ratio usually has a positive effect on the market spread. In other words, the effect of leverage on the company's risk is greater in the market's view than in the model. The significantly positive impact of the company's leverage on the probability of default, in addition to the effect of RNDP on this probability, was obtained by Tudela and Young (2003). At the same time, when the standard deviation of the assets, which, contrary to expectations, receives a significant negative coefficient, is added to the regression, the leverage ratio loses its effect, while the effect of the model spread on the market spread increases. The default probability as derived from the Merton Model is particularly sensitive to the value of the standard deviation of assets, while the negative coefficient of this variable in the regression indicates that the market is less sensitive to it.

Table 7C: Addition of Macro Variables

Explanatory Variable	The entire sample	High rating	Low rating	High liquidity
Model spread (%)	0.62***	0.51***	0.65***	0.51***
Leverage	0.18	0.11	0.03	1.09*
Duration	-1.92***	-1.09***	-3.31***	-1.15***
Standard deviation of assets (%)	-0.32***	-0.23***	-0.34***	-0.16***
Annual nominal risk-free interest rate (%)	-0.89***	-0.54***	-0.65*	-0.08
Term Premium slope	-0.75*	-0.73**	-0.33	0.54
Cumulative monthly return on the TA25 (%)	0.02	0.02	0.06	-0.03
Average monthly standard deviation on the TA25 (%)	0.41***	0.23***	0.58***	0.09***
Dummy variable for low rated debt	6.56***			
Interceptor	7.74***	7.32***	12.72***	6.55***
Observations	8,493	4,297	4,196	2,170
R^2	0.41	0.45	0.38	0.45

Tables 7A, 7B and 7C: These three tables show the results of the regressions on individual observations of market spreads (as opposed to medians of the spreads as we saw in Table 6), where gradually other explanatory variables are added to the model spread. In addition to the model spreads, Table 7B also firm-specific variables. In addition to the company characteristics, Table 7C also includes macro-economic variables.

Adding market variables significantly increases the explanatory power of the regressions for the observed spreads, especially for the group of high rated companies, and for bonds with the most frequently traded equity. The duration has a significant negative effect in all the regressions. The effect of the standard deviation of the assets is also negative and significant – a 1 percentage point increase in the standard deviation decreases the projected market spread by 0.32 percentage points, as indicated by the regression for the entire sample in Table 7C, while this effect is smaller for high rated companies and greater for low rated ones.

Changes in the standard deviation of the Tel Aviv 25 Index have a clear economic significance for the spread: a 1 percentage point rise in the standard deviation in the stock market increases the market spread of high rated bonds by 0.23 percentage points and the spread of low rated bonds by 0.58 percentage points. As with the regressions on the medians in Table 6, the slope of the term premium has no significant effect in most of the regressions, other than for high rated bonds. The one-year risk-free interest rate has a significant negative impact in almost all the regressions, meaning that when the risk-free interest rate rises, the spread between it and the yield on corporate bonds narrows. Longstaff and Shwartz (1995) and Dufee (1998) also found a negative connection between market bond spreads and interest rates.

The regressions show that the risks affecting the stock market and the interest rate risk are important in explaining bond prices. In the theoretical framework of the Merton Model (and in options theory in general), the risk of default of the company can be eliminated by maintaining an investment portfolio with suitable proportions of stocks and debt, and by continuously adjusting these proportions, with the possibility of borrowing and short selling in a sophisticated market. In other words, there is no theoretical justification for the existence of a risk premium. Obviously, these assumptions are not fulfilled in the real market; in particular, trading in stocks and bonds is not continuous, and features a lack of liquidity. For this reason, it is possible that some of the market spread in bonds is due to the premium required by investment managers, given their inability to completely eliminate the risk of default.

The tables also indicate that the spreads for the debt derived from the model have some ability to explain the bond spreads in the market. At the same time, while the coefficient of the model spread is significant in most of the regressions, it has little economic explanatory value for the market spreads, especially considering the differences in magnitude between the model spreads and the market spreads.

6. *What Characterizes the Companies Whose Debt Price Derived from the Model Behaves Similarly to the Market Price of the Debt?*

In this section of the study, I concentrated on the group of companies that exhibit a stronger link between the theoretical debt price and the market debt price, in order to see what characterizes this group and distinguishes it from the other companies.

For this purpose, I selected companies with a high correlation between the actual market spreads on bonds and the predicted spreads (a correlation greater than 0.75). 43 companies met this criterion. The following table shows a number of characteristics of these companies:

Table 8: Characteristics of Companies for Which the Model Prices Debt Similar to the Market Price

	Average in the population	Standard Deviation in the sample	Average in the group
Asset value (NIS million)	3,905.1	9,917.8	7,414.5
Total face value liabilities (NIS million)	3,133.7	9,673.1	7,342.6
Equity market value (NIS million)	1,191.5	3,718.5	1,318.0
Public debt to total debt ratio	0.3	0.5	0.2
Average rating	14.1 (B+)	8.2	9.8 (BB)
Average trading volume of bonds (NIS million)	1.7	3.4	2.3
Leverage ratio	0.73	0.33	0.81
Average number of trading days per month	15.2	5.0	17.8
Average number of observations (out of 67)	38.7	17.8	45.2
Market spread / Model spread	0.9	1.0	1.1

This table shows a number of characteristics of companies for which the model provides a good projection of the market price of the debt.

One of the most prominent characteristics is that the companies in this group are larger than average, with a large volume of liabilities, both in relative (leverage) and absolute terms. In addition, these companies are considered safer companies: they have a higher debt rating, and their securities (both equity and bonds) are more liquid. At the same time, they have relatively less public debt as a share of the entire debt, in comparison with the sample as a whole. There is a positive correlation between some of the characteristics, meaning that in general, larger companies also tend to have more liabilities, higher market values, a lower proportion of public debt, a higher rating, larger trading turnovers in bonds, more liquid equity, and a slightly higher ratio of the market spread to the model spread. It is reasonable that the information about these companies in the market is more available and up-to-date, is reflected in frequent price updates, and is less influenced by a liquidity premium. Among other things, the model’s success in explaining the prices is therefore related to the fact that the data are more reliable, with fewer sharp fluctuations in prices and a lower liquidity premium.

In contrast to the results displayed in the table for the group, no connection was observed in the general population between the size of a company and its leverage. Ceteris paribus, companies with high leverage have a greater risk of default, i.e. the market price should be affected by changes in this risk to a greater extent than other risks (e.g. liquidity), which explains why the model does a better job of explaining the market price for highly leveraged companies: the model prices in only the risk of default.

4. SUMMARY

This study applies the structural model proposed by Merton (1974) for pricing debt of a public company to the data for non-financial Israeli companies listed for trade during January 2005-July 2010. The study examines two aspects of the model's analytical and predictive power: the model's ability to assess a company's credit risk and discriminate between companies headed for difficulties in repaying their debt and other companies in the sample, and its ability to explain the market prices of corporate bonds.

The results obtained in the first section of the study show that the model is successful in identifying companies liable to enter into difficulties in repaying their debt, even two to three years in advance, but with Type I and Type II errors in identification amounting to about 30 percent. Such an error in magnitude is no worse than that of models based solely on accounting data, such as the Altman index adjusted to the Israeli market.

Not only is the Merton model's performance no worse than that of other models, it also has benefits over the others. It allows for the assessment of default risk for a wide variety of companies while relying on a small amount of data: RNDP can be calculated for any public company in comparison with a relatively small number of companies monitored by the rating companies, and is revised continuously, not just quarterly, as in the case of indices based solely on accounting data. In addition, the Merton Model can also be used to evaluate companies with non-traded debt, regarding which there is no information on market returns or spreads. Finally, in contrast with accounting data, which describe the past, the index derived from the model is forward looking and is also based on expectations.

The second section shows that the price of debt obtained from the model, based on the company's default risk, is significantly higher than its market price. The model spreads explain only about one fifth of the actual spread, and even that small result is obtained under conditions that assume bankruptcy costs amounting to 40 percent of the debt's value. This result is similar to the results obtained in overseas studies, which show that the default risk has little power to explain the prices of corporate bonds. The regressions conducted in the second section show that other factors contribute greatly to understanding the spread. These are related to the general market risk, and are reflected in a strong effect of the standard deviation from the stock market in the regressions. In addition, the risk of default explains a larger proportion of the spread for companies with frequently traded stocks and bonds—a result mostly accounted for both by the model's ability to estimate the spreads more accurately for this type of company and by lower liquidity premiums.

In applying the model in the article, a number of simplifying assumptions were made that affect the estimation results: calculation of the Merton Model is significantly influenced by the standard deviation of the equity. In the absence of options on individual stocks, the way to estimate this standard deviation is through historical data. Instead of using Equation (5) for calculating the standard deviation of the assets, which should hold only instantaneously and is based on the standard deviation of equity, a recursive method can be used that simultaneously solves the value of the company's assets and their standard deviation over time (Crosbie & Bohn (2003), Bharath & Shumway (2008)). This method is likely to greatly improve the model's estimation, because the fluctuations in the standard deviation of the assets obtained from the estimation of the model in this article and the fluctuations of the derived spreads are very large. In comparison with the results obtained from regression on the medians, the model's explanatory power for each company separately is very small due to the large number of outliers. The existence of more detailed information about the company's liabilities—such as a decomposition of the different maturities of the debt, collaterals, and priority rules—will also improve the model spread's estimation ability and the comparison with the market spreads.

The basic Merton Model prices the debt as a "European option", i.e. default is possible only at the time of maturity of the debt, T. In practice, when the company's financial situation deteriorates, the bondholders usually demand that the debt be put up for immediate repayment, or the company itself initiates measures and enters into negotiations with the bondholders. Pricing the company's liabilities using "barrier options" is therefore likely to improve estimation of the expected loss to the debt holders, because it allows a default event to take place at any time between the issue and T. Tudela and Young (2003), who used options of this type, obtained results facilitating a better discrimination between companies headed for financial difficulties and other companies.

In current research, there is renewed interest in using the model in the area of financial stability. The recent financial crisis showed how important it is to estimate not only the risk of default of a single company in the market, but mainly the connection between various bodies, their influence on each other, and their joint risk of default. The Merton Model allows us to estimate the distribution of a joint default of various companies, such as the financial system as a whole, by enabling us to examine the connection between individual risk indices over time (Gray et al. (2010)). The combination of these risk indices and their joint distribution in the framework of macroeconomic models enables us to examine scenarios that may lead to financial crises in the future (Gray et al. (2009)).

Appendix 1 – Theoretical and Market Spreads for Selected Companies

The companies shown below were selected at random in order to reflect a variety of industrial sectors, of company size, of market spreads and of tradable debt to total liabilities ratio.

Company characteristics[7]

Company name	Sector	Total assets	Total liabilities	Traded debt	Stock market value	Correlation with model spreads
1	94	5,316	5,698	601	245	0.50***
2	95	2,139	2,445	493	287	0.86***
3	95	2,139	1,361	700	1,031	0.72***
4	95	16,312	27,728	5,299	2,776	0.66***
5	97	122	67	37	64	-0.18***
6	94	671	409	72	274	0.36
7	95	451	335	39	244	0.15**
8	95	15,894	21,776	2,211	644	0.64***
9	95	1,440	1,110	775	525	0.48***
10	97	12,014	8,977	3,972	3,926	0.55***
11	95	187	233	86	18	0.71***
12	96	84	77	34	14	0.19

[7] All of the data are in NIS millions. The data in the table were sampled in August 2008.

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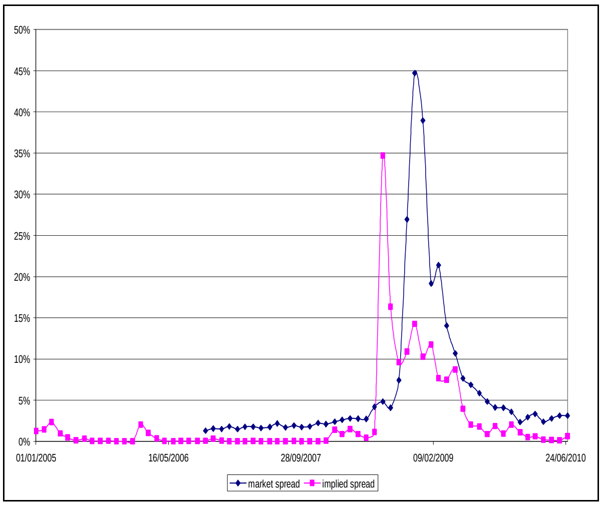

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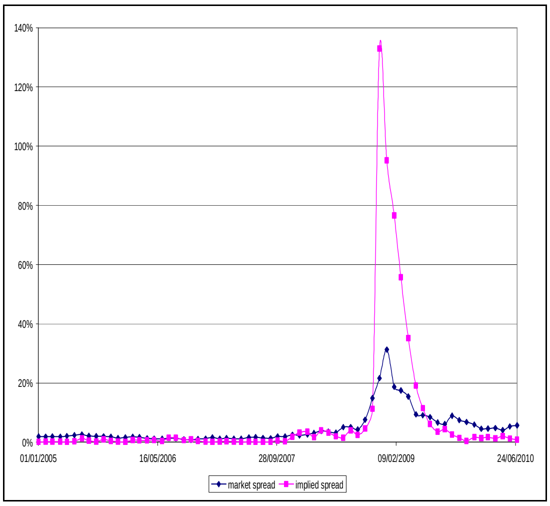

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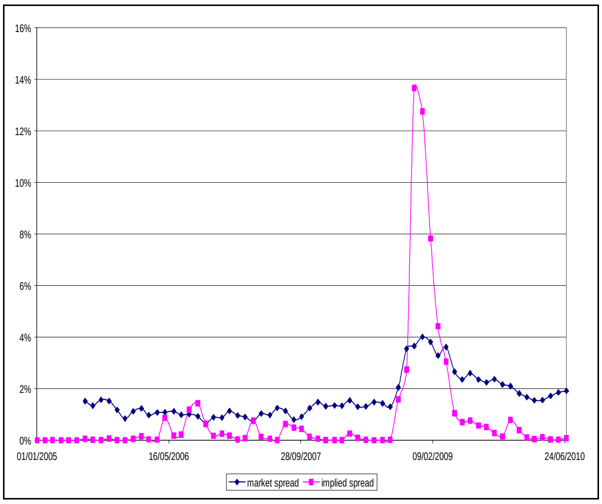

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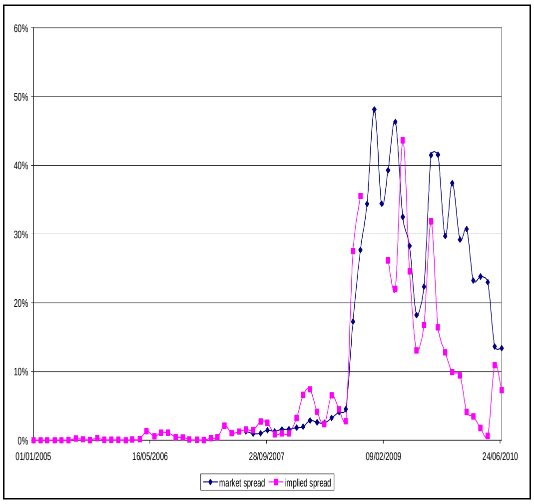

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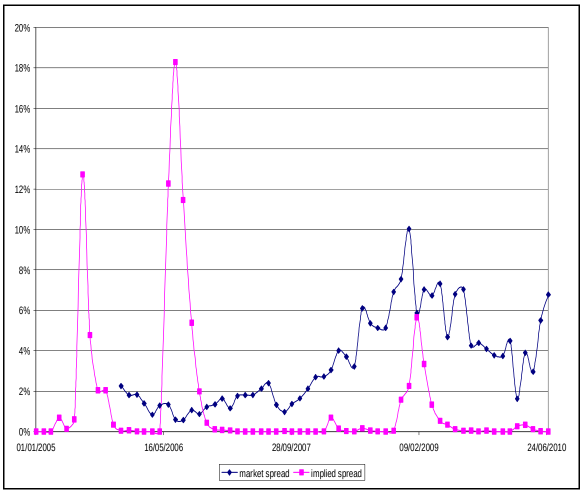

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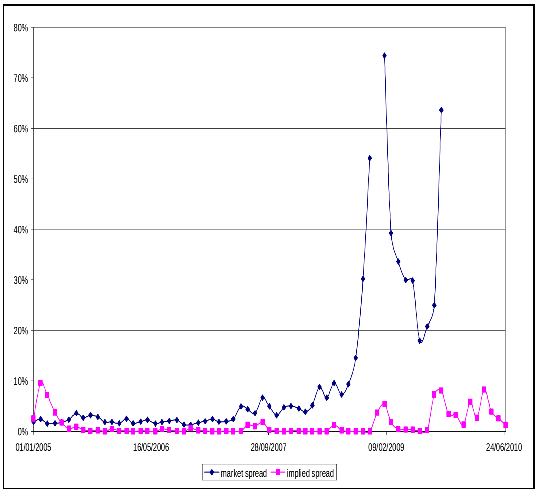

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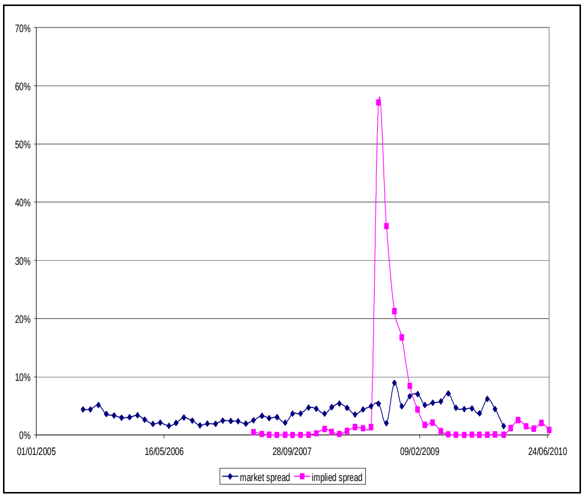

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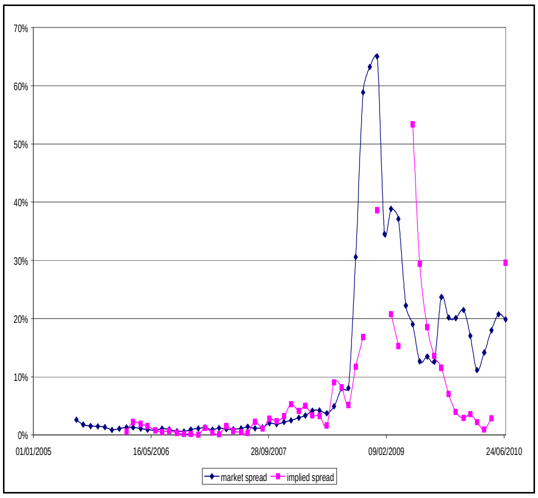

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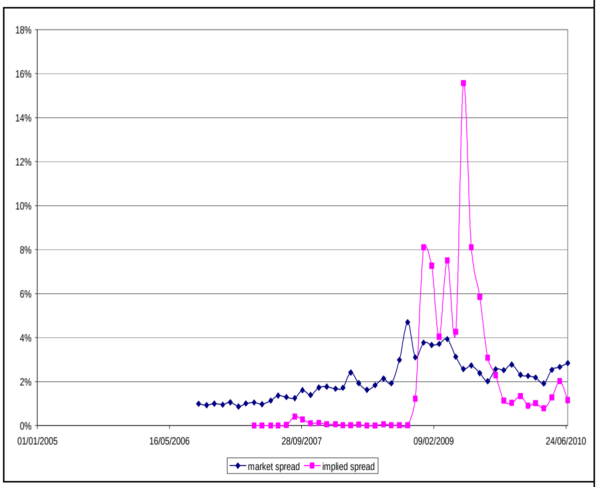

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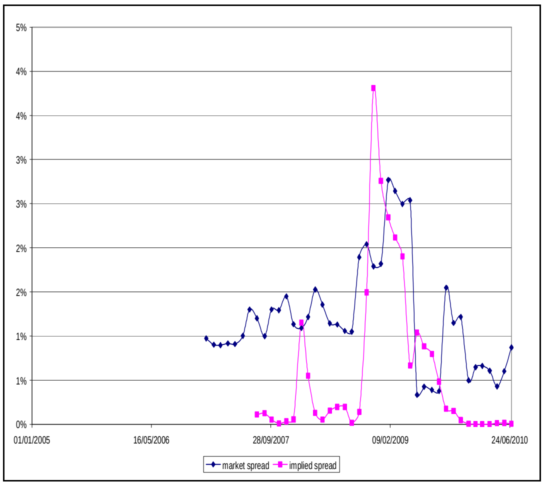

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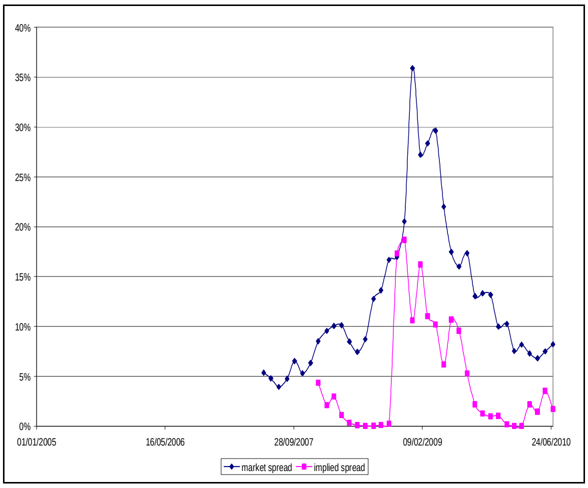

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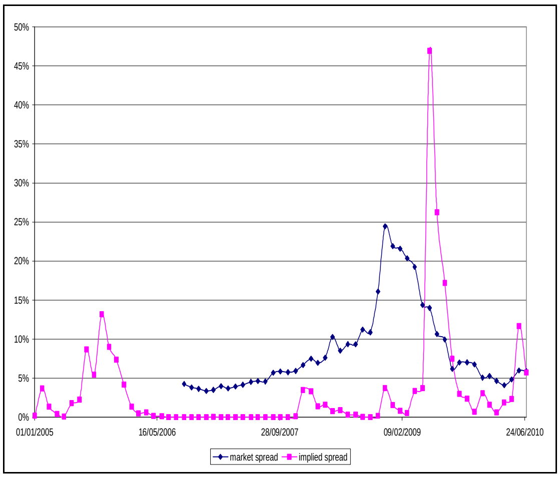

Appendix 2 – List of companies in the sample that encountered financial difficulties

Company name	Sector	Date entered into negotiations (dd/mm/yyyy)	Description of event
Litho Group	Investments and Holdings	19/08/2008	The company announced its desire for an arrangement with bondholders
Direct Capital	Real Estate and Construction	11/12/2008	The bondholders decided to appoint a representative to negotiate with the company
World Capital	Investments and Holdings	29/01/2009	The bondholders decided to appoint a representative to negotiate with the company
Arazim	Real Estate and Construction	04/12/2008	The bondholders decided to appoint a representative to negotiate with the company
Landmark Group	Real Estate and Construction	23/04/2009	The bondholders decided to appoint a representative to negotiate with the company
Eldan Tech	Real Estate and Construction	18/03/2010	The company announced that it was unable to meet its commitments
Rabintex	Manufacturing	11/05/2009	The company announced its desire for an arrangement with bondholders
Evrot	Manufacturing	09/03/2009	The company announced its desire for an arrangement with bondholders
Leader Investments	Investments and Holdings	07/05/2009	The company announced its desire for an arrangement with bondholders
Aloni Meitar	Real Estate and Construction	26/01/2009	The bondholders decided to appoint a representative to negotiate with the company
Sovereign Properties	Real Estate and Construction	08/09/2008	The bondholders decided to appoint a representative to negotiate with the company
Ofek Real Estate	Real Estate and Construction	06/11/2008	The bondholders decided to appoint a representative to negotiate with the company
Peleg Nye	Manufacturing	13/10/2009	The bondholders decided to appoint a representative to negotiate with the company
Boimelgreen Capital	Real Estate and Construction	19/03/2009	The company announced its desire for an arrangement with bondholders
Anter Holdings	Investments and Holdings	31/03/2009	The company filed a request in court for a stay of proceedings
Digel	Real Estate and Construction	20/04/2009	The bondholders decided to appoint a representative to negotiate with the company
Profit	Real Estate and Construction	13/05/2009	The company announced its desire for an arrangement with bondholders
Africa Investments	Real Estate and Construction	30/08/2009	The company announced its desire for an arrangement with bondholders

Elran Investments	Real Estate and Construction	15/02/2009	The bondholders agreed to the deferment of interest and principal payments on B bonds
Polar Communications	Investments and Holdings	26/07/2009	The company announced its desire for an arrangement with bondholders
Osif Investments	Real Estate and Construction	27/05/2009	The company announced the deferment of interest payments on D bonds and the convening of a meeting to assess an arrangement
Angel Resources	Real Estate and Construction	25/05/2009	The bondholders decided to appoint a representative to negotiate with the company
Syngery Cable	Manufacturing	24/01/2010	The company announced its desire for an arrangement with bondholders
Angel Europe	Real Estate and Construction	26/05/2009	The bondholders decided to appoint a representative to negotiate with the company
Asim Investments	Real Estate and Construction	07/04/2009	The company announced its desire for an arrangement with bondholders
Tesuah 10	Real Estate and Construction	22/04/2010	The company announced that it was unable to meet its commitments
Gmul Investments	Investments and Holdings	09/12/2008	The bondholders decided to appoint a representative to negotiate with the company
Polishek	Manufacturing	25/01/2009	The company announced that it was unable to meet its commitments
Globalicom Trade	Goods and Services	15/12/2008	The bondholders decided to appoint a representative to negotiate with the company
Intercolony	Real Estate and Construction	22/04/2009	The bondholders decided to appoint a representative to negotiate with the company
Nidar Construction	Real Estate and Construction	22/08/2008	The company announced that it was unable to meet its commitments
D&E Biomed / Laser Detect	Manufacturing	15/12/2009	The company announced its desire for an arrangement with bondholders
Intercure	Manufacturing	10/01/2010	The bondholders decided to appoint a representative to negotiate with the company
Top Image	Goods and Services	31/12/2009	The company announced its desire for an arrangement with bondholders
Leadcom	Goods and Services	24/06/2009	The bondholders decided to appoint a representative to negotiate with the company

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