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Labor Market Frictions and Optimal Monetary Policy

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Abstract

We study the properties of optimal monetary policy in an environment of nominal wage rigidity and unemployment. We show that nominal wage rigidity increases the sacrifice ratio, and therefore reduces the effectiveness of sacrificing employment in order to stabilize inflation. It follows that in response to higher nominal wage rigidity, it is optimal to allow for smaller fluctuations of unemployment at the expense of larger inflation fluctuations.

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שוק העבודה והמדיניות המוניטרית האופטימלית

אלון בנימיני

תקציר

עבודה זו בוחנת מהי המדיניות המוניטרית האופטימלית בסביבה המתאפיינת בשכר נומינלי קשיח ובאבטלה חיכוכית. אנו מראים שקשיחות של השכר הנומינלי מעלה את "יחס ההקרבה" (sacrifice ratio), ולכן מקטינה את הכדאיות של ייצוב האינפלציה על חשבון יציבות של שיעור האבטלה. לפיכך, ככל השכר הנומינלי יותר קשיח, כך המדיניות האופטימלית תפעל במידה רבה יותר לייצוב האבטלה סביב שיעורה ה"טבעי", גם אם הדבר כרוך בתנודתיות אינפלציה גבוהה יותר.

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1 Introduction

One of the many contributions of micro-founded models is their usefulness in supporting welfare analysis. New Keynesian (NK) models are therefore widely employed to derive optimal monetary plans and to learn their properties. We employ the emerging approach of integrating labor market search and matching frictions into NK models, to explore the properties of the optimal monetary policy under discretion in an environment of unemployment and staggered nominal wages. Our main result refers to the optimal response of unemployment and inflation deviations (from their natural rate and target, respectively) in response to business cycle shocks. More specifically, we explore the optimal ratio of such deviations, namely the optimal unemployment-inflation ratio, and show why it falls with the degree of nominal wage rigidity.

Earlier studies of labor market fluctuations, based on the Diamond-Mortensen-Pissarides (DMP)¹ search and matching setup, generated labor market behavior which was not consistent with observed moments: compared with the results suggested by theoretically based models, observed wages seemed to be much more stable and unemployment much more volatile. This led [Hall \(2005a\)](#) and [Shimer \(2005\)](#) to suggest additions to the equilibrium mechanisms of the labor market, thus introducing real wage rigidities to the search and matching literature.

Similarly, the standard approach taken by the earlier NK literature analyzed environments characterized by a neo-classical labor market, without imperfections such as wage rigidity or unemployment. In such an environment, standard models showed that productivity shocks don't induce any output-inflation tradeoff from the perspective of monetary policy—a result which [Blanchard and Galí \(2007\)](#) referred to as a *divine coincidence*. [Erceg et al. \(2000\)](#) were the first to show that staggered wage contracts break this *divine coincidence*, and were followed by later NK models which introduced such nominal wage rigidity. However, the first such models were still characterized by an otherwise neo-classical labor market, with endogenous adjustments on the intensive margins, but not on the extensive ones. That is, even when staggered nominal wages were assumed, the standard setup in-

¹See [Mortensen \(2011\)](#) for a review of the DMP framework and its evolution.

cluded endogenous working hours but full employment, similar to the original setup of [Erceg et al. \(2000\)](#).²

Recently, the DMP search and matching setup has been integrated into the NK model, enriching the structure by introducing extensive margins which influence the model's dynamics, both quantitatively and qualitatively. [Galí \(2010\)](#) describes some of the essential ingredients and properties of those models and their implications. Labor market frictions can contribute by improving the fit of otherwise standard NK models. But the literature's attention is also focused on theoretical analysis of optimal monetary policy under such labor market imperfections. [Faia \(2008\)](#) and [Blanchard and Galí \(2010\)](#) offer such an analysis based on a setup of real wage rigidity. Both works introduce real wage rigidity in an ad-hoc manner and analyze its implications for a welfare based monetary policy. Other recent papers use nominal wage rigidity, which yields different dynamics and more insights, compared with the case of real rigidity. However, such models with rigid nominal wages can be too complex to draw the optimal policy analytically and to learn its implications. Three relevant examples for such works are [Gertler et al. \(2008\)](#), [Christiano et al. \(2011\)](#) and [de Walque et al. \(2009\)](#). Simpler models, purely motivated by shedding some theoretical light on relevant economic mechanisms, include [Thomas \(2008\)](#) and [Blanchard and Galí \(2010\)](#).³ [Galí \(2010\)](#) enhances the argument made by [Hall \(2005a\)](#), that the most important contribution of labor market frictions lies in making room for wage rigidity which, in turn, has important consequences for business cycles dynamics, with implications for monetary policy.

In the present work we extend the model recently proposed by [Blanchard and Galí \(2010\)](#), hereinafter BG. In their work, BG introduce labor market frictions into an NK model in a very simple manner. Indeed, they emphasize that simplification is one of the

²See, for example, [Christiano et al. \(2005\)](#) and [Smets and Wouters \(2003\)](#).

³Another four related papers are [Trigari \(2009\)](#), [Walsh \(2005\)](#), [Hall \(2005a\)](#) and [Hall \(2005b\)](#). All of them draw on the lines of search and matching frictions in an NK setup. The first two study the response of real variables to monetary policy shocks through a theoretical setup ([Walsh \(2005\)](#)) and an estimation ([Trigari \(2009\)](#)), introducing price rigidities but no nominal wage rigidities. Hall introduces a model with matching frictions and wage stickiness, in which sticky wages do not interfere with the efficiency of matching. He uses this model to explain the observed high volatility of unemployment, vacancies and job-finding rates.

major contributions of their paper. They use a basic NK model into which they integrate real wage rigidity, as well as hiring cost that increases with the ratio of hires to the unemployment pool.⁴ Among its other merits, this simplification enables the implementation of the quadratic approximation of the household utility function, following the popular approach pioneered by [Rotemberg and Woodford \(1998\)](#).

Our extension of the BG model includes rigidity of nominal wages and an explicit solution to the optimal unemployment-inflation ratio under discretion. [Thomas \(2008\)](#) already presents a normative analysis of this kind, explaining that under rigidity of **real** wages (which is the case under the BG specification), the central bank loses most of its leverage over real wages. Along these lines, [Galí \(2010\)](#) also presents a similar extension and analysis. What we essentially add is an explicit solution with a comparative static with respect to some of the labor market frictions. We show that nominal wage rigidity reduces this optimal unemployment-inflation ratio. This seems to be the case since nominal wage rigidity increases the sacrifice ratio, that is the employment that has to be sacrificed in order to stabilize inflation. It follows that with higher nominal wage rigidity, the optimal unemployment-inflation ratio falls, and it is optimal to allow for smaller fluctuations of unemployment at the expense of larger inflation fluctuations.

In addition, we deviate from BG not only by introducing nominal wage rigidity (instead of real rigidity) but we also further endogenize the wage as an equilibrium mechanism, as in [Thomas \(2008\)](#) and [Galí \(2010\)](#). That is, while optimizing, economic agents account for the nominal rigidity and therefore make a forward looking decision, which leads to an equilibrium real wage which is both backward and forward looking. This dynamic nature of the perfectly endogenous real wage introduces an overshooting into the impulse response of the inflation and other variables. As a result, impulse responses under discretionary monetary policy involve an overshooting pattern of inflation which, in a standard and simple NK model, characterizes only the impulse response under commitment, but not under discretion ([Galí, 2008](#), Ch.5, among others). Through the expectations channel, this

⁴Under more standard specification of matching frictions, the relevant definition of labor market tightness is the ratio of vacancies—not hires—to the unemployment pool. BG simplify by assuming that all vacancies are immediately filled, essentially making hiring cost the only matching friction in their model.

overshooting can be very useful in stabilizing the economy following exogenous shocks. As a result, nominal wage rigidity not only affects the optimal unemployment-inflation ratio—by reducing it as discussed above—but it may also help to reduce the amplitudes of both unemployment and inflation, as a response to a given level of exogenous shock.

The rest of the paper is organized as follows. Section 2 presents the model which is then calibrated in Section 3. The mechanisms at work are discussed using impulse response analysis in Section 4. The effects of nominal wage rigidity on optimal monetary policy under discretion are explored in Section 5, which is followed by concluding remarks in Section 6.

2 The model

This section presents the formal setup of an NK model with two labor market imperfections: search and matching frictions and nominal wage rigidity. It then presents and discusses an analytical solution for the optimal policy under discretion.

2.1 The structure

2.1.1 Preferences and technology

The economy consists of large, infinitely lived households, with full risk sharing within each one of them. The representative household maximizes an infinite horizon utility function:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \chi \frac{N_t^{1+\phi}}{1+\phi} \right), \quad (2.1)$$

where E_t denotes expectations formed at period t , β is the time discount factor, $N_t \in [0, 1]$ is the fraction of representative household members who are employed at period t and C_t is a standard [Dixit and Stiglitz \(1977\)](#) sub-utility function of final goods with constant elasticity of substitution (CES), ε .

Households thus have two decisions to make: labor supply, which is discussed below, and a standard consumption-saving decision,

$$E_t \frac{C_{t+1}}{C_t} = \beta E_t \frac{(1+i_t)}{(P_{t+1}/P_t)}, \quad (2.2)$$

where P_t is a standard [Dixit and Stiglitz \(1977\)](#) consumer price index and i_t is the nominal interest rate set by the central bank.

Intermediate goods are produced competitively, using a constant returns to scale technology, with an exogenous productivity shock A_t which is common across firms. Thus, the production technology of the single and competitive intermediate good is

$$X_t(j) = A_t N_t(j), \quad (2.3)$$

where $N_t(j)$ is the employment in firm $j \in [0, 1]$. Assuming big enough families, employment within each family is distributed the same as in the economy, so that

$$N_t = \int_0^1 N_t(j) \cdot dj. \quad (2.4)$$

Finally, differentiated final goods are produced by a continuum of firms which simply brand name and differentiate intermediate goods, so that⁵

$$Y_t(j) = X_t(j). \quad (2.5)$$

2.1.2 The labor market

Employment in firm j evolves as:

$$N_t(j) = (1 - \delta) N_{t-1}(j) + H_t(j), \quad (2.6)$$

where $\delta \in (0, 1)$ is an exogenous separation rate and the variable $H_t(j)$ denotes new hiring by firm j , from the pool of those unemployed at the beginning of the period.⁶ Aggregating based on (2.4), we get

$$N_t = (1 - \delta) N_{t-1} + H_t, \quad (2.7)$$

where $H_t \equiv \int_0^1 H_t(j)$.

⁵We adopt the BG separation of intermediate and final goods firms, so as to avoid interaction between price setting and wage bargaining at the firm level.

⁶[Hall \(2005b\)](#), [Shimer \(2005\)](#) and [Shimer \(2007\)](#) report some findings supporting the simplifying assumption of exogenous and constant separation rate, according to which unemployment fluctuates mostly due to variations in hiring.

We assume full participation, from which it follows that end-of-period unemployment is:

$$u_t = 1 - N_t, \quad (2.8)$$

and unemployment at the beginning of the period is:

$$U_t = 1 - (1 - \delta) N_{t-1}. \quad (2.9)$$

Labor market tightness is defined as:⁷

$$x_t = H_t/U_t, \quad (2.10)$$

where only those unemployed at the beginning of the period can be hired—that is, $H_t \leq U_t \implies x_t \in [0, 1]$ —and they start working at the same period. We thus assume that vacancies are filled immediately. We also follow the BG assumption, that shocks are small enough, so that hires are positive at all times. Finally, the producer of the intermediate good is subject to hiring cost, $G_t H_t(j)$, where⁸

$$G_t = A_t B x_t^\alpha; \quad B > 0, \quad \alpha \geq 0. \quad (2.11)$$

This hiring cost is expressed in terms of the CES bundle of goods. We therefore obtain the following market clearing condition:

$$C_t = A_t (N_t - B x_t^\alpha H_t). \quad (2.12)$$

2.2 Flexible wage benchmark

We begin this subsection assuming both wage and price flexibility. We close it by explaining why the following results are satisfied as long as wages are flexible, in the case of price rigidity as well.

⁷Following BG, our definition is slightly different than the standard market tightness definition, which is usually the vacancies to unemployment ratio. See review by [Yashiv \(2007\)](#).

⁸BG discuss differences and analogies between this specification and the standard one, which includes a different definition for labor market tightness and uncertainty with respect to filling vacancies.

Let W_t^m denote the real wage of the marginal worker, and P_t^I the price of intermediate goods. It then follows that profit maximization by the intermediate-goods firms leads to the following equality between their marginal cost and price:

$$\frac{P_t^I}{P_t} = \frac{MC_t^I}{P_t} = \frac{W_t^m + G_t - \beta(1 - \delta) E_t \left(\frac{C_t}{C_{t+1}} G_{t+1} \right)}{A_t}, \quad (2.13)$$

where MC_t^I denotes the real marginal cost for the intermediate-goods firms. Here the standard marginal cost is generalized by the addition of hiring cost, net of the discounted-saved hiring cost next period.

Under flexible prices, optimal price setting by retailers yields the standard markup over their marginal cost:

$$P_t = M P_t^I, \quad (2.14)$$

where the optimal markup is $M = \varepsilon / (\varepsilon - 1)$. Under rigid prices, however, the actual markup fluctuates and therefore deviates from the optimal one, M .

We use the Nash bargaining solution to characterize the real wage in equilibrium. By definition, the labor market tightness, x_t , is also the job finding rate. Hence, letting W_t^{Flex} denote the flexible real wage, which is therefore symmetric across the economy, the value of an employed member to the household is:

$$V_t^N = W_t^{Flex} - \chi C_t N_t^\phi + \beta E_t \left\{ \frac{C_t}{C_{t+1}} [(1 - \delta(1 - x_{t+1})) V_{t+1}^N + \delta(1 - x_{t+1}) V_{t+1}^U] \right\}, \quad (2.15)$$

where, V_t^U , the value of an unemployed member is:

$$V_t^U = \beta E_t \left\{ \frac{C_t}{C_{t+1}} [x_{t+1} V_{t+1}^N + (1 - x_{t+1}) V_{t+1}^U] \right\}. \quad (2.16)$$

The Nash bargaining solution under flexible wages is the wage level that solves

$$\max (S_t^H)^\eta (S_t^F)^{(1-\eta)}, \quad (2.17)$$

where the parameter η denotes the bargaining power of the households, the surplus from employment is

$$\begin{aligned} S_t^H &\equiv V_t^N - V_t^U \\ &= W_t^{Flex} - \chi C_t N_t^\phi + (1 - \delta) \beta E_t \left\{ \frac{C_t}{C_{t+1}} (1 - x_{t+1}) S_{t+1}^H \right\}, \end{aligned} \quad (2.18)$$

and the firm surplus from an established employment relationship is simply the hiring cost saved at the margins,

$$S_t^F = G_t = A_t B x_t^\alpha. \quad (2.19)$$

With $\vartheta \equiv \frac{\eta}{1-\eta}$ denoting the relative bargaining power of the households, the solution to the Nash bargaining problem (2.17) is $S_t^H = \vartheta S_t^F$. Using (2.18) and (2.19), the real wage that satisfies this solution under the flexible wages benchmark is

$$W_t^{Flex} = \chi C_t N_t^\phi + \vartheta \left\{ A_t B x_t^\alpha - \beta (1 - \delta) E_t \left[\frac{C_t}{C_{t+1}} (1 - x_{t+1}) A_{t+1} B x_{t+1}^\alpha \right] \right\}. \quad (2.20)$$

Recall that at this point we assume both wage and price flexibility. Also note that under flexible wages we can substitute $W_t^m = W_t^{Flex}$. Substituting the pricing condition of the final good firm (2.14) into the one of the intermediate good firm (2.13), we can solve for the wage level which is consistent with the pricing decision. From (2.20) we get the wage level consistent with Nash bargaining. Comparing the two, and substituting the market clearing condition (2.12) to eliminate the consumption levels, we get an expression which does not include the productivity level, A_t . Using (2.7, 2.9 and 2.10) as well, we can solve for all the labor market pools and flows, which are therefore constant under flexible prices and wages. This is a result of the specific BG preferences structure, in which income and substitution effects on labor supply cancel each other out. Under this specification, productivity shocks are fully absorbed by real wages and quantities of goods—output, consumption and hiring costs—while labor market pools and flows remain constant. More generally, this result reflects what [Blanchard and Galí \(2007\)](#) called the *divine coincidence*: that under flexible wages there is an equivalence between the efficient and the natural employment (or output in their paper), where the second is defined as the one consistent with price stability.

Let x denote the constant labor market tightness under the benchmark of flexible wages and prices. It follows from the marginal cost and optimal pricing (2.13 and 2.14), that the real wage under this benchmark is:

$$W_t^{Flex} = \Theta A_t, \quad (2.21)$$

where $\Theta \equiv 1/M - [1 - \beta (1 - \delta)] B x^\alpha$.

Now, substituting (2.11), (2.12) and (2.21) into (2.13), we get

$$\frac{MC_t^I}{P_t} = \frac{P_t^I}{P_t} = \Theta + Bx_t^\alpha - \beta(1 - \delta) E_t \left(\frac{(N_t - Bx_t^\alpha H_t)}{(N_{t+1} - Bx_{t+1}^\alpha H_{t+1})} Bx_{t+1}^\alpha \right).$$

This means that, even without assuming flexible prices, as long as the wage level satisfies (2.21), the real marginal cost can be expressed in terms of labor market pools alone (without any direct effect of the productivity shock). That is, as long as the wage level is flexibly set according to (2.21), there is no tradeoff between inflation and labor market pools. It follows that, as long as wages are set flexibly, a welfare maximizing central bank would continuously keep inflation at zero, which is enough for the above results to be satisfied: under flexible wages, the wage level would evolve according to (2.21), so that productivity shocks would drive wages and quantities of goods (consumption, output and hiring costs), leaving labor market pools unchanged.⁹

2.3 Equilibrium with nominal wage rigidities

Here, while introducing nominal wage rigidity, we depart from the BG specification. We use the Calvo (1983) setup, assuming that every period a randomly selected portion of the existing wage contracts, $\theta^w \in (0, 1)$, are not renegotiated,¹⁰ whereas new contracts are assigned to the average wage.¹¹ Arguably, it might be more empirically appealing to impose wage update constraints on the firm level, rather than on the contract level. For instance, Thomas (2008) uses the Calvo (1983) formalization, assuming that randomly selected *firms* renegotiate wage contracts, so that—within each firm—newly hired workers receive the same wage as continuing workers. While Thomas (2008) also justifies his choice

⁹Appendix A shows another BG result: if $M = 1$ and $\vartheta = \alpha$, the flexible wage equilibrium renders the optimal allocation. The first condition, $M = 1$, can be satisfied with an employment subsidy in place, that would completely offset the monopolistic distortion. The externalities relevant to the second condition, $\vartheta = \alpha$, are familiar from Hosios (1990).

¹⁰We assume that this random selection is realized only after separations take place. Thus, whether wages are renegotiated or not is not a consideration that affects (the exogenous) separations. Also note that, due to the Calvo-style wage rigidity, there is wage dispersion. However, with complete risk sharing within each one of the big households, such wage dispersion does not cause consumption dispersion and we are therefore left with a standard, representative household investment-saving decision.

¹¹Galí (2010) reviews some of the literature and arguments concerning alternative wage setting schemes for new hires. He suggests that the empirical evidence on its relevance seems controversial.

based on empirical findings from the literature, our modeling choice is a simplifying one: even though we use linear technology, our wage scheme enables treating all intermediate-goods producers symmetrically, assuming they share the same marginal wage, W_t^m , and therefore the same marginal cost (2.13).¹² Applying the law of large numbers, we get the law of motion for the average nominal wage:

$$w_t = \theta^w w_{t-1} + (1 - \theta^w) w_t^*, \quad (2.22)$$

where small letter denotes the nominal wage. Thus, the variable w_t denotes the weighted average of nominal wage in the economy as a whole and the variable w_t^* denotes the equilibrium nominal wage of the subset of wages renegotiated at period t . Whenever renegotiated, the nominal wage schedule based on Nash bargaining ends up satisfying the following condition:

$$E_t \sum_{i=0}^{\infty} \left\{ [\beta \theta^w (1 - \delta)]^i (w_t^* - \Theta A_{t+i} P_{t+i}) \right\} = 0. \quad (2.23)$$

That is, whenever firms and workers renegotiate wages, they account for the expected path of productivity and price levels. Thus, the negotiated wage ends up being a weighted average of future expected Nash bargaining results under the flexible wages benchmark.¹³ The weights consist of the relevant time-discount factor, β^i , the job survival probability, $(1 - \delta)^i$, and the survival probability of the negotiated wage, $(\theta^w)^i$.

Combining equations 2.22 and 2.23, together with the definition of the aggregate real wage, $W_t \equiv w_t/P_t$, we get a dynamic expression for the aggregate real wage, W_t (step by step derivation is provided in appendix B below):

$$W_t = \gamma_{bw} \cdot W_{t-1} \frac{P_{t-1}}{P_t} + \gamma_{fw} \cdot E_t \left[W_{t+1} \frac{P_{t+1}}{P_t} \right] + \gamma_{flex} \cdot W_t^{Flex}. \quad (2.24)$$

¹²Alternatively, Galí (2010) assumes decreasing returns to scale, which enables equating marginal costs across firms even when they face heterogenous marginal wages. The specification of Galí (2010), however, complicates the welfare based criterion of the central bank, by making it also a function of wage inflation.

¹³For this Nash bargaining result to hold, it should be verified that both households and firms have non-negative surplus under the new real wage, so that neither of them has an incentive to terminate the employment relationship (see Hall (2005a) and Galí (2010)). Therefore, the following condition should hold: $\chi C_t N_t^\phi \leq W_t \leq A_t/M$. Like BG, we assume that the economy fluctuates due to productivity shocks that are small enough for this condition to hold.

Here the backward looking coefficient is $\gamma_{bw} \equiv \theta^w / \gamma$ (where we define $\gamma \equiv 1 + \beta (\theta^w)^2 (1 - \delta)$), the forward looking coefficient is $\gamma_{fw} \equiv \beta \theta^w (1 - \delta) / \gamma$, and the elasticity with respect to the flexible wage benchmark is $\gamma_{flex} \equiv (1 - \theta^w) [1 - \beta \theta^w (1 - \delta)] / \gamma$. That is, the real wage is a function of the contemporaneous productivity shock, but due to nominal frictions it is also a function of past and future-expected real wage and inflation.

2.4 Log linearization, Phillips curve and Fischer equation

Before solving for the optimal policy rule, we log-linearize the model around its purely deterministic steady state. Appendix C presents the log-linearized system of equations, where hat over a variable denotes logarithmic deviation from the purely deterministic steady state.¹⁴ In addition to the policy rule, which is treated in the next subsection, the log-linearized system includes two equations not described in the preceding text. The first one is a standard NK Phillips curve (NKPC) for inflation ($\hat{\pi}_t \equiv \ln(P_{t+1}/P_t)$),

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \lambda \cdot \widehat{mc}_t. \quad (2.25)$$

Here $\lambda \equiv (1 - \beta \theta^p) (1 - \beta) / \theta^p$, where θ^p is the degree of goods price rigidity, defined in an analogous way to its nominal wage counterpart θ^w (see Woodford (2003) or Galí (2008), among others, for a step by step derivation). The second is the Fischer equation, connecting real and nominal interest rates (\hat{r}_t and \hat{i}_t , respectively):

$$\hat{r}_t \equiv \hat{i}_t - E_t \hat{\pi}_{t+1}.$$

All together, the log-linearized system includes nine equations and the following nine variables:

$$\left\{ \hat{x}_t, \hat{n}_t, \hat{c}_t, \widehat{mc}_t, \hat{w}_t, \hat{\pi}_t, \hat{r}_t, \hat{i}_t, \hat{u}_t \right\}.$$

¹⁴Unemployment in the log linearized version of the model is an exception, being expressed in terms of percentage points. The motivation is technical, and is mentioned in appendix C.

2.5 A welfare based policy

We close the model using an optimal policy rule. The central bank sets its policy instrument, the nominal interest rate, so as to minimize the following utility based loss function:¹⁵

$$Loss_t = E_t \sum_{i=0}^{\infty} \beta^i (\widehat{\pi}_{t+i}^2 + \alpha_u \widehat{u}_{t+i}^2), \quad (2.26)$$

where $\alpha_u \equiv \frac{\lambda}{\varepsilon} (1 + \phi) \chi (1 - u)^{\phi-1} > 0$, the parameter u denotes the constant unemployment rate under flexible wages and $\widehat{u}_t \equiv u_t - u$ is the deviations from this rate, expressed in terms of percentage points (pp). Under slightly different assumptions, as in [Thomas \(2008\)](#) and [Galí \(2010\)](#), the welfare based criterion of the central bank would also be a function of wage inflation.¹⁶ Note that, while the degree of price rigidity enters the loss function,¹⁷ the degree of wage rigidity does not. Yet, the degree of wage rigidity will end up affecting the optimal policy. Derivation of the loss function (2.26) as a quadratic approximation of the household utility function (2.1) is provided by appendix D. In a setup which includes nominal wage rigidity, [Erceg et al. \(2000\)](#) showed that nominal *wage* inflation is also welfare reducing. However, simplifying assumptions adopted by the present work yielded a loss function (2.26) without this feature: since we use the same structural equations for preferences and technology as do BG, and considering our particular assumptions concerning the wage structure, the loss function (2.26) ends up being identical to theirs.

As discussed in Section 2.2, under flexible wages productivity shocks do not induce an unemployment-inflation tradeoff and, therefore, both variables are stable at their optimal rates. Nominal wage rigidity is therefore welfare reducing: it induces a tradeoff, and therefore fluctuations, of unemployment and inflation as a response to productivity shocks.¹⁸

¹⁵Like BG, we simplify by assuming that a constant employment subsidy is in place, so as to offset the monopolistic distortion, and that the [Hosios \(1990\)](#)-like condition, $\vartheta = \alpha$, is satisfied. These assumptions are justified by the results in appendix A: they assure that deviations from the steady-state allocation are indeed deviations from the optimal one.

¹⁶Which is not the case in our setup, thanks to the linear technology we assume. This linearity, in turn, requires the particular assumptions we make with regard to the wage setting scheme (see footnote 12).

¹⁷ α_u contains λ , which consists of the the goods price rigidity, θ^p .

¹⁸Although household utility (2.1) is not a direct function of inflation, its quadratic approximation (2.26) is. The welfare-reducing effect of inflation fluctuations has the standard NK explanation, and is related to its distorting effect on allocation: due to the interaction of staggered pricing and finite elasticity of substitution

Now, using some straightforward substitutions of the log-linearized system presented in appendix C, and ignoring all terms irrelevant for discretion (past, future-expected and exogenous values) the Lagrangian under discretion takes the following form:

$$\mathfrak{L} = \frac{1}{2} (\widehat{\pi}_t^2 + \alpha_u \cdot \widehat{u}_t^2) - \psi (\widehat{\pi}_t + \Upsilon \cdot \widehat{u}_t), \quad (2.27)$$

where ψ is a Lagrange multiplier and

$$\Upsilon \equiv \lambda \frac{gM}{(1-u)\delta} \left[\alpha + \delta\beta(1-\delta) \frac{(\alpha g + g - 1)}{(1-\delta g)} \right] / \left[1 + \lambda \bar{w} M \frac{\theta^w}{1 + \beta(\theta^w)^2(1-\delta)} \right]$$

is a positive value under our calibration. Roughly speaking, the Lagrangian (2.27) implies that the reduced-form parameter Υ can be intuitively interpreted as being inversely related to the sacrifice ratio.

It then follows that discretionary monetary policy aimed at minimizing the loss function (2.26) leads to the following policy rule:

$$\widehat{u}_t = \frac{\Upsilon}{\alpha_u} \widehat{\pi}_t. \quad (2.28)$$

The central bank sets the interest rate i_t so as to satisfy the optimality condition (2.28). It is straightforward from the optimal rule (2.28) that the optimal unemployment-inflation ratio falls with the sacrifice ratio (which is inversely related to Υ) and with the degree of aversion to unemployment fluctuations. A comparative static concerning this condition is discussed in Section 5 below.

3 Calibration

We consider every period to be a quarter. Accordingly, we calibrate the model based on quarterly series for the period 1998:Q1-2011:Q3.¹⁹ The parameters are calibrated based on

across final goods (and therefore across differentiated types of labor efforts), inflation increases the aggregate labor effort required to achieve a given utility from consumption. The presence of the unemployment rate in the quadratic approximation (2.26) is more straightforward: households utility (2.1) is a function of labor effort, which is related to the unemployment rate by a structural identity (2.8). Since the optimal allocation involves constant employment (as implied by the discussion in Section 2.2), deviations from this constant are welfare reducing.

¹⁹Starting from the first quarter of 1998, there is available data about separations and hiring in the business sector, based on a review by the Ministry of Industry, Trade and Labor.

either first moments observed in the data or robust results from the literature. We use BG calibration as our baseline. Table 3.1 presents the calibrated values of the parameters.

Table 3.1: Calibrated values

Parameter	Description	Value
α	Hiring cost elasticity	1.00
β	Discount factor	0.99
δ	Separation	0.09
ε	CES	6.00
ϕ	Labor supply elasticity	1.00
θ^p	Prices Calvo	0.60
θ^w	Wages Calvo	0.75
B	Hiring cost constant	0.20
u	Unemployment	0.084
x	Tightness	0.50

Preference parameters are calibrated using common practice: $\beta = 0.99$, $\phi = 1$ and $\varepsilon = 6$.

Based on first moments observed in the data, we calibrate $u = 8.4\%$ and $\delta = 9.0\%$. Note that while u is expressed in terms of percents of the labor force, δ is expressed in terms of percents of the working population. With some straightforward algebra we get the following steady state linkage:

$$x = \frac{\delta(1-u)}{1-(1-\delta)(1-u)},$$

which implies the calibration $x = 0.5$.

Based on estimation results by [Argov et al. \(2012\)](#), we calibrate the price rigidity parameter to be $\theta^p = 0.60$. The wage rigidity parameter is calibrated to be $\theta^w = 0.75$, a value that reflects a wage update frequency of once a year on average.^{20,21}

We are left with the parameters α and B to be calibrated. Following BG, we parameterize $\alpha = 1$, and we calibrate B so that the steady-state hiring cost would be lower than

²⁰[Galí \(2010\)](#) reviews papers that find evidence for similar, and even stronger, nominal wage rigidity based on micro data and surveys from U.S. and European economies.

²¹Note that BG choose to calibrate $\theta = 0.75$. This implies a price update frequency of once a year on average, in line with our assumption regarding the degree of nominal wage rigidity. We choose $\theta = 0.60$, assuming that the price of goods are less sticky than nominal wages.

1% of GDP. This implies $\frac{ABx^\alpha H}{AN} = \frac{ABx^\alpha \delta N}{AN} = Bx^\alpha \delta < 0.01 \implies B < \frac{0.01}{x^\alpha \delta} = \frac{0.01}{0.5 \cdot 0.09} = 0.22$.

We therefore use the calibration $B = 0.20$.

4 Impulse response

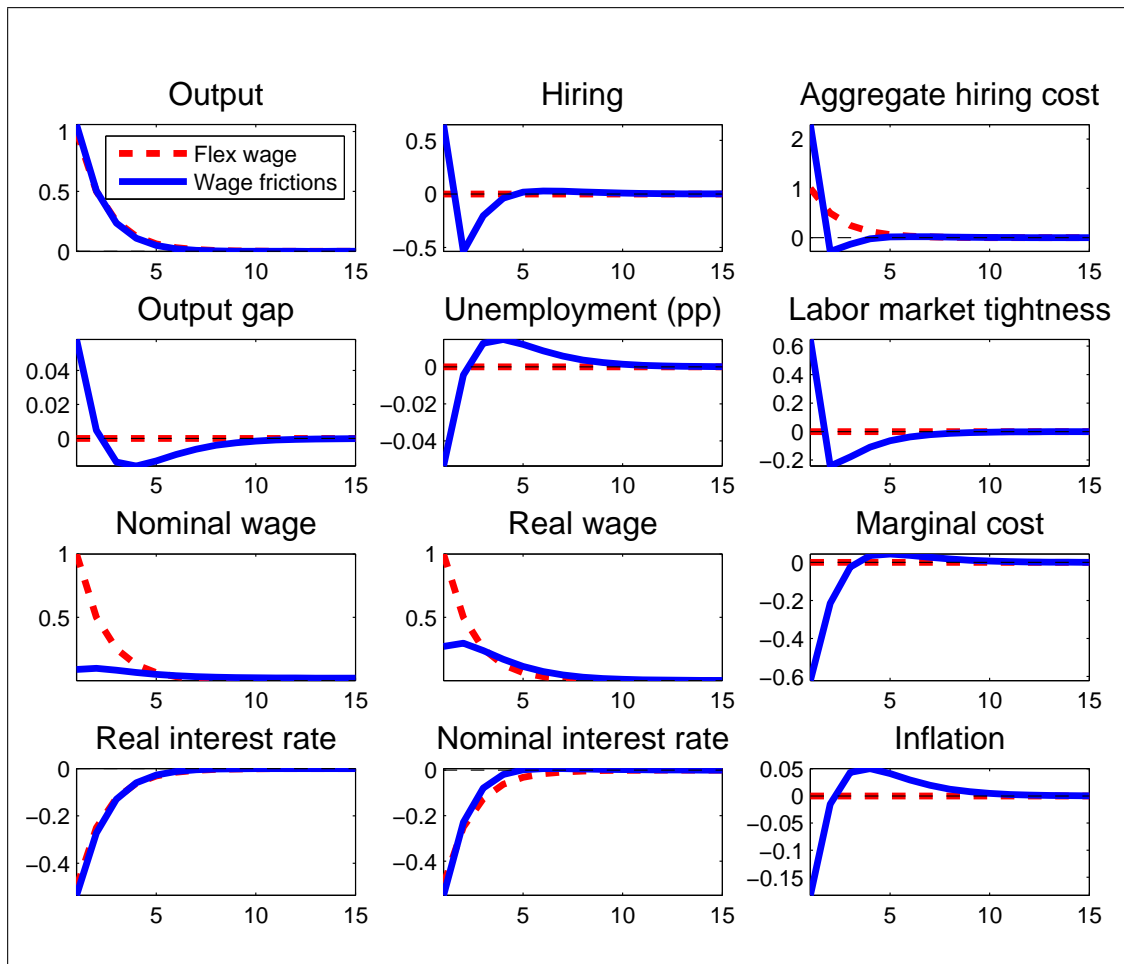


Figure 4.1: Impulse response to a serially correlated productivity shock.

The productivity shock is an AR(1) process, with the degree of persistence being equal to 0.5.

Dashed line — a flexible wage benchmark. Solid line — dynamics under nominal wage frictions.

Since under the flexible wage benchmark productivity shock does not induce any trade-off between inflation and unemployment, they both remain constant under this benchmark.

Figure 4.1 demonstrates that, under the flexible wage benchmark, productivity shock in-

creases output and wages, leaving other labor market conditions unchanged. From the (labor) supply side, the logarithmic utility from consumption is responsible for this result, by inducing income and substitution effects that cancel each other out. The increase of wage and hiring cost combined with the increase in productivity leave the marginal cost unchanged. Therefore, inflation remains in line with its target as well. Interest rate falls however, both nominal and real, so as to clear a good market with higher output.

Under nominal wage rigidity this is no longer the case, and productivity shock does induce an inflation-unemployment tradeoff. The on-impact response to a productivity shock under nominal wage rigidity is that the real wage ends up being lower than its flexible-benchmark counterpart. Hence, labor effort increases on impact due to higher demand, but also due to a higher supply of labor.

Considering the lower wage, increased labor *demand* is straightforward. The on-impact increase in labor *supply* is driven by income effect and by continuation value, which dominate a negative substitution effect. While the substitution effect is subject only to an intra-temporal consideration, the first two are forward looking: permanent income falls due to lower present and future wages, and continuation value increases since the future labor market is less tight and future wages are higher (compared with the flexible benchmark). Such continuation-value consideration is unique to a labor market with frictions on the external margins, and is absent in a neo-classical labor market which characterizes the simple NK model. On impact we thus end up with higher labor effort relative to the flexible wage benchmark, and therefore with lower unemployment. The increased labor effort is reflected by a positive output gap, that is, a gap between the rigid-wage output and the flexible-wage output.

At the same time there is also a negative inflation on impact: the real wage, being lower than its flexible wage counterpart, drives real marginal cost down.²² It is impossible to increase inflation without further reducing unemployment. Hence, the inflation-unemployment tradeoff under nominal wage rigidity is reflected by this impulse response to productivity shock, with both unemployment and inflation deviating from their steady-

²²While the direct effect of lower inflation on the real wage is positive, this effect is obviously not the dominant one.

state rates.

Figure 4.1 shows that the convergence under nominal wage rigidity is characterized by the overshooting of some variables. This is so since the wage rigidity, the same rigidity that on impact attenuates the wage increase, also delays the convergence back to the steady-state wage. That is, while on impact the rigidity causes a wage which is *lower* than the benchmark, down the road the same rigidity causes a wage which is *higher* than the benchmark. Thus, it is the dynamic nature of the wage that is responsible for the overshooting of inflation and other variables. In turn, the dynamic nature of the wage results from the rigidity being specified as part of the structure, that is, the rigidity is accounted for during optimization and bargaining by the economic agents. This is one of the added values from extending the BG specification. With regard to inflation, such overshoot implies a path that under the basic NK model is typical to monetary policy under commitment, not to discretionary policy which characterizes the present analysis. With inflation being forward looking, which is the case under our standard NKPC (2.25), the overshooting is helpful in stabilizing inflation and, by extension, the economy as a whole. It means that nominal wage rigidity, in addition to its effect on the optimal unemployment-inflation ratio, is also helpful in reducing the volatility of the entire system in response to a given shock.

To shed some more light on the economic mechanisms at work, appendix E depicts the response to a cost-push shock.²³

5 Optimal monetary policy

Provided that productivity shocks induce an unemployment-inflation tradeoff under price and wage rigidity, we saw that an optimal, discretionary monetary policy (2.28) should accommodate such shocks by allowing fluctuations of both unemployment and inflation. Figure 5.1 presents the optimal ratio between unemployment and inflation deviations from their steady-state levels, henceforth the $\hat{u}_t/\hat{\pi}_t$ ratio, as a function of selected structural parameters.

²³That is, an ad hoc shock to the NKPC (2.25).

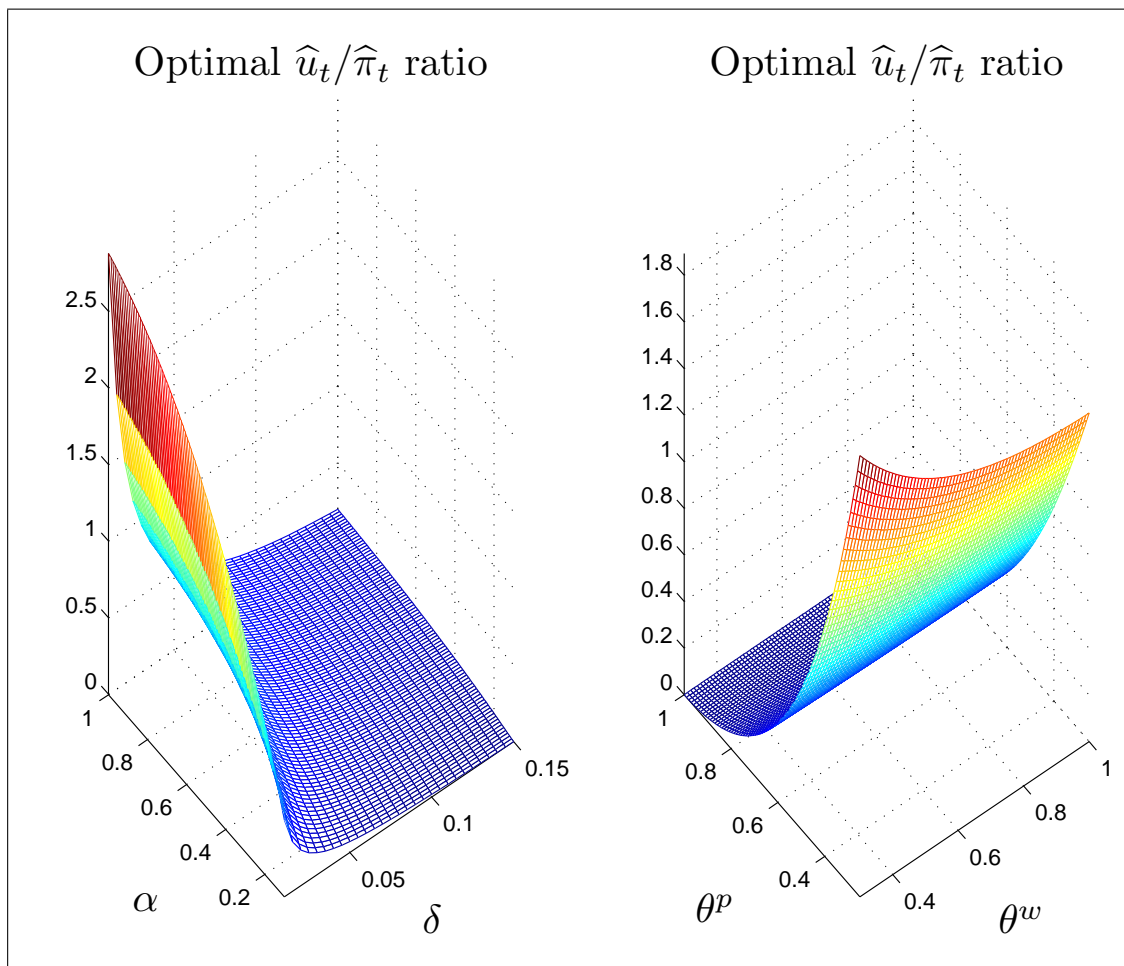


Figure 5.1: Optimal unemployment-inflation ratio as a function of structural parameters.

The plot on the right side of Figure 5.1 presents the optimal $\widehat{u}_t/\widehat{\pi}_t$ ratio as a function of price and wage rigidities— θ^p and θ^w , respectively. Since these parameters denote the probability that an update signal is not received, as they grow from zero to one nominal prices and wages become more rigid. Hence, the right side of the figure shows that the optimal $\widehat{u}_t/\widehat{\pi}_t$ ratio falls with nominal rigidity. That is, when nominal rigidity increases, it is optimal to have smaller deviations of unemployment (from its natural rate) at the expense of inflation deviations. Here, both wage and price rigidity increase the sacrifice ratio, which is inversely related to the reduced-form parameter in the Lagrangian (2.27), Υ . The associated economic mechanism is simple — when nominal rigidity increases, wages are less responsive to activity and, therefore, so are marginal costs which drive inflation. Thus, as nominal rigidity increases, it is less effective to sacrifice unemployment for stabilizing inflation. This effect dominates another one which works in the opposite direction: higher goods-price rigidity increases the importance of inflation stabilization.²⁴

The left side of Figure 5.1 presents the optimal $\widehat{u}_t/\widehat{\pi}_t$ ratio with respect to two other structural parameters: the separation rate and the hiring cost elasticity with respect to labor market tightness— δ and α , respectively. These two parameters are related to real labor market frictions. The figure shows that the optimal $\widehat{u}_t/\widehat{\pi}_t$ ratio falls with separation but increases with the hiring cost elasticity. To understand the influence of these parameters on the optimal ratio, it is useful to consider their sacrifice ratio effect. Higher separation rate, δ , reduces the continuation value of established employment relationships, thus making the actual real wage more responsive to the contemporaneous flexible one.²⁵ Hence, higher separation means a higher sacrifice ratio, which leads to a lower optimal $\widehat{u}_t/\widehat{\pi}_t$ ratio. The hiring cost elasticity, α , works in the other direction—it increases marginal cost elasticity,

²⁴Being (negatively) related to λ , price rigidity θ^p directly enters the welfare criterion (2.26) through α_u , thus increasing the importance of inflation stabilization. This is a standard NK result, reflecting the idea that the distorting effect of inflation grows with nominal rigidity and therefore motivates a higher, not lower $\widehat{u}_t/\widehat{\pi}_t$ ratio. In the standard NK model, as in Woodford (2003) for instance, the two forces—namely, the sacrifice ratio effect and the direct effect on the welfare criterion—cancel each other out and the optimal ratio between inflation and the so called output gap under discretion is invariant to the degree of price rigidity. Here, however, with the existence of both price and wage rigidity, the sacrifice ratio effect dominates the direct effect on the welfare criterion, so that the optimal $\widehat{u}_t/\widehat{\pi}_t$ ratio falls with nominal rigidity.

²⁵This is evident, most clearly, by the result of Nash bargaining under rigid wages (2.23).

hence inflation elasticity, with respect to labor market tightness. It follows that the sacrifice ratio falls with α (indeed, it is easy to verify that $\partial\Upsilon/\partial\alpha > 0$), which therefore increases the optimal $\widehat{u}_t/\widehat{\pi}_t$ ratio.

6 Concluding remarks

We present a simple new Keynesian model with two labor market imperfections—search and matching frictions and nominal wage rigidity. In such case, it is no longer possible to achieve what [Blanchard and Galí \(2007\)](#) referred to as a *divine coincidence*: constant employment and zero inflation when the economy is subject to productivity shocks, which is shown to be the optimal equilibrium result under flexible wages. We then discuss how monetary policy under discretion is influenced by these labor market imperfections. That is, we discuss how the optimal ratio between deviations of unemployment and inflation—from their natural rate and target, respectively—changes with the intensity of these imperfections.

Similar to the conclusions of [Thomas \(2008\)](#) and [Blanchard and Galí \(2010\)](#), we show that labor market frictions imply that optimal monetary policy should accommodate some inflation and limit the degree of unemployment fluctuations. We show that labor market imperfections affect the sacrifice ratio, which in turn affects the optimal ratio between unemployment and inflation deviations. Imperfections that reduce the slope of the Phillips curve, thus increasing the sacrifice ratio, reduce the optimal unemployment-inflation ratio and *vice versa*. The idea is that using unemployment to stabilize inflation is less efficient when the sacrifice ratio increases.

We further show that while some labor market imperfections increase the sacrifice ratio, and therefore reduce the optimal unemployment-inflation ratio, other imperfections may have the opposite effect. Two of the imperfections we discuss, namely nominal wage rigidity and the separation rate, increase the sacrifice ratio, therefore reducing the optimal ratio between unemployment and inflation deviations. At the same time, a third imperfection has the opposite effect—hiring cost elasticity with respect to labor market tightness.

While this work examines a monetary policy under discretion, it would be interesting to explore the mechanisms at work under commitment. The interaction between policy

and the various expectation channels of the economy is richer under commitment, compared with the case of discretion. Nominal wage rigidity further enriches these channels by contributing both backward- and forward-looking channels. For instance, analyzing the impulse response to productivity shock under *discretionary* monetary policy (Section 4), we saw that wage rigidity causes inflation to have an overshooting response which, under the basic NK model, is only typical of monetary policy under *commitment*. Through the expectations channel, such overshooting can be very useful in stabilizing the economy, by reducing the amplitudes of both unemployment and inflation as a response to a given level of exogenous shock. It is therefore a natural next step to explore the interactions between nominal wage rigidity and monetary policy under commitment.

Appendices

Appendix A Optimal allocation

In this appendix we derive an implicit solution to the optimal labor market tightness, x . The result will be useful to compare the flexible wage to the optimal equilibria, as well as while deriving the quadratic loss function (2.26) in appendix D below.

The optimality condition denoted by the planner's problem is:²⁶

$$\chi C_t N_t^\phi \leq A_t - (1 + \alpha) A_t B x_t^\alpha + \beta (1 - \delta) E_t \left\{ \frac{C_t}{C_{t+1}} A_{t+1} B x_{t+1}^\alpha [1 + \alpha (1 - x_{t+1})] \right\}.$$

Substituting the resource constraint (2.12) in, we get:

$$\chi (N_t - B x_t^\alpha H_t) N_t^\phi \leq 1 - (1 + \alpha) B x_t^\alpha + \beta (1 - \delta) E_t \left\{ \frac{(N_t - B x_t^\alpha H_t)}{(N_{t+1} - B x_{t+1}^\alpha H_{t+1})} B x_{t+1}^\alpha [1 + \alpha (1 - x_{t+1})] \right\}.$$

Note that the exogenous productivity shock is cancelled out in the last expression, which includes only labor market pools. This is consistent with Subsection 2.2, from which we know that under flexible wages there is no unemployment-inflation tradeoff and that, as a

²⁶Interpretations are discussed by BG.

result, labor market pools and flows end up being constant. Therefore, the last condition can be further simplified to:

$$\chi(1 - Bx^\alpha\delta)N^{1+\phi} = 1 - (1 + \alpha)Bx^\alpha + \beta(1 - \delta)Bx^\alpha[1 + \alpha(1 - x)], \quad (\text{A.1})$$

where we also used (2.7) and drop the time subindex t to denote constant levels.

We can compare the optimal allocation to the market solution under flexible wages. The optimal allocation is reflected by either one of the last two expressions. The second, the market solution achieved under flexible wages, is reflected by the solution to the Nash bargaining problem under flexible wages, with (2.21) substituted into (2.20). It is easy to verify that the two solutions are equivalent if $M = 1$ and $\vartheta = \alpha$.

Now using (2.7) and (2.9), we get:

$$N = \frac{x}{\delta + (1 - \delta)x},$$

which we also used in the calibration (Section 3). Substituting into (A.1), we get an implicit expression for the optimal labor market tightness, x .

Appendix B The real wage

Rearranging the nominal wage consistent with Nash bargaining (2.23) we get:

$$\begin{aligned} w_t^* &= [1 - \beta\theta^w(1 - \delta)] E_t \sum_{i=0}^{\infty} \left\{ [\beta\theta^w(1 - \delta)]^i \Theta A_{t+i} P_{t+i} \right\} = \\ & [1 - \beta\theta^w(1 - \delta)] \Theta A_t P_t + [1 - \beta\theta^w(1 - \delta)] E_t \sum_{i=1}^{\infty} \left\{ [\beta\theta^w(1 - \delta)]^i \Theta A_{t+i} P_{t+i} \right\} = \\ & [1 - \beta\theta^w(1 - \delta)] \Theta A_t P_t + [1 - \beta\theta^w(1 - \delta)] \beta\theta^w(1 - \delta) E_t \sum_{i=0}^{\infty} \left\{ [\beta\theta^w(1 - \delta)]^i \Theta A_{t+i+1} P_{t+i+1} \right\}. \end{aligned}$$

From which it follows that:

$$w_t^* = [1 - \beta\theta^w(1 - \delta)] \Theta A_t P_t + \beta\theta^w(1 - \delta) E_t w_{t+1}^{Opt}.$$

Now rearranging the aggregate nominal wage (2.22), we can express it as a weighted average between past and present aggregate nominal wages:

$$w_t^* = \frac{1}{1 - \theta^w} w_t - \frac{\theta^w}{1 - \theta^w} w_{t-1}.$$

Comparing the last two equations and rearranging, we get the nominal wage as a function of past and future-expected nominal wages, and of present productivity and price levels:

$$w_t = \frac{\theta^w w_{t-1} + \beta \theta^w (1 - \delta) E_t [w_{t+1}] + (1 - \theta^w) [1 - \beta \theta^w (1 - \delta)] \Theta A_t P_t}{1 + \beta (\theta^w)^2 (1 - \delta)}.$$

We then divide through by the price level, P_t , and use the definition linking real and nominal wages, $W_t \equiv w_t/P_t$, to get the expression for the real wage (2.24).

Appendix C Log linearization

Log linearizing the labor market tightness (2.10) we obtain:

$$\delta \hat{x}_t = \hat{n}_t - (1 - \delta) (1 - x) \hat{n}_{t-1}, \quad (\text{C.1})$$

where we used (2.9).

Log linearizing the clearing condition in the goods market (2.12) and substituting a log linearized version of the expression for hiring (2.7), we get:

$$\hat{c}_t = \hat{a}_t + \frac{1 - g}{1 - \delta g} \hat{n}_t + \frac{(1 - \delta) g}{1 - \delta g} \hat{n}_{t-1} - \frac{\alpha g}{1 - \delta g} \delta \hat{x}_t, \quad (\text{C.2})$$

where $g \equiv Bx^a$.

The marginal cost (2.13) is expressed, in log linearized terms, as follows:

$$\hat{m}c_t = \bar{w} M \hat{w}_t - M \Theta \hat{a}_t + \alpha g M \hat{x}_t - \beta (1 - \delta) g M E_t [(\hat{c}_t - \hat{a}_t) - (\hat{c}_{t+1} - \hat{a}_{t+1}) + \alpha \hat{x}_{t+1}], \quad (\text{C.3})$$

where the steady-state real wage is $\bar{w} = \Theta \bar{A}$ (as in the period-by-period Nash bargaining result), and the real wage (2.24) after log linearization is:

$$\hat{w}_t = \frac{\theta^w (\hat{w}_{t-1} - \hat{\pi}_t) + \beta \theta^w (1 - \delta) E_t [\hat{w}_{t+1} + \hat{\pi}_{t+1}] + (1 - \theta^w) [1 - \beta \theta^w (1 - \delta)] \frac{\Theta}{\bar{w}} \hat{a}_t}{1 + \beta (\theta^w)^2 (1 - \delta)}. \quad (\text{C.4})$$

The New Keynesian Phillips curve (2.25) is:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \lambda \cdot \hat{m}c_t; \quad \lambda \equiv \frac{(1 - \beta \theta^p) (1 - \beta)}{\theta^p}. \quad (\text{C.5})$$

The intertemporal Euler condition (2.2) is:

$$\widehat{c}_t = E_t \widehat{c}_{t+1} - \widehat{r}_t, \quad (\text{C.6})$$

where the real interest rate is defined as:

$$\widehat{r}_t \equiv \widehat{i}_t - E_t \widehat{\pi}_{t+1}. \quad (\text{C.7})$$

Defining $\widehat{u}_t \equiv u_t - u$ allows us to use the following connection between employment to end-of-period unemployment:

$$\widehat{n}_t = -\widehat{u}_t/N. \quad (\text{C.8})$$

Finally, the optimal policy rule (2.28) is

$$\widehat{u}_t = \frac{\Upsilon}{\alpha_u} \widehat{\pi}_t. \quad (\text{C.9})$$

We thus end up with a log linearized system of nine equations (C.1-C.9) to describe the law of motion of the nine variables,

$$\left\{ \widehat{x}_t, \widehat{n}_t, \widehat{c}_t, \widehat{m}c_t, \widehat{w}_t, \widehat{\pi}_t, \widehat{r}_t, \widehat{i}_t, \widehat{u}_t \right\}.$$

Appendix D Utility based loss function

This appendix derives the quadratic approximation (2.26) to the household utility loss function. The derivation is essentially identical to the one in Blanchard and Gali (2010), and is brought here for the work to be self-contained, but also to show that the different setup of the wage rigidity does not affect the approximation. Throughout the derivation, we assume that the decentralized equilibrium yields the optimal one. That is, we assume that an employment subsidy is in place to fully offset the monopolistic distortion, so that $M = 1$, and that a Hosios (1990)-like condition is satisfied, so that $\vartheta = \alpha$.

D.1 Derivation

A second order approximation of the household utility function (2.1) yields:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \ln C + \frac{(C_t - C)}{C} - \frac{1}{2} \frac{(C_t - C)^2}{C^2} \\ - \\ \left[\chi \frac{N^{1+\phi}}{1+\phi} + \chi N^\phi (N_t - N) + \frac{1}{2} \chi \phi N^{\phi-1} (N_t - N)^2 \right] \end{array} \right\} + o(\|\cdot\|^3), \quad (\text{D.1})$$

where $o(\|\cdot\|^3)$ represents terms of third or higher order.

Define small letters with hat as logarithmic deviations from steady state, so that $\hat{z}_t \equiv \ln(Z_t/Z)$. It follows that²⁷:

$$\frac{Z_t - Z}{Z} \simeq \hat{z}_t + \frac{1}{2} \hat{z}_t^2. \quad (\text{D.2})$$

Now expressing (D.1) in terms of logarithmic deviations from a purely deterministic steady state, using the approximation (D.2), we get:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \hat{c}_t + \frac{1}{2} \hat{c}_t^2 - \frac{1}{2} \hat{c}_t^2 \\ - \\ \left[\chi N^{1+\phi} (\hat{n}_t + \frac{1}{2} \hat{n}_t^2) + \frac{1}{2} \chi \phi N^{1+\phi} \hat{n}_t^2 \right] \end{array} \right\} + t.i.p. + o(\|\cdot\|^3),$$

where *t.i.p.* stands for *Terms Independent of (monetary) Policy*. The last expression can be simplified to:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[\hat{c}_t - \chi N^{1+\phi} \cdot \hat{n}_t - \frac{1}{2} \chi N^{1+\phi} (1 + \phi) \hat{n}_t^2 \right] + t.i.p. + o(\|\cdot\|^3). \quad (\text{D.3})$$

Next we have to find a quadratic approximation for the relation between \hat{c}_t and \hat{n}_t . Market clearing condition (2.12), together with the standard demand under a [Dixit and Stiglitz \(1977\)](#) setup, implies:

²⁷From the definition $\hat{z}_t \equiv \ln(Z_t/Z)$ we get that $Z e^{\hat{z}_t} = Z \cdot Z_t/Z = Z_t$. It then follows, using a second order Taylor expansion, that $(Z_t - Z)/Z = (Z e^{\hat{z}_t} - Z)/Z \simeq (Z e^{\ln(1)} - Z)/Z + e^{\ln(1)} \hat{z}_t + \frac{1}{2} e^{\ln(1)} \hat{z}_t^2 = \hat{z}_t + \frac{1}{2} \hat{z}_t^2$.

$$\begin{aligned}
A_t [N_t - Bx_t^\alpha H_t] &= \int_0^1 C_t(j) dj \\
&= C_t \int_0^1 \frac{C_t(j)}{C_t} dj \\
&= C_t \int_0^1 \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon} dj \\
&= C_t D_t.
\end{aligned} \tag{D.4}$$

where $j \in [0, 1]$ is a good index and we use the definition $D_t \equiv \int_0^1 \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon} dj$.

Lemma 1 *Assuming that Bx^α is small enough so that terms involving $Bx^\alpha \widehat{n}_t$ are of second order, we can show that:*

$$\begin{aligned}
N_t - Bx_t^\alpha H_t &\simeq (1 - \delta Bx^\alpha) N + \frac{1}{2} N \widehat{n}_t^2 \\
&\quad + N [1 - Bx^\alpha (1 + \alpha)] \widehat{n}_t \\
&\quad + (1 - \delta) Bx^\alpha N [1 + (1 - x) \alpha] \widehat{n}_{t-1}.
\end{aligned}$$

The proof is in Subsection D.2 below.

Using the Lemma and (D.4), we get:

$$\begin{aligned}
\frac{C_t D_t}{A_t (1 - \delta Bx^\alpha) N} &= \frac{N_t - Bx_t^\alpha H_t}{(1 - \delta Bx^\alpha) N} \\
&\simeq 1 + \frac{1}{2} \left[\xi_0 + \frac{Bx^\alpha (1 + \alpha)}{1 - \delta Bx^\alpha} \right] \widehat{n}_t^2 + \xi_0 \cdot \widehat{n}_t + \xi_1 \cdot \widehat{n}_{t-1},
\end{aligned}$$

where $\xi_0 \equiv \frac{1 - Bx^\alpha (1 + \alpha)}{1 - \delta Bx^\alpha}$ and $\xi_1 \equiv \frac{(1 - \delta) Bx^\alpha [1 + (1 - x) \alpha]}{1 - \delta Bx^\alpha}$.

Logarithmic transformation yields:

$$\ln C_t + \ln D_t - \ln A_t - \ln N - \ln (1 - \delta Bx^\alpha) \simeq \ln \left[\begin{array}{l} 1 + \frac{1}{2} \left[\xi_0 + \frac{Bx^\alpha (1 + \alpha)}{1 - \delta Bx^\alpha} \right] \widehat{n}_t^2 \\ + \xi_0 \cdot \widehat{n}_t + \xi_1 \cdot \widehat{n}_{t-1} \end{array} \right]. \tag{D.5}$$

Second order Taylor expansion, using the approximation (D.2) for the left hand side and $\ln(1 + \widehat{z}_t) \simeq \widehat{z}_t - \frac{1}{2}\widehat{z}_t^2$ for the right hand side, we get²⁸:

$$\widehat{c}_t \simeq \widehat{a}_t - d_t + \xi_0 \widehat{n}_t + \xi_1 \widehat{n}_{t-1}. \quad (\text{D.6})$$

Lemma 2 *Up to a second order approximation, $d_t \equiv \ln D_t \simeq \frac{\varepsilon}{2} \text{var}_j [p_t(j)]$, where $p_t \equiv \ln P_t$. The proof is in Subsection D.2 below.*

Using (D.6) and Lemma 2, we can rewrite (D.3) as:

$$\mathcal{L} = -\frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} \varepsilon \cdot \text{var}_j [p_t(j)] + \chi N^{1+\phi} (1 + \phi) \widehat{n}_t^2 \\ - (\xi_0 + \beta \xi_1 - \chi N^{1+\phi}) \widehat{n}_t \end{array} \right] + t.i.p. + o(\|\cdot\|^3).$$

But from the optimal allocation (A.1) we get that $\xi_0 + \beta \xi_1 - \chi N^{1+\phi} = 0$, so we get:

$$\mathcal{L} = -\frac{1}{2} \sum_{t=0}^{\infty} \beta^t \{ \varepsilon \cdot \text{var}_j [p_t(j)] + \chi (1 + \phi) N^{1+\phi} \widehat{n}_t^2 \} + t.i.p. + o(\|\cdot\|^3).$$

Lemma 3 $\sum_{t=0}^{\infty} \beta^t \{ \text{var}_j [p_t(j)] \} = \lambda^{-1} \sum_{t=0}^{\infty} \beta^t \widehat{\pi}_t^2$. The proof is in Subsection D.2 below.

Substituting Lemma 3 we get:

$$\mathcal{L} = -\frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\varepsilon}{\lambda} \widehat{\pi}_t^2 + [\chi (1 + \phi) N^{1+\phi}] \widehat{n}_t^2 \right\} + t.i.p. + o(\|\cdot\|^3).$$

Finally, from the definition of end-of-period unemployment (2.8) we get that $\widehat{n}_t = -(u_t - u)/N \equiv -\widehat{u}_t/N$. Substituting into the last expression we get:

$$\mathcal{L} = -\frac{1}{2} \frac{\varepsilon}{\lambda} \sum_{t=0}^{\infty} \beta^t \left\{ \widehat{\pi}_t^2 + \underbrace{\frac{\lambda}{\varepsilon} (1 + \phi) \chi (1 - u)^{\phi-1}}_{\alpha_u} \cdot \widehat{u}_t^2 \right\} + t.i.p. + o(\|\cdot\|^3).$$

²⁸To use this approximation for $\ln(1 + \widehat{z}_t)$, take $1 + \widehat{z}_t$ to be the entire expression inside the \ln from the right hand side of (D.5). We also have to use $\widehat{z} = \ln(Z/Z) = 0$ and the assumption employed by Lemma 1, suggesting that terms involving $Bx^\alpha \widehat{n}_t$ are of second order.

D.2 Proofs of Lemmas

Proof of Lemma 1. First note that, based on the steady-state representation of (2.7) we get that $H = \delta N$. Now a second order approximation of the expression $N_t - Bx_t^\alpha H_t$ from the left hand side of (D.4) yields:

$$N_t - Bx_t^\alpha H_t \simeq (1 - \delta Bx^\alpha) N + N \left(\frac{N_t - N}{N} \right) - \alpha Bx^\alpha \delta N \left(\frac{x_t - x}{x} \right) - Bx^\alpha H \left(\frac{H_t - H}{H} \right),$$

where, similarly to Blanchard and Galí (2010), we assume that Bx^α is small enough so that the terms involving $Bx^\alpha \hat{n}_t$ are of second order (which means that terms involving $Bx^\alpha \hat{n}_t^2$ are of higher order and therefore dropped).

Now based on (2.7) we get $\frac{H_t - H}{H} = \frac{1}{\delta} \frac{(N_t - N)}{N} - \frac{(1 - \delta)}{\delta} \frac{(N_{t-1} - N)}{N}$. Substituting in, we get:

$$\begin{aligned} N_t - Bx_t^\alpha H_t &\simeq (1 - \delta Bx^\alpha) N + N \left(\frac{N_t - N}{N} \right) - \alpha Bx^\alpha \delta N \left(\frac{x_t - x}{x} \right) \\ &\quad - Bx^\alpha N \left(\frac{(N_t - N)}{N} - (1 - \delta) \frac{(N_{t-1} - N)}{N} \right). \end{aligned}$$

Substituting in the linear approximation (C.1) as well, we get:

$$\begin{aligned} N_t - Bx_t^\alpha H_t &\simeq (1 - \delta Bx^\alpha) N + N \left(\frac{N_t - N}{N} \right) \\ &\quad - \alpha Bx^\alpha N [\hat{n}_t - (1 - \delta)(1 - x)\hat{n}_{t-1}] \\ &\quad - Bx^\alpha N \left(\frac{(N_t - N)}{N} - (1 - \delta) \frac{(N_{t-1} - N)}{N} \right). \end{aligned}$$

Now using the approximation (D.2) and the above-mentioned assumption about the term $Bx^\alpha \hat{n}_t$, we can rewrite as:

$$\begin{aligned} N_t - Bx_t^\alpha H_t &\simeq (1 - \delta Bx^\alpha) N + N \left(\hat{n}_t + \frac{1}{2} \hat{n}_t^2 \right) \\ &\quad - \alpha Bx^\alpha N [\hat{n}_t - (1 - \delta)(1 - x)\hat{n}_{t-1}] \\ &\quad - Bx^\alpha N [\hat{n}_t - (1 - \delta)\hat{n}_{t-1}]. \end{aligned}$$

Collecting terms and rearranging we get the expression in Lemma 1. ■

Proof of Lemma 2. Optimal allocation across differentiated goods under a Dixit and Stiglitz (1977) setup yields a price index that satisfies:

$$\begin{aligned}
1 &= \int_0^1 \left(\frac{P_t(j)}{P_t} \right)^{1-\varepsilon} \cdot dj \\
&= \int_0^1 \exp \{ (1-\varepsilon) [p_t(j) - p_t] \} \cdot dj \\
&\simeq 1 + (1-\varepsilon) \int_0^1 [p_t(j) - p_t] \cdot dj + \frac{(1-\varepsilon)^2}{2} \int_0^1 [p_t(j) - p_t]^2 \cdot dj,
\end{aligned}$$

where the last expression is a second order approximation. Solving for p_t it follows that:

$$p_t \simeq \int_0^1 p_t(j) \cdot dj + \frac{(1-\varepsilon)}{2} \int_0^1 [p_t(j) - p_t]^2 \cdot dj. \quad (\text{D.7})$$

It therefore follows that

$$\begin{aligned}
D_t &\equiv \int_0^1 \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon} \cdot dj \\
&= \int_0^1 \exp \{ -\varepsilon [p_t(j) - p_t] \} \cdot dj \\
&\simeq 1 - \varepsilon \int_0^1 [p_t(j) - p_t] \cdot dj + \frac{\varepsilon^2}{2} \int_0^1 [p_t(j) - p_t]^2 \cdot dj \\
&\simeq 1 + \frac{\varepsilon}{2} \int_0^1 [p_t(j) - p_t]^2 \cdot dj,
\end{aligned}$$

where the third row is a second order approximation and the fourth one follows from a substitution of (D.7). Therefore, up to a second order approximation, we see that $d_t \simeq \frac{\varepsilon}{2} \text{var}_j [p_t(j)]$, which proves Lemma 2. ■

Proof of Lemma 3. Combining the Dixit and Stiglitz (1977) style demand and Calvo (1983) style rigid prices gives the following linearized expression for the average price:

$E_j [p_t(j)] = \theta E_j [p_{t-1}(j)] + (1 - \theta) p_t^{Opt}$. Substituting into $\hat{\pi}_t = E_j [p_t(j) - p_{t-1}]$, we get $\hat{\pi}_t = \theta E_j [p_{t-1}(j) - p_{t-1}] + (1 - \theta) (p_t^{Opt} - p_{t-1})$. But, since $E_j [p_{t-1}(j)] = p_{t-1}$, it reduces to

$$\hat{\pi}_t = (1 - \theta) (p_{H,t}^{Opt} - p_{H,t-1}). \quad (\text{D.8})$$

Now using $\text{var}(x - \text{constant}) = \text{var}(x)$ and $\text{var}(x) = E(x^2) - (Ex)^2$, we can write:

$$\begin{aligned} \Delta_t &\equiv \text{var}_j [p_t(j)] = \text{var}_j [p_t(j) - p_{t-1}] \\ &= E_j \{ [p_t(j) - p_{H,t-1}]^2 \} - \{ E_j [p_t(j) - p_{H,t-1}] \}^2. \end{aligned}$$

Substituting again the above linearized expression for the average price we get:

$$\begin{aligned} \Delta_t &= \theta E_j [p_{t-1}(j) - p_{t-1}]^2 + (1 - \theta) (p_t^{Opt} - p_{t-1})^2 \\ &\quad - \{ E_j [p_t(j) - p_{t-1}] \}^2. \end{aligned}$$

Note that, based on the law of the unconscious statistician, the weights in the last expression are θ and $(1 - \theta)$, and not their squares. Now since $E_j [p_t(j)] = p_t$, we can rewrite as:

$$\Delta_t = \theta \Delta_{t-1} + (1 - \theta) (p_t^{Opt} - p_{t-1})^2 - \hat{\pi}_t^2.$$

Using Lemma (D.8) we get $\Delta_t = \theta \Delta_{t-1} + \frac{1}{(1-\theta)} \hat{\pi}_t^2 - \hat{\pi}_t^2$. This, after rearranging, becomes:

$$\Delta_t = \theta \Delta_{t-1} + \frac{\theta}{(1 - \theta)} \hat{\pi}_t^2.$$

By recursive substitution we get:

$$\Delta_t = \theta^{s+1} \Delta_{t-s-1} + \frac{\theta}{(1 - \theta)} \sum_{i=0}^s \theta^i \hat{\pi}_{t-i}^2.$$

From this equation, neglecting historical terms which are therefore *t.i.p.*, we can get:

$$\sum_{t=0}^{\infty} \beta^t \Delta_t = \frac{\theta}{1 - \theta} \sum_{t=0}^{\infty} \left[\beta^t \sum_{i=0}^t \theta^i \hat{\pi}_t^2 \right].$$

Opening the sigmas, rearranging, and collecting again, we can get:

$$\sum_{t=0}^{\infty} \beta^t \Delta_t = \frac{\theta}{1 - \theta} \sum_{t=0}^{\infty} \left[\beta^t \hat{\pi}_t^2 \sum_{i=0}^{\infty} (\beta^i \theta^i) \right].$$

Using the formula for converging infinite series, we can rewrite as:

$$\sum_{t=0}^{\infty} \beta^t \Delta_t = \frac{\theta}{(1-\theta)(1-\beta\theta)} \sum_{t=0}^{\infty} [\beta^t \widehat{\pi}_t^2],$$

which proves Lemma 3, since $\lambda \equiv (1-\theta)(1-\beta\theta)/\theta$. ■

Appendix E Impulse response to a cost-push shock

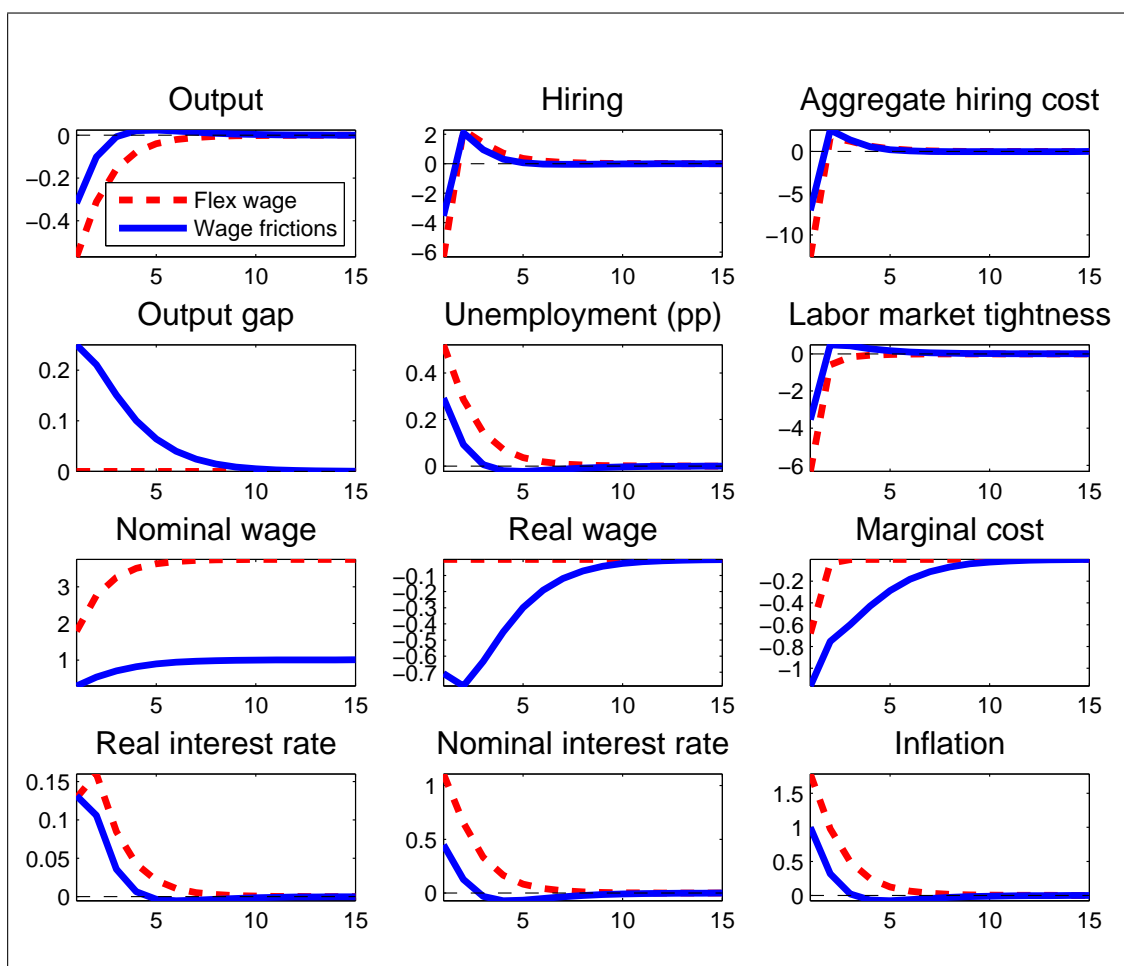


Figure E.1: Impulse response to a serially correlated cost-push shock.

Shock to the NKPC (C.5). The shock is an AR(1) process, with the degree of persistence being equal to 0.5. Red dashed line — a flexible wage benchmark. Blue solid line — dynamics under nominal wage frictions.

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