Fiscal Policy, Wealth Effects, and Markups*

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Abstract

We study the role of the wealth effect in the transmission of government spending shocks. To this end, we build an otherwise standard business cycle model with price rigidity, in which preferences can be consistent with an arbitrarily small wealth effect on labor supply, and show that such effect is linked to the degree of complementarity between consumption and hours. Different assumptions about the intensity of the wealth effect can span the whole set of theoretical results on the responses of private consumption and the real wage to a government spending shock, from negative to positive. The latter outcome, in particular, happens when the preferences are such that the positive wealth effect on labor supply is small and therefore the negative wealth effect on consumption is, somewhat counterintuitively, large.

Keywords: Government spending, private consumption, wealth effect, markup.

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1 Introduction

The responses of several macroeconomic variables to government spending shocks are interesting for their obvious policy implications, but also because some of these responses - notably those of private consumption and of the real wage - are potentially powerful tools to discriminate between different models. Empirically, some authors, like Fatas and Mihov (2001), Blanchard and Perotti (2002), Galí, López-Salido and Vallés (2007), Perotti (2007) and Ravn, Schmitt-Grohé and Uribe (2008), have found that government spending shocks cause private consumption and the real product wage to rise; others, like Ramey and Shapiro (1998), Edelberg, Eichenbaum, and Fisher (1999), Burnside, Eichenbaum, and Fisher (2004), and Ramey (2008) argue instead that the data suggest the opposite effects.

As first pointed out in the seminal contribution of Barro and King (1984), a negative response of private consumption and of the real product wage to a shock to wasteful, unproductive government spending is the hallmark of the neoclassical model. The key mechanism in that model is the wealth effect of a positive government spending shock, that increases the net present value of taxes paid by the individuals. This depresses their consumption and shifts out their labor supply, thereby also causing a decline in the real product wage along a fixed labor demand curve.

A number of recent papers, that we discuss briefly in the concluding section, have highlighted several alternative mechanisms by which government spending shocks can affect private consumption and the real wage, mostly positively. These mechanisms are based on several departures from the standard neoclassical model, including some forms of market incompleteness or specific assumptions about time variations in the fiscal and monetary policy reactions functions. While we think these mechanisms are empirically and theoretically important, in this paper we want to explore further the role of the simple wealth effect on labor supply in the most standard neoclassical model, except possibly for the presence of price rigidity. We show that different assumptions about the intensity of this wealth effect can span the whole set of theoretical results on the responses of private
consumption and the real wage to a government spending shock, from negative to positive.

We start from the Barro-King observation that the driving force behind any model with rational, forward-looking consumers is the wealth effect of the taxes associated with government spending shocks. Hence, a positive response of private consumption and of the real wage can arise only if one overcomes the negative wealth effect of government spending shocks on consumption; rather counterintuitively, we show that this can happen if preferences are such that the wealth effect on labor supply is small, which from the budget constraint implies that the negative wealth effect of the higher taxes on consumption is large.

We illustrate all this by assuming preferences, recently popularized by Jaimovich and Rebelo (2008), that allow for an arbitrarily weak wealth effect on labor supply. These preferences nest two polar specifications that have featured prominently in the business cycle literature: the one used in King, Plosser and Rebelo (1988) (KPR henceforth) and the one introduced by Greenwood, Hercowitz and Huffman (1988) (GHH henceforth). Using these preferences Jaimovich and Rebelo (2008) and Schmitt-Grohé and Uribe (2008) show that anticipated future news (on productivity) can drive the business cycle; the latter authors also estimate the parameters of this utility function, and argue that the evidence indicates that indeed the wealth effect on labor supply is practically zero. GHH preferences have been used extensively in the business cycle literature as a helpful device to match a series of empirical regularities, especially in response to productivity shocks. See, for instance, Raffo (2008), Schmitt-Grohe and Uribe (2008), Garcia-Cicco et al. (2009).

Both KPR and GHH preferences imply non-separability between consumption and leisure. A key result of this paper is to show that the strength of the wealth effect on labor supply is directly proportional to the degree of complementarity between consumption and labor hours. Interestingly, non-separable preferences feature prominently in the labor-search business cycle literature (see, e.g., Shimer (2009)).

Our paper relates also to previous work by Basu and Kimball (2003) who study the transmission of government spending (and monetary) shocks in a standard New Keynesian model. In particular, they show how different assumptions about adjustment costs on
capital can alter the sign of the effect of government spending shocks on output. We show that adjustment costs on capital have drastic implications also for the effects of government spending shocks on private consumption. More fundamentally, in Basu and Kimball (2003) all the effects of government spending are still driven by the standard negative wealth effect on consumption and leisure, hence they have the opposite sign than in our model.

The plan of the paper is as follows. In section 2 we present the empirical evidence. Section 3 discusses briefly the intuition of our model. Section 4 sets up the model. Section 5 illustrates its working in a simple two-period version with flexible and with fixed prices. Section 6 discusses the full, infinite horizon version of the model, with flexible and with sticky prices. Sections 7 and 8 present two important extensions: habit persistence and capital accumulation. Section 9 discusses the match between the estimated impulse responses and the impulse responses from a calibrated version of the model. Section 10 concludes.

2 Intuition

As a benchmark, it is useful to start with the standard neoclassical model with infinitely-lived forward-looking agents, flexible prices, complete asset markets, and lump-sum taxation, as in Baxter and King (1993). In this model, when government spending increases, expected taxation increases by the same present value, and the representative household experiences a negative wealth shock; as a consequence, she consumes less and works more: the labor supply curve shifts out along an unchanged labor demand curve. Thus, in a neoclassical model the positive Hicksian wealth effect\(^1\) of a government spending shock on labor supply plays the key role, and leads to a rise in hours and output and a decline

\(^1\)For future reference, it is useful to distinguish between a "wealth shock" and a "wealth effect". The former is the change in wealth caused by a given shock, the latter is the Hicksian wealth effect on a given variable generated by the wealth shock. Thus, a negative wealth shock is a decline in wealth; the positive Hicksian wealth effect on labor supply of a given negative wealth shock is the rightward shift in labor supply caused by the decline in wealth. The Hicksian wealth effect on labor supply (of any given shock to an exogenous variable) is the change in hours that would obtain if the household received a lump sum generating the same change in utility caused by the shock, but at unchanged (pre-shock) real prices.
in private consumption and in the real wage.\(^2\)

Despite the importance of the wealth effect on labor supply in this and virtually all recent dynamic general equilibrium models with government spending (see e.g., Smets and Wouters (2007)), surprisingly little macro evidence is available on its strength. Schmitt-Grohé and Uribe (2008) estimate the parameters of the Jaimovich and Rebelo preferences, and conclude that the Hicksian wealth effect on labor supply is virtually zero.

Thus, we consider the standard KPR preferences commonly used in business cycle analyses, but also GHH preferences, and show how the latter can modify radically the mode of operation of fiscal policy in an otherwise standard neoclassical model. In this section, we provide an intuitive argument of how a low Hicksian wealth effect on labor supply can generate a positive response of private consumption to government spending shocks.

When government spending increases, the consumer faces a negative wealth shock from the associated higher taxes (in present value terms). As a consequence, the labor supply curve shifts down in the case of KPR preferences: it stays still in the case of GHH preferences, which feature no wealth effect on labor supply. If the real wage decreased, the consumer would also experience a negative substitution effect on consumption (note that in equilibrium there is no wealth effect from changes in the real wage, because in our model of monopolistic competition changes in labor income and in profits cancel out, given hours). Hence, a necessary condition for private consumption to increase is that the real wage should increase.

A positive real wage response requires labor demand to shift out. This happens in our model because of rigidities in price setting by monopolistically competitive firms. When government spending increases, firms face an outward shift in the demand curve for the variety they produce; those firms that cannot change their prices meet this extra demand

\(^2\)If taxation were distortionary (see Ohanian (1997), Cooley and Ohanian (1997), and Ludvigson (1996)), one could think of specific time paths of taxes inducing a pattern of intra- and inter-temporal substitution that generates temporary increases in consumption or the real wage; however, because of the negative wealth shock, the present value of consumption and of the real product wage must fall at some point.
by increasing production, hence shifting out the derived demand for labor.\(^3\)

But because of price stickiness, movements in the real interest rate are limited. From the Euler equation, this also limits changes in the marginal utility of consumption. For illustrative purposes, consider an extreme case: the marginal utility of consumption is fixed in the short run. Assume, initially, that hours and consumption are complements, in the usual sense that the cross-derivative of the utility function is positive.

When labor demand shifts out and hours increase along the labor supply curve, the marginal utility of consumption increases; to restore the initial value, consumption too must increase (the derivative of the marginal utility of consumption with respect to consumption is negative). This induces a new outward shift in the demand facing each firm, and therefore a new outward shift in the derived demand for labor.

When does the process stop? From the aggregate resource constraint of the economy we know that, in equilibrium, the difference between hours (and therefore output) and private consumption must increase exactly by the change in government spending. Clearly, the higher the complementarity between hours and consumption, the higher the needed increase in hours and consumption until the above equilibrium condition is realized.

Below we show, as an implication of the Slutsky equation, that the degree of complementarity between hours and consumption is inversely related to the strength of the wealth effect on labor supply: hence complementarity is highest under GHH preferences (which feature no wealth effect on labor supply), and declines as one moves towards KPR preferences. This result explains why the multiplier effect on private consumption is highest under GHH. In fact, under KPR preferences, consumption falls if hours and consumption are substitutes instead of complements.

The above discussion clarifies the role of the low wealth effect on labor supply to generate an increase in consumption. This argument might be counterintuitive at first,

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\(^3\)In what follows, for expositional purposes it is useful to think of changes in the real wage as the result of shifts in the supply of labor and in the (derived) demand for labor. The expression "labor demand" has to be understood as the contemporaneous relation between the real wage and hours stemming from the first order conditions for profit maximization, holding everything else (including, possibly, current and future values of some endogenous variables) constant.
because from the budget constraint a low wealth effect on labor supply implies a large negative wealth effect of the higher taxes on consumption. But a low wealth effect on labor supply also means a small (or zero, in the case of GHH preferences) downward shift in labor supply; because of this, when labor demand shifts out the real wage increases more, inducing a larger substitution effect from leisure into consumption. Hence the degree of complementarity and the substitution effect of a higher wage interact in producing a multiplier effect on consumption.

3 The model

The representative household maximizes the expected discounted sum of utilities:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \right\}$$

under the sequence of budget constraints:

$$C_t + E_t \{Q_{t,t+1}B_{t+1}\} \leq W_tN_t - T_t + B_t + \int_0^1 \Gamma_t(i)$$

where $B_{t+1}$ is a portfolio of real state contingent assets, $Q_{t,t+1}$ is the stochastic discount factor, $C_t$ is consumption of a composite final good which assembles a continuum of differentiated varieties produced by monopolistically competitive firms, $W_t$ is the real wage (the nominal wage divided by the consumption-based price index $P_t$, defined below), $N_t$ is labor hours, $T_t$ is real lump-sum taxes, and $\Gamma_t(i)$ are the real profits of monopolistic firm $i$, whose shares are owned by the households.\(^4\)

Final goods producers are perfectly competitive. They assemble the differentiated varieties for the production of the final good $Y_t$ according to the following technology:

$$Y_t = \left[ \int_0^1 Y_t(i) \frac{(1-\varepsilon)}{1-\varepsilon} di \right]^{\frac{1}{\varepsilon-1}}$$

where $\varepsilon > 1$ is the elasticity of substitution across differentiated varieties. As standard, profit maximization by the final good producer yields the following demand function for

\(^4\)Each domestic household owns an equal share of the monopolistic firms.
the differentiated variety:

$$Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t$$  \hspace{1cm} (4)

where $P_t(i)$ is the price of differentiated variety $i$, and $P_t = \left[ \int_0^1 P_t^{1-\varepsilon}(i) \, di \right]^{\frac{1}{1-\varepsilon}}$ is the price of the composite good consistent with the final good producer earning zero profits.

In turn, each differentiated good is produced with the technology

$$Y_t(i) = N_t(i)^{\alpha} \quad i \in [0, 1], \quad \alpha \leq 1$$  \hspace{1cm} (5)

In the absence of any form of nominal price rigidity each monopolistic producer $i$ sets its relative price as a markup over the marginal cost

$$\frac{P_t(i)}{P_t} = \mu_t \frac{W_t}{MPN_t}$$  \hspace{1cm} (6)

where $MPN_t$ is the marginal product of labor and $\mu_t$ is the (possibly time-varying) markup. In a symmetric equilibrium the above expression reduces to

$$W_t = \frac{MPN_t}{\mu_t}$$  \hspace{1cm} (7)

which, once a decision rule is given for the markup, could be interpreted as an aggregate labor demand function.

The government buys the composite final good and throws it away, paying for it with lump-sum taxes $T_t$. As it is well known, government debt makes no difference to the equilibrium variables in this case, hence we will assume that the government budget is balanced on a period-by-period basis: $T_t = G_t$, where $G_t$ is real government spending.

The household’s optimization problem implies the following conditions:

$$-\frac{U_n(C_t, N_t)}{\lambda_t} = W_t$$  \hspace{1cm} (8)

$$U_c(C_t, N_t) = \beta E_t \left\{ \frac{U_c(C_{t+1}, N_{t+1}) R_t}{\Pi_{t+1}} \right\}$$  \hspace{1cm} (9)

where $U_j$ denotes the first derivative of the period utility function with respect to the argument $j = C, N, R_t$ is the nominal interest rate, $\Pi_{t+1}$ is the inflation rate between
periods $t + 1$ and $t$, and $\lambda_t$ is the marginal utility of (real) wealth. In equilibrium, $\lambda_t$ equals the marginal utility of consumption. As usual, equation (8) can be interpreted as defining a labor supply relation between the real wage $W_t$ and hours $N_t$. In addition, optimization requires that (2) holds with equality and that bonds accumulation satisfies the no-Ponzi condition:

$$\lim_{T \to \infty} E_t Q_{t,T} B_T = 0$$

where $Q_{t,T} \equiv \Pi_{s=t+1}^T Q_{s-1,s}$.

Because the Hicksian wealth effect on labor supply is so central to the operation of government spending shocks, we use a general utility specification that allows for different intensities of this effect. Specifically, we use the period utility function introduced by Jaimovich and Rebelo (2008):

$$U(C_t, N_t) = \frac{(C_t - \psi N_t^\gamma X_t)^{1-\sigma}}{1-\sigma}, \quad \sigma > 0$$  \hspace{1cm} (10)

where $X_t$ is an index variable evolving according to

$$X_t = C_t^{\gamma} X_{t-1}^{1-\gamma}$$

For $\gamma = 1$ this expression nests the utility function of KPR, which is standard in business cycle analysis:

$$U(C_t, N_t) = \frac{C_t^{1-\sigma} e^{-(1-\sigma)f(N_t)}}{1-\sigma}$$  \hspace{1cm} (11)

with $f(N_t) = -\log(1 - \psi N_t^\zeta)$. Under these preferences, equation (8) implies:

$$\frac{\psi \zeta N_t^{\zeta-1} C_t}{1 - \psi N_t^\zeta} = W_t$$  \hspace{1cm} (12)

The key feature of the KPR utility function is that the marginal rate of substitution between consumption and hours is a multiplicative function of $C_t$, so that hours remain constant on a balanced-growth path where the real wage and consumption grow at the
same rate. In other words, with this utility function the substitution and income effects of a permanent change in the real wage cancel out.\footnote{See also Basu and Kimball (2002).}

In the case \( \gamma = 0 \) the utility function (11) nests the preferences in GHH, which display no wealth effect on labor supply. To see this, notice that when \( \gamma = 0 \) (and after normalizing \( X_t = 1 \)) the marginal rate of substitution between consumption and leisure is independent of consumption:

\[
\psi \zeta N_t^{ \zeta - 1} = W_t
\]  

(13)

Hence, at an unchanged real wage (the relevant case to measure the Hicksian wealth effect) hours do not change.

For our purposes, two more points should be noted about the utility function (10). First, exactly because consumption does not appear in the expression for the marginal rate of substitution between consumption and leisure (equation 13), GHH preferences fail to be consistent with the balanced-growth fact emphasized above. Nevertheless, to provide intuition we will focus on the two polar cases of \( \gamma = 0 \) and \( \gamma = 1 \); we will then look at the intermediate case of \( 0 < \gamma < 1 \), which does satisfy the balanced-growth condition.

Second, while the KPR specification exhibits (log) separability between hours and consumption when \( \sigma = 1 \), the GHH specification is always non separable: higher values of \( \sigma \) merely imply stronger complementarity between hours and consumption.

### 4 A two-period version

To gain insight into the working of the model, we first consider a two-period, perfect foresight version. We compare a flexible-price equilibrium with a fixed-price equilibrium. When prices are flexible, the markup is constant at \( \varepsilon / (\varepsilon - 1) \); from equation (7) it follows that, if the production function has constant returns to scale \((\alpha = 1)\), also the real wage is constant. We henceforth focus on this case, which implies that under flexible prices any substitution effect from changes in the real wage is eliminated. In turn this allows us to concentrate specifically on the role of the wealth effect.
The representative household maximizes the expected discounted sum of utilities:

\[ U(C_1, N_1) + \beta U(C_2, N_2) \]  

subject to the budget constraints

\[ C_1 + B \leq W_1 N_1 - T_1 + \Gamma_1 \]  
\[ C_2 \leq W_2 N_2 + (1 + r)B - T_2 + \Gamma_2 \]

where \( B \) is real bonds, \( r \) is the real interest rate, and \( \Gamma \) is real profits; note that, by our assumption of a balanced budget on a period by period basis, each period taxes are equal to government spending.

In what follows we will refer, somewhat improperly, to the pre-shock flexible price equilibrium as the "steady-state", so that we will not have to change notation and terminology when we move to the infinite horizon model. Let an upper bar indicate a steady state value, and small letters, like \( w_t, c_t, \) and \( n_t \), indicate log deviations from steady state values. The exception is \( g_t \), which is defined as the percentage points deviation of the share of \( G_t \) in steady state output: \( g_t = (G_t - \bar{G})/\bar{N} \). Let \( \bar{g} \) indicate the ratio of government spending to GDP in the initial steady state: \( \bar{g} = \bar{G}/\bar{N} \). The economy starts in the steady state with flexible prices, then at the beginning of period 1 an unexpected shock \( g_1 > 0 \) to government spending occurs; if the shock is temporary \( g_2 = 0 \); if the shock is permanent, \( g_2 = g_1 > 0 \).

We assume a feedback interest rate rule that relates the short-term nominal interest rate to inflation:

\[ \frac{R}{\bar{R}} = \pi^{\phi_n} \]  

where \( \pi = (P_2 - P_1)/P_1 \) and \( \bar{R} = 1/\beta \) is the nominal interest rate in a steady state without inflation; we assume \( \phi_n > 1 \), which as it is well known is a necessary and sufficient condition for the price level to be uniquely determined.\(^6\) Note that according to this monetary policy rule the inflation rate must be positive for the real interest rate to rise above its steady state level \( 1/\beta \).

\(^6\)See Woodford (2003).
4.1 Flexible prices

We start with the flexible price case. In equilibrium, profits are just production less labor income: \( \Gamma_t = (1 - W_t)N_t \), hence \( W_tN_t + \Gamma_t = N_t \). By combining this condition with the time \( t \) budget constraints of the consumer (equations (15) and (16)) and with the government budget constraint \( T_t = G_t \), and imposing that in equilibrium net private debt \( B_t = 0 \), we obtain the aggregate resource constraint:

\[
N_t = C_t + G_t
\]  

(18)

With the real wage fixed at \( \bar{W} = (\varepsilon - 1)/\varepsilon \), in both periods equilibrium hours are pinned down by the labor market condition:

\[
-\frac{U_n(N_t - G_t, N_t)}{U_c(N_t - G_t, N_t)} = \frac{\varepsilon - 1}{\varepsilon} = \bar{W}
\]  

(19)

whereas consumption is determined by (18). Hence the analysis that follows applies to the effects of both temporary and permanent shocks.

Log-linearizing the resource constraint \( (18) \) yields

\[
n_t = c_t(1 - \bar{g}) + g_t
\]  

(20)

Log-linearizing the labor market condition \( (19) \) under GHH and using \( (20) \) yields

\[
n_t = 0, \quad c_t = -\frac{g_t}{1 - \bar{g}}
\]  

(21)

while under KPR

\[
n_t = \frac{1 - \bar{g}}{1 + \bar{W} + (\zeta - 1)(1 - \bar{g})} g_t > 0, \quad c_t = -\frac{\bar{W} + (\zeta - 1)(1 - \bar{g})}{1 + \bar{W} + (\zeta - 1)(1 - \bar{g})} g_t < 0
\]  

(22)

Because the real wage is constant, the key effect at work here is the wealth effect of higher government spending. With flexible prices, the main feature distinguishing the two utility functions is how the negative wealth effect of an increase in government spending is distributed between consumption and hours.
Under KPR, both leisure and consumption are normal goods; since the real wage is constant, both must fall in each period in which government spending is above its steady state value.\(^7\) If the shock is temporary, the real interest rate, which is determined by the Euler equation, increases, for the marginal utility of consumption in period 1 increases (both because consumption falls and because hours increase). As discussed above, this implies that goods price inflation must be positive.

Under GHH, there is no wealth effect on hours; hence, consumption absorbs all the wealth shock, and falls by more than in the KPR case; in fact, from (18) it falls by exactly the same increase in \(G\). Note that, since the real wage is constant, the elasticity of substitution between consumption and hours \(\sigma\) plays no role in determining the equilibrium value of these variables, both in and out of the steady state.

The result that consumption falls might seem to contrast with Linnemann (2005), who presents a model with flexible prices where hours and consumption are complements, and consumption increases after a shock to \(G\). The reason, however, is that he uses a utility function that cannot be nested into (10), and has the feature that consumption is an inferior good and the labor supply is downward sloping, as shown by Bilbie (2006).

It should be clear by now that, since consumption is a normal good under both KPR and GHH, it can increase only if the real wage increases, so that by the substitution effect hours can increase.\(^8\) Of course this is only a necessary condition for consumption to rise: hours must increase beyond the level needed to pay for the higher government spending. This is where nominal price rigidities (and therefore variable markup and real wage) enter the picture.

### 4.2 Fixed prices

Note first that, in this perfect foresight model, if goods prices were fixed only in period 1, the flexible price allocation would obtain in both periods. The reason is that in period 2

\(^7\)If the production function exhibited decreasing returns to scale to labor \((\alpha < 1)\), the decline in consumption would be even stronger, as the real wage would have to decline when hours increase.

\(^8\)Recall that, in equilibrium, there is no wealth effect on labor supply from a change in the real wage, since changes in labor income are offset by changes in profits, given hours.
the allocation would be identical to the flexible price equilibrium; and even though prices are fixed in period 1, prices in period 2 can move in order to obtain the exact real interest rate that supports the flexible price allocation; the nominal wage in period 2 would then adjust to obtain a real wage equal to \((\varepsilon - 1)/\varepsilon\). In other words, fixing goods prices only in period 1 would be equivalent to fixing the composite goods price in period 1 as the numeraire, but letting all relative prices in both periods and the inflation rate free to adjust.

Hence, for illustrative purposes we consider an extreme version, with prices fixed in both periods 1 and 2. In turn, given the interest rate rule (17), this implies that the real interest rate is constant.

Since a permanent shock has the same effect under flexible and fixed prices, we consider only temporary shocks. From the Euler equation, the marginal utility of consumption in period 1 must be equal to the (constant) marginal utility of consumption in period 2:

\[
U_c(C_1, N_1) = U_c(N(1 - \bar{y}), \bar{N})
\]

for both GHH and KPR. Differentiating with respect to \(C_1\) and \(N_1\) yields:

\[
\frac{dC_1}{dN_1} = -\frac{U_{cn}}{U_{cc}} \equiv \chi_j \leq 1; \quad j = GHH, KPR \tag{24}
\]

where

\[
\chi_{GHH} = \bar{W}; \quad \chi_{KPR} = \frac{\sigma - 1}{\sigma} \bar{W}
\]

Note that \(\chi_j\) can be interpreted as a specific "index of complementarity" between hours and consumption: it captures the strength of the comovement between \(C\) and \(N\), holding the marginal utility of consumption constant. Expressing the derivative in terms of log deviations from steady-states, and using the resource constraint (20) we can write:

\[
\begin{align*}
n_1 &= \frac{1}{1 - \chi_j} g_1; \\
c_1 &= \frac{\chi_j}{1 - \chi_j} \frac{1}{1 - \bar{y}} g_1
\end{align*}
\]

Clearly, the hours multiplier \(n_1/g_1\) is positive under both specification of preferences. Under GHH it is also always greater than 1, hence the consumption multiplier \((c_1/g_1)\) is
positive too. Under KPR, $n_1/g_1$ is greater than 1 and $c_1/g_1$ is positive only if consumption and hours are complements, i.e., $\sigma > 1$.

Importantly, the complementarity index $\chi_j$ is inversely related to the wealth effect on hours. From the Slutski equation the income effect of a change in the real wage on hours is:

$$\left( \frac{dN}{dW} \right)_{\text{income}} = -\frac{WU_{cc} + U_{cn}}{\Phi}$$

where $\Phi$ is a negative term by the second order conditions. Thus, if leisure is a normal good, it must be the case that $(WU_{cc} + U_{cn}) \leq 0$, which implies (from 24) that $\chi \leq W$.

Notice also that the index $\chi$ is highest under GHH preferences, which feature no wealth effect on hours, and then it falls under KPR preferences; in fact, $\chi_{KPR}$ is negative if hours and consumption are substitutes ($\sigma < 1$). Appendix C provides an alternative interpretation of $\chi$, based on the slopes of the indifference curves.

Thus, and to summarize, the fixed-price model reverses the conclusions of the flexible-price model: now private consumption can increase in response to a government spending shock, and the response of consumption is larger, the stronger the negative wealth effect on consumption (or, alternatively, the larger the complementarity index $\chi_j$). Below we provide an intuitive explanation of this result.

Unlike in standard consumption/leisure choice models, where non-labor income is given, here in equilibrium we have $C = N - G$ regardless of $W$, since a higher $W$ means higher labor income but lower profits (for simplicity, in the remaining part of this section we omit the time indices, but it should be understood that all variables refer to period 1). Thus, in equilibrium, from (24) the rate of substitution between $N$ and $C$ is driven by the index of complementarity $\chi_j$:

$$\frac{c}{n} = \frac{\chi_j}{1 - g}$$

while from the resource constraint the equilibrium rate of transformation between consumption and hours is

$$\frac{c}{n} = \frac{1}{1 - g} \left( 1 - \frac{g}{n} \right)$$

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Equating (28) and (29) yields the set of expressions in (26).

Another way to interpret (26) is that, in equilibrium, the difference between \( dN \) and \( dC \) must be equal to \( dG \), and since \( dC = \chi_j dN \) regardless of \( W \), this yields \( \frac{dN}{dG} = \frac{1}{1-\chi_j} \) and \( \frac{dC}{dG} = \frac{\chi_j}{1-\chi_j} \); once expressed in log deviations from the steady state, these conditions give exactly the set of expressions (26). Hence, the percentage increase in consumption is higher the higher the complementarity index.

How is the equilibrium brought about? When \( G \) increases, firms reduce the markup to meet the extra demand at the given prices; as the real wage increases, hours increase.\(^9\) Think of a notional "first round" where \( dN = dG \), both under GHH and KPR; because \( \chi_{GHH} > \chi_{KPR} \), \( C \) increases more under GHH; hence \( (dN - dC)_{GHH} < (dN - dC)_{KPR} < dG \). Firms reduce the markup further to meet the extra demand, and so on; the process stops when \( dN - dC = dG \); clearly, in equilibrium, \( dC \) is higher the higher \( \chi \). Thus, the degree of complementarity between consumption and hours captures a multiplier effect of the change in \( G \). An additional implication is that the markup will fall more under GHH relative to KPR preferences in response to a rise in government spending.

The key conclusion from this section is that, with flexible prices, and even if there is strong complementarity between hours and consumption, the latter cannot increase in response to a government spending shock, as long as leisure and consumption are normal goods; in addition, the response of consumption is more negative, the stronger the negative wealth effect of the initial increase in government spending on consumption. But with fixed prices the opposite holds true: consumption can increase, and it will increase more, the stronger the negative wealth effect of government spending on consumption. In fact, with fixed prices consumption always increases under GHH; and it increases under KPR if hours and consumption are complements. In any case, the response under KPR is always smaller than under GHH.

The two period model is obviously extreme in that it fixes the real interest rate. If the real interest rate were allowed to move, then an additional intertemporal substitution

\(^9\) Under KPR preferences, hours increase also because of the downward shift in the labor supply curve due to the wealth effect.
effect would come to play. But the basic principle remains: by limiting changes in the
real interest rate, price stickiness forces a stronger comovement between consumption and
hours when the utility function is non-separable; because hours increase, the degree of
complementarity determines the response of consumption.

5 Infinite horizon

Armed with the above intuition, we now go back to the infinite horizon version of our
model. Once again we contrast the case of flexible prices to the case of sticky prices.

5.1 Flexible prices

We assume that the evolution of the exogenous variable $g_t$ is governed by the AR(1)
process

$$g_t = \rho g_{t-1} + \eta_t \quad \rho < 1$$

(30)

where $\eta_t$ is white noise. Thus, the case $\rho = 1$ corresponds to a permanent shock. We start
in a steady state, and then assume that at time 0 a positive realization of $\eta_0$ occurs. We
study the impulse responses of selected endogenous variables to this government spending
shock.

It is easy to trace out the dynamics under flexible prices. In this case, the real wage
is constant at $(\varepsilon - 1)/\varepsilon$. Hence at any time $t$ the evolution of $n_t$ and $c_t$ is described by
(21) under GHH and by (22) under KPR. In turn, the Euler condition (9) pins down the
process for the real interest rate residually.

If the shock is permanent, the system moves immediately to the new steady state,
with no change in hours under GHH preferences, and with some increase under KPR
preferences, but not enough to prevent a decline in consumption. When the shock is
temporary, at time $t = 0$ consumption falls under both specifications (and by more under
GHH), and then goes back monotonically to the initial steady state, as the wealth effect
declines and resources are freed up by the decline in $G$ after the initial jump. This is
exactly analogous to the two-period model studied above.
5.2 Sticky prices

We now introduce sticky prices à la Calvo, whereby each period firms face a given probability of being able to adjust their prices. As it is well known, in equilibrium, this feature generates a log-linear Phillips curve of the type\(^{10}\)

\[
\pi_t = \beta E_t \{\pi_{t+1}\} + \kappa mc_t
\]  

(31)

where \(mc_t\) is the log deviation of the real marginal cost from its steady state level, and \(\kappa\) is a positive function of the probability of adjusting the price (a negative function of the degree of price stickiness).

Under both types of preferences, we are able to provide a closed-form solution. We start with the GHH case. Using the log-linearized version of the labor market condition (13) yields (recall that with constant returns to scale \(mc_t = w_t\)):

\[
mc_t = (\zeta - 1) n_t
\]  

(32)

Combining this with the log-linearized resource constraint (20), the Phillips curve becomes

\[
\pi_t = \beta E_t \{\pi_{t+1}\} + \kappa (\zeta - 1) [(1 - \bar{g})c_t + g_t]
\]  

(33)

Log linearizing the consumption Euler equation (9), and again using the resource constraint (18) to substitute for hours, we obtain:

\[
c_t (1 - \chi_{GHH}) = E_t \{c_{t+1}\} (1 - \chi_{GHH}) - \frac{\chi_{GHH}}{1 - \bar{g}} E_t (g_{t+1} - g_t) - \frac{\sigma_r}{\sigma} (r_t - E_t \{\pi_{t+1}\})
\]  

(34)

where \(\sigma_r \equiv (1 - \frac{\chi_{GHH}}{\zeta (1 - \bar{g})})\). Hence the rational expectations sticky-price equilibrium under GHH can be described as a set of processes for \(\{c_t, \pi_t, r_t\}\), for any given exogenous process \(\{g_t\}\), solving (33), (34) and (the log-linear version of) (17).

We guess the following solution to the above system:

\[
c_t = A^G_{c} g_t; \quad \pi_t = A^G_{\pi} g_t
\]  

(35)

\(^{10}\)See Yun (1996), Clarida et al. (1999), Woodford (2003).
Applying the method of undetermined coefficients one obtains an expression for $A_{c}^{GHH}$ which we provide in Appendix D. In the case of purely flexible prices ($\kappa \to \infty$), it is easy to show that $A_{c}^{GHH}$ coincides with the consumption multiplier in the two-period version of the model, equation (21). Conversely, in the case of permanently fixed prices ($\kappa \to 0$), $A_{c}^{GHH}$ reduces to its counterpart in the two-period model, equation (26).

Consider now the KPR case. Using the log linearized version of the labor market condition (12)

$$mc_t = (\zeta - 1 + \frac{1}{1 - \bar{g}})n_t + c_t$$

and the log-linearized resource constraint (20), the Phillips curve becomes

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \kappa \left[\frac{1 + \chi_{cHH}}{1 - \bar{g}} + \zeta - 1\right] (1 - \bar{g})c_t + (\zeta - 1 + \frac{\chi_{cHH}}{1 - \bar{g}})g_t$$

Log linearizing the consumption Euler equation we obtain an expression which is isomorphic to its GHH counterpart (34), except that the elasticity of intertemporal substitution is now $1/\sigma$ instead of $\sigma_r/\sigma$ (as we know, under KPR the parameter $\sigma$ indexes both the degree of complementarity between consumption and hours and the elasticity of substitution in consumption).

Like in the GHH case, applying the method of undetermined coefficients one can obtain an expression for $A_{c}^{KPR}$, which we also provide in Appendix D. Once again, in the extreme cases of flexible and fixed prices, this expression reduces to its two-period counterparts, equations (22) and (26), respectively. Exactly like in the two period model, it is easy to show that $A_{c}^{KPR} < A_{c}^{GHH}$. This implies that, relative to GHH, a larger degree of price stickiness is necessary under KPR to obtain a positive consumption multiplier. In addition, $A_{c}^{KPR}$ is certainly negative if $\chi_{KPR}$ is negative, i.e., if $\sigma < 1$. But unlike in the two period model, $A_{c}^{KPR}$ can be negative even if $\sigma > 1$, if there is not enough price stickiness.

Appendix D shows formally how $A_{c}^{KPR}$ and $A_{c}^{GHH}$ depend on the various parameters of the model. Here we provide the main intuition; we do not display the impulse responses because all endogenous variables are linear functions of $g_t$, hence they are AR(1) processes with persistence parametrized by $\rho$. 
The multiplier $A^t_c$ is decreasing in the intertemporal elasticity of substitution in consumption, which is captured by $\sigma_r/\sigma$ under GHH and by $1/\sigma$ under KPR. Intuitively, the larger that elasticity, the stronger the incentive to postpone consumption into the future, for any given variation in the real interest rate (recall that real interest rate movements were somehow artificially restricted in the two-period model presented above).

$A^t_c$ is a positive function of the elasticity of the labor supply function, which is inversely related to $\zeta$. A flatter labor supply function means a bigger increase in hours given the shift in labor demand, hence a stronger complementarity effect on consumption.

$A^t_c$ is also positively related to the degree of price stickiness (inversely related to $k$): a higher degree of price stickiness implies that more firms will respond to a shock by increasing production rather than their price; it follows that markups will respond more strongly, and the derived demand for labor will shift out more.

Finally, $A^t_c$ is decreasing in $\rho$, the degree of persistence of the government spending process. Intuitively, the larger $\rho$, the stronger the impact on lifetime wealth of the required increase in taxes, and therefore the stronger the negative wealth effect on consumption.

### 6 Habit persistence

Our model-based responses of private consumption decline monotonically towards the steady state after the shock, while the VAR-based impulse responses typically build up slowly and then go back to trend. We now introduce external habit persistence in consumption. As it is well known from the recent literature\(^{11}\) habit persistence helps dynamic general equilibrium models capture the gradual buildup of many (real) variables in response to alternative shocks. But in addition, we show below that with GHH preferences habit persistence has surprising effects in our model.

The period utility function now reads

$$U \left( \tilde{C}_t, N_t \right) = \frac{\left( \tilde{C}_t - \psi N_t \zeta X_t \right)^{1-\sigma}}{1-\sigma}$$

\(^{11}\)See, among many others, Smets and Wouters (2007) and Christiano, Eichenbaum and Evans (2005).
where
\[ X_t = \tilde{C}_t^{\gamma} X_{t-1}^{1-\gamma} \]  
and
\[ \tilde{C}_t \equiv C_t - hC_{t-1} \]  
with \( h > 0 \) being the habit persistence parameter.

From the first order condition the marginal utility of wealth \( \lambda_t \) is no longer equal to the marginal utility of consumption:
\[ \lambda_t = U_{C,t} - h \beta E_t \{ U_{C,t+1} \} \]  
where \( U_{C,t} \equiv U_t(\tilde{C}_t, N_t) \). The marginal rate of substitution between consumption and leisure on the l.h.s. of the intratemporal condition under GHH preferences becomes
\[ \frac{U_{C,t} \psi N_t^{-1}}{U_{C,t} - h \beta E_t \{ U_{C,t+1} \}} = W_t \]  
Therefore, due to external habits the marginal rate of substitution is no longer independent of consumption. As a consequence, the Hicksian wealth effect of a change in the real wage on labor supply is now different from zero; in other words, habit persistence reintroduces the wealth effect on labor supply under GHH preferences. This has a surprising implication for the equilibrium effects of government spending on consumption, even under flexible prices.

We have seen above that, when prices are flexible, under GHH preferences private consumption falls in response to a rise in government spending, just like under KPR preferences. This is no longer the case with habit persistence. Intuitively, when government spending rises temporarily, the consumer is not willing to decrease consumption by as much as before, because she knows that consumption will have to go back to the initial steady-state, and changes in consumption are costly. But then, given that with flexible prices the real wage is constant, labor supply must increase; this in turn can induce an increase in private consumption through the complementarity between hours and
consumption; in fact, we show below that this effect is stronger the higher $\sigma$.\footnote{Also intuitively, this effect is stronger the less persistent the shock is: with a very persistent shock the consumer is willing to adjust consumption downward even with habit persistence, hence we are close to the case of no habit persistence.}

Figures 2 and 3 display impulse responses to a government spending shock from the model with habit persistence, with flexible and sticky prices respectively, assuming a AR(1) process for government spending (30) with autoregressive parameter $\rho = .8$. The shock to government spending is normalized to 1 percentage point of steady state GDP; the response of consumption is expressed as share of steady state GDP by multiplying the log response by the steady state share of consumption in GDP; the responses of hours, the real wage and the markup are expressed in percentage terms. We assume a value of the habit parameter, $h$, equal to .7.\footnote{This is the same as the posterior mode estimate of $h$ for the US in Smets and Wouters (2007).} The other baseline values of the parameters used for calibration are reported in Table 1:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>autoregressive parameter of $g$ process</td>
<td>0.8</td>
</tr>
<tr>
<td>$\beta$</td>
<td>discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>price elasticity of demand</td>
<td>6</td>
</tr>
<tr>
<td>$\phi_\pi$</td>
<td>coefficient on inflation in monetary policy rule</td>
<td>1.5</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>slope of NK Phillips curve</td>
<td>consistent with 3.5 qrt price stickiness</td>
</tr>
<tr>
<td>$\bar{g}$</td>
<td>steady state share of govt. spending in output</td>
<td>0.2</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>inverse of elasticity of substitution in consumption</td>
<td>1.5</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>parameter governing Frisch elasticity of labor supply</td>
<td>1.8</td>
</tr>
<tr>
<td>$h$</td>
<td>parameter of habit persistence</td>
<td>0.7</td>
</tr>
</tbody>
</table>

These parameter values are typical of the macro business-cycle literature, hence we limit a discussion to $\zeta$, the parameter governing the elasticity of labor supply, which as it is frequently the case in these models bears important consequences. The Frisch elasticity of labor supply is defined as the elasticity of labor supply to the real wage holding constant the marginal utility of wealth. In our model with non-separable utility
such elasticity is well defined only in the case of GHH preferences, and equals $1/(\zeta - 1)$. In the KPR case with $\sigma \neq 1$ this elasticity is a more complicated function of $\zeta$, and it also depends on consumption. Existing empirical estimates of the Frisch elasticity in the macro literature are usually based on KPR-type preferences that are separable in consumption and leisure. The typical estimates in this literature vary between $1/3$ and $1$.\textsuperscript{14} In the by now classic DSGE studies of Smets and Wouters (2007) and Christiano et al. (2005) the implied estimated values of the Frisch elasticities are respectively $0.54$ and $1$, although again derived within the context of a model with separable utility. When mapped into our GHH-based definition of Frisch elasticity such estimates imply a value for $\zeta$ equal to $2.85$ and $2$, respectively. These values for $\zeta$ are substantially higher than those employed in Jaimovich and Rebelo (2008), who calibrate $\zeta = 1.4$, and Schmitt-Grohé and Uribe (2008), who report a posterior mode estimate for $\zeta = 1.16$. In our exercise we strike a balance between these estimates and set $\zeta = 1.8$, implying a value for the GHH-based Frisch elasticity of $1.25$. We illustrate further insights by means of numerical simulations.

As discussed above, Figure 2 shows that with flexible prices private consumption and hours now rise in the GHH case. Thus, if one takes an otherwise fully neoclassical model and only eliminates the wealth effect on labor supply, habit persistence \textit{per se} can generate a positive response of private consumption to a government spending shock, even when prices are perfectly flexible.

Interestingly, Figure 3 shows that under sticky prices the opposite happens: habit persistence \textit{dampens} the positive response of private consumption and of the real wage under GHH preferences. Intuitively, when there are no habits, private consumption increases a lot on impact, and then goes down monotonically to the steady state: to satisfy the initial demand producers reduce the markup. But with habits, consumption increases slowly; hence, producers do not need to reduce the markup by as much as under no habits to meet the extra demand. The real wage then increases less; this produces a smaller substitution effect, hence it dampens the responses of consumption relative to the no habits

\textsuperscript{14}As it is well known, the typical estimate in the micro literature is lower: see e.g. Domeij and Floden (2006).
The next three figures study the sensitivity of the responses of the main variables to the key parameters of the model. We know already that the consumption response is increasing in the intertemporal elasticity of substitution (decreasing in $\sigma$, Figure 4) and increasing in the elasticity of labor supply (decreasing in $\zeta$, Figure 5); the same holds for the response of hours and the markup.

So far, we have illustrated the polar cases of $\gamma = 0$ and $\gamma = 1$. As we discussed, the former case, while displaying no wealth effect on labor supply, is inconsistent with the secular growth of the real wage and consumption at almost constant labor supply. In Figure 6 we display impulse responses of the main variables to a government spending shock, for alternative values of $\gamma$ under GHH preferences. Specifically, along with the two polar cases of $\gamma = 0$ and $\gamma = 1$, we display the cases of $\gamma = 0.01$, as estimated by Schmitt-Grohé and Uribe (2008) and implying almost no wealth effect on labor supply, and of $\gamma = 0.25$, the value used by Jaimovich and Rebelo (2008). As one can see, the response of consumption and hours declines monotonically as $\gamma$ increases. It is essentially 0 for $\gamma = 0.25$, and it is negative for $\gamma = 1$.

### 7 Capital accumulation

So far we have abstracted from the presence of capital accumulation. In this section we show that this feature too has important consequences for the effects of fiscal policy on consumption and the labor market.

We assume that physical capital is accumulated by the households. The final investment good has the same composition as the consumption good. The households’s budget constraint modifies to

$$(C_t + I_t) + E_t \{Q_{t,t+1}B_{t+1}\} \leq W_t N_t + Z_t K_t + T_t + B_t + \int_0^1 \Gamma_l(i)$$

where $K_t$ is the capital stock at the beginning of time $t$, and $Z_t$ is the real rental rate of capital.
Capital accumulation evolves according to the law of motion

\[ K_{t+1} = (1 - \delta)K_t + I_t(1 - \Phi_t) \]  \hfill (44)

where \( \delta \) is the rate of physical depreciation, and \( \Phi_t \) is a function that captures the existence of adjustment costs on capital.

We will compare two alternative types of adjustment cost on capital, that have figured prominently in the recent literature. As we will see, the type of adjustment cost has important consequences, and for intuitive reasons.

In the first specification, adjustment costs are proportional to the rate of change in investment (as in Christiano et al. 2005):

\[ \Phi_t \equiv \Phi \left( \frac{I_t}{I_{t-1}} - 1 \right) \]  \hfill (45)

with the function \( \Phi(\cdot) \) satisfying \( \Phi = \Phi' = 0 \) and \( \Phi''(1) > 0 \) in steady-state. We label this case \( \Delta I \) adjustment costs. In the second specification, the costs of adjusting capital are proportional to the investment-capital ratio as follows:

\[ \Phi_t \equiv \phi \left( \frac{I_t}{K_t} \right) \frac{K_t}{I_t} \]  \hfill (46)

where \( \phi(\cdot) \) is increasing and convex. We label this case convex adjustment costs.

The key difference between the two types of frictions is well known: while under convex adjustment costs it is costly to change the stock of capital, under \( \Delta I \) adjustment costs it is costly to change the flow of investment. This difference bears key implications for the dynamic behavior of investment. Under convex adjustment costs, the dynamic of investment resembles that under frictionless capital accumulation, with its response being simply more muted. Conversely, under \( \Delta I \) adjustment costs, the evolution of investment resembles the one under planning costs: investment is inertial, namely, it is largely unresponsive in the short run, and then starts to build up its response gradually over time.

On the firm’s side, we assume a standard Cobb-Douglas production function:

\[ Y_t(i) = N_t(i)^a K_t(i)^{1-a} \]  \hfill (47)
The optimal choice of labor and capital implies

\[ W_t = MC_t \alpha \left( \frac{K_t(i)}{N_t(i)} \right)^{1-\alpha} \tag{48} \]

\[ Z_t = MC_t (1 - \alpha) \left( \frac{N_t(i)}{K_t(i)} \right)^{\alpha} \tag{49} \]

Notice that the real marginal cost continues to be common across firms as we implicitly assume a rental market for capital.\(^{15}\) In a symmetric equilibrium, combining (48) and (49) the expression for the real marginal cost \(MC_t\) reads:

\[ MC_t = \frac{W_t^\alpha Z_t^{1-\alpha}}{\alpha^\alpha (1 - \alpha)^{1-\alpha}} \tag{50} \]

For expository purposes, it is useful to interpret movements in investment as additional shifters of the labor demand schedule. To see this, notice that log-linearizing (44), (48), and (50), and combining, yields (under both types of adjustment costs) the following linear labor demand schedule (as usual, lower case letters denote percentage deviations from steady state):

\[ n_t^d = -(w_t - z_t) + (1 - \delta)^{-1} (k_{t+1} - \delta i_t) \tag{51} \]

Hence, on impact, a fall in investment causes, ceteris paribus (and in particular, holding constant the future capital stock), a leftward shift in the labor demand schedule. As capital builds up over time, though, this effect will be reversed.

The implications of capital accumulation for the transmission of government spending shocks in models with KPR preferences are well known. Consider first the case of flexible prices, as in Baxter and King (1993). The expansion in labor supply implied by the negative wealth effect induces a rise in the marginal product of capital. In equilibrium, this induces a rise in the rental cost of capital, and therefore a fall in investment. The strength of this negative effect on investment depends on the persistence of the shock. If the government spending shock is very short-lived, wealth is barely affected: hence

\[^{15}\text{We do not explore here the effects of the opposite extreme hypothesis, namely capital being firm specific.}\]
consumption and leisure, and therefore hours, barely respond. Given that capital is predetermined, output is also barely affected on impact, hence even if the effect on the rental cost is muted (due to the muted effect on hours) a temporary rise in government spending crowds out private investment almost one-to-one on impact. In general, the higher the persistence of the shock, the larger the negative wealth effect on consumption, and hence the smaller the crowding-out effect on investment.

As shown in Linnemann and Schabert (2003) the crowding-out effect on investment survives with nominal price stickiness, in fact it is even stronger. The reason is that the outward shift in labor demand (due to the fall in the markup) causes a stronger response of hours, and therefore a stronger increase in the rental cost of capital.

With KPR preferences (not shown) frictions in capital accumulation do not affect the response of consumption significantly. But things are different under GHH preferences. Figure 7 displays impulse responses for the two types of adjustment costs, with habit persistence and in the case of sticky prices. In the case of ΔI adjustment costs we specify the function Φ(·) as follows:

\[ \Phi (I_t, I_{t-1}) = \frac{\xi}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \] (52)

In the case of convex adjustment costs we calibrate the function Φ(·) as

\[ \Phi (I_t, K_t) = \frac{\xi}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 \frac{K_t}{I_t} \] (53)

In this exercise we set \( \xi = 2 \) in the case of convex costs and \( \xi = 3 \) in the case of ΔI adjustment costs (as estimated in Christiano et al. 2005 and more recently by Schmitt-Grohé and Uribe 2008). Like in the previous section we assume a value of the habit persistence parameter, \( \varphi \), of 0.7. The share of labor in output, \( \alpha \), is set to .66. The other parameters are as in Table 1.

Investment falls below baseline in response to the shock, as in the standard model with KPR. But consumption increases (by about .3 percentage points of GDP at peak) only under ΔI adjustment costs, which generates a weaker decline in investment. The intuition is as follows. As illustrated above, a decline in investment generates a leftward movement
in the labor demand schedule. In the case of flexible prices, and under GHH preferences, this invariably generates a fall in the real wage and therefore a negative substitution effect on consumption. In the case of sticky prices, the leftward movement in the labor demand schedule competes with the rightward movement due to the fall in the markup. Under ΔI adjustment costs the initial decline in investment is smaller, hence the rightward shift in labor demand prevails, leading to higher real wage and private consumption; the opposite occurs under convex costs of adjustment.

Basu and Kimball (2003) show that in a standard model with KPR preferences the type of friction in capital accumulation has implications for the output response to government spending shocks. It is interesting to notice that in the same model, with GHH preferences, capital accumulation frictions have powerful implications not only for the output multiplier but also for both the sign and the magnitude of the consumption multiplier.

8 Empirical evidence on consumption, the real wage, and the markup

We focus on the joint responses to government spending shocks of private consumption, private investment, the real wage (both the consumption and the product wage), and the markup. We start by estimating a reduced form VAR in six variables: government spending on goods and services, GDP, private consumption, private investment (all in logs of real, per capita terms), the Barro-Sahasakul 17 average marginal tax rate on labor income, and one of the following three variables in turn: the markup, the real consumption wage, and the real product wage, all three again in logs. The VAR also includes a constant and a linear trend. Government spending on goods and services is defined as government consumption plus defense equipment investment, following international guidelines. The

16 In fact, ΔI adjustments costs, by generating investment inertia, limit the crowding-out effect on investment of a government spending shock, allowing employment and output to expand relatively more than under convex costs of adjustment.

sample runs from 1947:1 to 2003:4 (the constraint on the end date is the availability of data for the Barro-Sahasakul tax rate). Appendix A describes the data in more detail.

We identify government spending shocks via two alternative methodologies. The first is the "SVAR approach" of Blanchard and Perotti (2002): this is based on the idea that, due to decision and implementation lags, there is no automatic or discretionary response of government spending to output and other shocks within a quarter. Thus, government spending shocks are identified via a Choleski decomposition in which government spending comes first (see Blanchard and Perotti (2002)). If one, like us, is interested only in the response to government spending shocks, this is all is needed.

The second method is based on the "Ramey-Shapiro approach" of Ramey and Shapiro (1998), Edelberg, Eichenbaum, and Fisher (1999), and Burnside, Eichenbaum, and Fisher (2004), in turn a variant of the Romer and Romer (1989) "event study approach" to the identification of monetary policy shocks. In essence, the method starts by defining a dummy variable capturing the main episodes of military buildups due to foreign policy crises (which can be argued to be exogenous and unforecastable), and then it traces the effects over time of a shock to this dummy variable on several endogenous variables.

Based on a careful reading of the weekly press, Ramey and Shapiro (1998) identify the exact quarter when the expectations of the Korean, Vietnam and Carter-Reagan military buildups first took hold, and define the "war dummy variable" $D_t$ as taking the value of 1 on 1950:3, 1965:1, 1980:1; to these dates we (like Eichenbaum and Fisher (2004), Ramey (2006) and Perotti (2007) before us) add 2003:1 to capture the expectation of the post September-11 military buildup. We include lags 0 to 6 of the dummy variable $D_t$ in the reduced form equations for government spending and taxation (the two fiscal policy variables) and only lag 0 in the other reduced form equations. Formally, and assuming for illustrative purposes that the VAR includes only two variables, government spending
and output, we estimate the reduced form VAR

\[
g_t = \kappa_{10} + \kappa_{11} t + \sum_{i=1}^{4} a_{1i} g_{t-i} + \sum_{i=1}^{4} a_{12i} y_{t-i} + \sum_{i=0}^{6} B_{1i} D_{t-i} + u^g_t \tag{54}
\]

\[
y_t = \kappa_{20} + \kappa_{21} t + \sum_{i=1}^{4} a_{21i} g_{t-i} + \sum_{i=1}^{4} a_{22i} y_{t-i} + B_{21} D_t + u^y_t \tag{55}
\]

By including six lags of the war dummy variable in the fiscal policy equation, we allow the military episodes to explain a large part of the deviation from normal of the policy variable for seven periods in each episode; but by including only lag 0 in the output equation, we assume that, after the impact period, the dynamic response of output to these military buildups follows the "normal" pattern. This specification captures the notion that we learn from these military buildup episodes because they are large, exogenous and (arguably) unforecasted, not because the response of the economy is special. In other words, we estimate the "normal" dynamic response of the economy to these "abnormal" policy events (see Perotti (2007) for a more extended discussion of this issue).\(^\text{18}\)

We consider two different measures of the markup, in the non-financial corporate business sector and in manufacturing. The former, a "value added markup", is constructed as the share of labor in value added of the non-financial corporate business sector (net of indirect taxes, see Rotemberg and Woodford (1999)); the latter, a "gross output markup", is constructed as the share of labor in manufacturing national income. One advantage of value added data is that there is a linear relation between the log of the price markup and the log of the labor share in value added\(^\text{19}\) (see Appendix B); this is not the case with gross output data, where the relation might depend on the share and the relative price of

\(^{18}\)In contrast to our specification, Edelberg, Eichenbaum and Fisher (1999) and Burnside, Eichenbaum and Fisher (2004) include lags 0 to 6 in all equations of the reduced form VAR. Thus, implicitly these authors assume that the military buildup episodes explain a large part of the deviation from normal of the dynamics of all endogenous variables for six quarters after each episode. To make this clearer, suppose there were only one military buildup episode: then including lags 0 to 6 of the war dummy variable in all reduced form equations would cause the residuals of all equations to be 0 for seven quarters: in other words, the episode would explain all the deviation from normal of all variables for seven quarters. The war dummy variable consists of four episodes, hence the problem is less extreme, but the underlying logic remains the same.

\(^{19}\)Possibly after appropriate corrections for the presence, for instance, of non-Cobb-Douglas production functions, of overhead labor, of labor adjustment costs, or of market power in the labor market (see, e.g., Rotemberg and Woodford 1999).
materials and energy.

We do not have data on the hourly wage in the non-financial corporate business sector, hence we use hourly compensation in the business sector; for manufacturing, we use the average hourly earnings of production workers in manufacturing. To construct the product wage, we divide the nominal wage measures by the implicit price deflator of the business sector\(^\text{20}\) and by the producer price index of manufacturing, respectively. To construct the consumption wage, we divide the nominal wage measures by the CPI net of food prices.

Figure 1 displays the impulse responses of several variables to a government spending shock in the SVAR specification (columns 1 and 2) and to a shock to the war dummy variable in the RS specifications (columns 3 and 4). In columns 1 and 3 the markup refers to the non-financial corporate business sector, the wage variables to the entire business sector; in columns 2 and 4, both the markup and the wage refer to the manufacturing sector.

The responses of government spending, private consumption and private investment are expressed as changes in the share of GDP relative to the pre-shock path, by multiplying the original log response by the average share of each variable in GDP. The responses of GDP, of the markup, and of the two wage variables are expressed in percentage point deviations from the pre-shock path. In the SVAR specification, the initial shock to government spending is normalized to 1 percentage point of GDP. All responses are displayed with one standard error bands above and below the point estimate.

Note first that the patterns of all responses are qualitatively similar in the SVAR and RS specifications, for virtually all variables, except partially private investment and the product wage. In particular, the peak response of government spending is similar in the SVAR and the RS specifications, about 1 percent of GDP. We then highlight five results that will be useful in discussing our model.

First, in both specifications and with both manufacturing and non-financial corporate business sector data, private consumption increases; the peak response is remarkably similar in the SVAR and RS specifications, about .5 percentage points of GDP, and

\(\text{20}\) We use the gross output deflator from BLS since we do not have the value added deflator.
occurs at roughly the same horizon. Second, investment falls in the SVAR specification, particularly with the non-financial corporate business sector data, while it is basically flat (after a small initial increase) in the RS specification. Third, in the SVAR specification the markup falls, by about .5 percent in the non-financial corporate business sector, and by more than double this amount in manufacturing. The countercyclical response of the markup is slightly more muted in the RS specification (in fact, the markup rises for the first three quarters in manufacturing). Fourth, the real product wage also rises, by about 1 percentage point at the peak in the non-financial corporate business sector and again by much more in manufacturing, where the markup response is also stronger. Fifth, the real consumption wage rises, by less than 1 percentage point at the peak in both sectors, although it is not always precisely estimated.

It is important to notice that, by including zero to six lags of the dummy variable also in the output equation of the system (54), Burnside, Eichenbaum and Fisher (2004) and Ramey (2008) obtain respectively a virtually flat and a positive response of consumption to an exogenous war dummy shock. Hence the literature remains divided on the likely sign of the consumption multiplier in response to an unanticipated government spending shock.\footnote{The key divisive point concerns whether, possibly due to decision and implementation lags in fiscal policy, variations in government (defense or non-defense) spending are anticipated. If this feature is important, spending shocks estimated with VAR techniques may be a combination of disturbances at all lags and leads, possibly leading to distorted inferences (see Leeper et al. 2009 on this point). Assessing whether anticipation effects are indeed sizeable is the object of a current intense research (Ramey 2008, Mertens and Ravn 2009, Leeper et al. 2009). An intriguing hypothesis (although not yet tested) is that anticipation effects may be important when government spendings run-ups are particularly large, for instance in "exceptional" times such as wars, but less so during "normal" times.}

In the following sections we build a model that is capable of delivering both positive and negative consumption multipliers, depending on the intensity of the wealth effect on labor supply embedded in the specification of preferences.

9 Matching the SVAR impulse responses

In this section we evaluate the ability of the more quantitative version of our model, featuring adjustment costs on capital of the $\Delta I$ type and habit persistence in consumption,
to fit the estimated impulse responses in our VAR exercise. We proceed as follows. Starting from the initial deterministic steady state, we feed through the model the estimated impulse response process for government spending obtained from the SVAR estimate. The parameter values are the same we assumed for the simulation exercise in the previous section. We set the parameter governing the wealth effect in the GHH preferences, \( \gamma \), to 0.01, the value estimated by Schmitt-Grohé and Uribe (2008).

Figures 8 and 9 display the results of our simulation exercise over an horizon of ten quarters. Each panel features the response of a selected variable obtained from the model (solid line) compared to the point estimates from the SVAR exercise based on the non-farm business sector data (dashed line). Each panel also reports one standard error bands from the SVAR exercise.

For illustrative purposes, we start in Figure 8 with the model without capital. This model clearly matches well the response of consumption, the real consumption wage and the markup. In particular, it matches well the gradual and persistent build-up of consumption following the shock.

Figure 9 displays the responses from the model with capital. As argued above, the SVAR impulse response features a very prolonged response of consumption (which reaches the peak at about fifteen quarters), whereas in the model consumption starts to revert back to baseline after about seven quarters. A similar problem applies to the comparison between estimated and simulated paths for the real consumption wage and the markup. The latter in particular responds too strongly on impact relative to the data, although it falls within the estimated bands between three and ten quarters. This general problem has to do with the role of investment in the model. In a nutshell, whereas in the data the response of investment is relatively quick and short-lived, in the model it is inertial in the short-run and subsequently very persistent. After a few quarters, the prolonged fall in investment (which reflects the very persistent nature of the estimated process for government spending) tends to dampen the positive labor demand effect induced by counter-cyclical markups, therefore also dampening the response of the real wage and consumption.
10 Conclusions: alternative mechanisms

In this paper, we have shown that the intensity of the wealth effect of government spending on labor supply by itself can span the whole range of responses of private consumption and the real wage to a government spending shock, from negative to positive. Of course, a negative response is the typical response of the neoclassical model with the standard KPR preferences largely used in growth and in real business cycle models. The contribution of this paper is therefore to show that in the same model a weak wealth effect can generate a positive response of these variables. We now discuss a few recent models that also deliver a positive response of private consumption to a government spending shock. All use standard KPR-type, yet separable, preferences.\textsuperscript{22}

The first two models rely crucially on a positive response of the real wage to generate a positive response of private consumption. They differ from ours either in the way they generate a positive real wage response, or in the way they translate a higher real wage in higher private consumption. Like us, Galí, Lopez-Salido and Vallés (2007) assume nominal rigidities to generate a rise in the real wage, but they rely on an extreme form of market incompleteness to generate the increase in private consumption. A fraction of the population cannot borrow nor lend and must consume all their labor income each period. Thus, when the real consumption wage increases, their labor income also increases, and so does their private consumption; if the share of hand-to-mouth consumers is large enough, aggregate private consumption can increase.

Also like us, Ravn, Schmitt-Grohé and Uríbe (2006) assume complete asset markets, but they differ from us in the mechanism they use to generate a countercyclical markup. They assume the existence of customer markets with good-specific habit persistence, part of a wide class of models with the common feature that the elasticity of demand perceived by the producers falls when aggregate demand increases.\textsuperscript{23} In their model, the demand

\textsuperscript{22}Schmitt-Grohé and Uríbe (2008) experiment with GHH preferences but do not investigate the effects of government spending shocks on private consumption and the real wage.

\textsuperscript{23}An earlier example of this class is Rotemberg and Woodford (1992), that relies instead on strategic interactions between producers.
function facing each producer has a price-elastic component that is a function of aggregate demand, and a price-inelastic component that is a function of the good-specific habit. An increase in aggregate demand, caused for instance by a shock to government spending, increases the share of the price-elastic component and thus the elasticity of demand, which in turn makes the markup countercyclical. Once the real wage increases, the model generates an increase in private consumption via the standard intra-temporal substitution effect on the demand for leisure and consumption.

The next two papers rely instead on an inter-temporal substitution effect, namely variations in the real interest rate. In Corsetti et al. (2009), government spending falls below its steady state after the initial rise. Thus, in the future the short-term interest rate must fall; the long-term real interest rate falls now, and from the Euler equation consumption increases as well. A different inter-temporal substitution mechanism is emphasized by Davig and Leeper (2009). In contrast to all the models studied so far, they allow for a passive monetary policy rule and an active fiscal policy rule. The former implies that the nominal interest rate reacts little to the increase in prices following the government spending shock; as a result, the real interest rate declines; in addition, the active fiscal policy rule means that the higher government spending will not be met fully by higher taxes, thus mitigating the negative wealth effect on consumption. The combination of these two effects can generate a positive response of private consumption to the government spending shock.

By emphasizing a different mechanism, we do not mean to imply that the mechanisms described in these papers are implausible or empirically irrelevant. The goal of this paper is to better understand the transmission mechanism of government spending shocks in a neoclassical model with price rigidity. In particular, we wish to highlight the role of the wealth effect on labor supply and its relationship with non-separability in preferences. A low wealth effect has been used recently to show that anticipated productivity shocks can have positive effects on current economic activity (see Jaimovich and Rebelo (2008) and Schmitt-Grohé and Uríbe (2008)). We show that it also has rather surprising implications for the effects of government spending in an otherwise standard model of the business
cycle.
Appendix A: The data

**Government spending**: constructed as \( \log[(A955RA3*GC1_2000+B873RA3*GC2_2000)/\text{pop}] \), where:

- A955RA3: Government consumption expenditure, quantity index of chained 2000 dollars
- B873RA3: Federal gross investment in equipment and software, national defense, quantity index of chained 2000 dollars
- GC1_2000: value in 2000 of A955RC1 (Government consumption expenditure, nominal)
- GC2_2000: value in 2000 of B873RC1 (Federal gross investment in equipment and software, national defense, nominal)

**GDP**: constructed as \( \log(B191RA3/\text{pop}) \), where

- B191RA3: Gross domestic product, index of chained 2000 dollars

**Private consumption**: \( \log(a794rx0) \)

- a794rx0: personal consumption expenditure, chained 2000 dollars per capita

**Private investment**: \( \log(B007RA3/\text{pop}) \)

- B007RA3: Gross private fixed domestic investment, quantity index of chained 2000 dollars


**Markup, Non-financial corporate business**: constructed as \( \log[(A455RC1-A325RC1)/A460RC1] \), where

- A455RC1: Gross value added of non-financial corporate business, BEA Table 1.14
- A325RC1: Taxes on production and imports less subsidies paid by non-financial corporate business, BEA Table 1.14
A460RC1: Compensation of employees in non-financial corporate business, BEA Table 1.14

**Markup, Manufacturing:** constructed as \( \log(J426RC1/A552RC1) \), where

- J426RC1: National income without capital consumption adjustment, Manufacturing, BEA Table 6.1B
- A552RC1: Wage and salary disbursements, Manufacturing, BEA Table 2.2A

**Wage, Business sector:** PRS84006103: Hourly compensation, Business sector, BLS

**Wage, Manufacturing:** CES3000000006: Average hourly earnings of production workers in manufacturing, BLS

**Deflator, Business sector:** PRS84006143: Implicit price deflator, Business sector, BLS

**Deflator, Manufacturing:** WPUDUR0200: Producer price index, manufacturing goods, BLS

**CPI deflator:** CPIULFSL: CPI for all urban consumers: All items less food, BLS

### Appendix B: Markups and the labor share

Consider a Leontief production function for gross output

\[
Q = \min \left[ \frac{\tilde{Q}}{1 - s}, \frac{M}{s} \right]
\]  \hspace{2cm} (56)

where \( \tilde{Q} \) is valued added, \( M \) is intermediate inputs, and \( s \) is the share of intermediate inputs in gross output (in what follows, a tilde will indicate a value added variable).

The value added production function is Cobb-Douglas in labor and capital:

\[
\tilde{Q} = N^\alpha K^{1-\alpha}
\]  \hspace{2cm} (57)

The value added price markup is

\[
\bar{\mu} = \frac{\tilde{P}\tilde{Q}N}{W}
\]  \hspace{2cm} (58)
which, with a Cobb-Douglas production function, reduces to:

\[ \tilde{\mu} = \alpha \tilde{s}_N^{-1} \]  

(59)

Thus, there is a linear relation between the logs of the value added markup and of the labor share in value added. We now show that this is no longer the case for the gross output markup and labor share.

The nominal marginal cost of producing an extra unit of gross output is

\[ MC = (1 - s) \frac{W}{Q_N} + s P_M \]  

(60)

where \( P_M \) is the price of intermediates and \( W/Q_N \) is the nominal marginal cost of producing an extra unit of value added. The inverse of the gross output markup is thus

\[ \frac{1}{\mu} = (1 - s) \frac{W}{P Q_N} + \frac{P_M}{P} s \]  

(61)

\[ = (1 - s) \alpha^{-1} \tilde{s}_N \tilde{P} + \frac{P_M}{P} s \]  

(62)

Now,

\[ s_N = \frac{W N}{P Q_Q} = \frac{W N \tilde{P} Q}{P Q Q} \]  

(63)

\[ = \tilde{s}_N \frac{P Q - P_M M \tilde{Q}}{P Q} \]  

(64)

\[ = \tilde{s}_N \frac{P Q - P_M M}{P Q} \]  

(65)

\[ = \tilde{s}_N (1 - \frac{P_M}{P} s) \]  

(66)

where \( \frac{P Q - P_M M}{Q} \) is the value added deflator.\(^{24}\) Hence, the inverse of the gross output markup becomes

\[ \frac{1}{\mu} = \frac{1 - s}{1 - \frac{P_M}{P} s} \alpha^{-1} s_N + \frac{P_M}{P} s \]  

(67)

\(^{24}\)With imperfect competition and substitutability between intermediates and primary inputs, the correct value added deflator is \( \frac{P Q - \mu P_M M}{\tilde{Q}} \) since firms also mark up intermediate costs (see Rotemberg and Woodford (1999, p. 1090). The formula used in the text is the Divisia value added deflator as calculated for instance by BEA, and would be correct under perfect competition.
which shows that the relationship between the labor share in gross output $s_N$ and the gross output markup $\mu$ is not linear, and it is influenced by movements in $s$ and $\frac{P_M}{P}$. In particular, by replacing (66) into (67), it is easy to see that, if $P_M/P$ falls after a fiscal policy shocks that raises demand, the gross output markup could increase while the value added markup falls.

**Appendix C: An interpretation of the index of complementarity between consumption and hours**

Suppose we represent the indifference curve and the budget constraint in the $C, N$ space, with $C$ on the vertical axis. Importantly, $\gamma$ is directly related to the wealth effect on hours. Under GHH preferences, there is no wealth effect on hours: graphically, this means that the slope of the indifference curve is constant on any vertical line

$$\frac{d}{dC} \left( \frac{U_n}{U_c} \right) = 0$$

which implies

$$\frac{U_{nc}}{U_{cc}} = \frac{U_n}{U_c} \implies \chi_{GHH} = W$$  \hspace{1cm} (69)

If instead there is a positive wealth effect of $W$ on hours (i.e., leisure is a normal good) as under KPR preferences, we have

$$\frac{d}{dC} \left( \frac{U_n}{U_c} \right) > 0$$

so that the tangency point of the new equilibrium indifference curve with a budget constraint that has the same slope of the old budget constraint is to the right of the old equilibrium; this implies

$$\frac{U_{nc}}{U_{cc}} < \frac{U_n}{U_c} \implies \chi_{KPR} < W$$  \hspace{1cm} (71)

**Appendix D. Sticky price model: closed form solution**

We illustrate in this section the closed-form solution of the infinite horizon model with staggered prices.

39
**GHH preferences** Log linearizing the consumption Euler under GHH yields initially

\[
    c_t - \psi \frac{N^\zeta}{C} \zeta n_t = E_t \{c_{t+1}\} - \psi \frac{N^\zeta}{C} \zeta E_t \{n_{t+1}\} - \sigma^{-1} \left(1 - \psi \frac{N^\zeta}{C}\right) (r_t - E_t \{\pi_{t+1}\})
\]

In the steady state (without habits)

\[
    \psi = \frac{\mu^{-1}}{\zeta N^{\zeta-1}}
\]

where \(\mu = \overline{W}^{-1}\) is the steady state markup. Let’s define

\[
    \sigma_r \equiv 1 - \psi \frac{N^\zeta}{C} = 1 - \frac{\overline{W}}{\zeta(1-\overline{g})}
\]

where \(\overline{g}\) is the share of government spending in output. Notice that \(\sigma_r < 1\). We also assume \(\sigma_r > 0\), so that the intertemporal elasticity of substitution in consumption is positive. Hence we can re-write:

\[
    c_t - (1 - \sigma_r) \zeta n_t = E_t \{c_{t+1}\} - (1 - \sigma_r) \zeta E_t \{n_{t+1}\} - \frac{\sigma_r}{\sigma} (r_t - E_t \{\pi_{t+1}\})
\]

Notice that since \(\sigma_r < 1\) the GHH preferences are always non separable.

Log linearizing the consumption-leisure condition, and using the log-linearized aggregate resource constraint:

\[
    mc_t = (\zeta - 1) n_t = (\zeta - 1) [(1 - \overline{g}) c_t + g_t]
\]

Using the aggregate resource constraint to eliminate employment, we can express the equilibrium in terms of the system

\[
    \pi_t = \beta E_t \{\pi_{t+1}\} + \kappa (\zeta - 1) [(1 - \overline{g}) c_t + g_t]
\]
\[ c_t (1 - \chi_{GHH}) = \left(1 - \chi_{GHH}\right) E_t \{c_{t+1}\} - \frac{\chi_{GHH}}{1 - \bar{g}} E_t \{g_{t+1} - g_t\} - \frac{\sigma_r}{\sigma} (r_t - E_t \{\pi_{t+1}\}) \]

We can guess the pair of solutions:
\[ c_t = A^{GHH}_c g_t; \quad \pi_t = A^{GHH}_\pi g_t \]

Using the method of undetermined coefficients one obtains
\[ A^{GHH}_c \equiv \frac{\chi_{GHH} (1 - \rho) (1 - \beta \rho) - \kappa \frac{\sigma_r}{\sigma} (\phi - \rho)(\zeta - 1)(1 - \bar{g})}{(1 - \chi_{GHH}) (1 - \rho) (1 - \beta \rho) + \kappa \frac{\sigma_r}{\sigma} (\phi - \rho)(\zeta - 1)(1 - \bar{g})} \frac{1}{1 - \bar{g}} \] (72)

In the case of purely flexible prices \((\kappa \rightarrow \infty)\) we obtain:
\[ A^{GHH}_c \rightarrow - \frac{1}{1 - \bar{g}} < 0 \] (73)

which coincides with (21)) in the two-period version of the model. Conversely, in the case of permanently fixed prices \((\kappa \rightarrow 0)\) (72) reduces to:
\[ A^{GHH}_c \rightarrow \frac{\chi_{GHH}}{1 - \chi_{GHH}} \frac{1}{1 - \bar{g}} > 0 \] (74)

which once again coincides with its counterpart in the two-period model (see equation (26)).

**KPR case** Log-linearizing the consumption-leisure condition yields:
\[ m c_t = (\zeta - 1 + \frac{\bar{W}}{1 - \bar{g}}) n_t + c_t \] (75)

Log-linearizing the consumption Euler equation yields
\[ c_t - \left[\frac{(\sigma - 1) (1 - \sigma_r)}{\sigma}\right] n_t = E_t \{c_{t+1}\} - \left[\frac{(\sigma - 1) (1 - \sigma_r) \zeta}{\sigma}\right] E_t \{n_{t+1}\} - \frac{1}{\sigma} (r_t - E_t \{\pi_{t+1}\}) \]

Using the resource constraint the above equation reduces to:
\[ c_t [1 - \chi_{KPR}] = E_t \{c_{t+1}\} [1 - \chi_{KPR}] - \frac{\chi_{KPR}}{1 - \bar{g}} E_t (g_{t+1} - g_t) - \frac{1}{\sigma} (r_t - E_t \{\pi_{t+1}\}) \] (76)
which is isomorphic to the Euler equation in the GHH case, except that the elasticity of intertemporal substitution is now $1/\sigma$ instead of $\sigma_r/\sigma$.

Like in the GHH case, applying the method of undetermined coefficients one obtains

$$ A_e^{KPR} \equiv \frac{X_{KPR} (1 - \rho) (1 - \beta \rho) - (\phi - \rho) \frac{\sigma}{\sigma - 1} [((\zeta - 1)(1 - \bar{g}) + \frac{\sigma}{\sigma - 1} X_{KPR})]}{(1 - X_{KPR})(1 - \rho) (1 - \beta \rho) + (\phi - \rho) \frac{\sigma}{\sigma - 1} [1 + ((\zeta - 1)(1 - \bar{g}) + \frac{\sigma}{\sigma - 1} X_{KPR})]} \frac{1}{1 - \bar{g}} \quad (77) $$

In the case of purely flexible prices ($\kappa \to \infty$) one obtains:

$$ A_e^{KPR} = - \frac{W + (\zeta - 1)(1 - \bar{g})}{1 + W + (\zeta - 1)(1 - \bar{g})} \frac{1}{1 - \bar{g}} < 0 $$

which again is the same formula as in the two-period model, equation (22). Similarly, under permanently fixed prices ($\kappa \to 0$) one obtains:

$$ A_e^{KPR} = \frac{X_{KPR}}{1 - X_{KPR}} \frac{1}{1 - \bar{g}} > 0 \quad (78) $$

evertywhere as in the two-period fixed-price model, equation (26).
References


Figure 1: SVAR and RS responses
Figure 2: No capital, flexible prices
Figure 3: No capital, sticky prices
Figure 4: No capital, sticky prices, habit persistence: sensitivity to $\sigma$
Figure 5: No capital, sticky prices, habit persistence: sensitivity to $\zeta$
Figure 6: No capital, sticky prices, habit persistence: sensitivity to $\gamma$
Figure 7: Capital, sticky prices, habit persistence
Figure 8: No capital, sticky prices, habit persistence: matching SVAR and model-based impulse responses
Figure 9: Capital, sticky prices, habit persistence: matching SVAR and model-based impulse responses